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Key Points:
- A typology of five generic categories of resilience responses is proposed
- Those resilience categories are reframed into a generic metric using the viability modeling
- By shifting from resistance to absorptive, adaptive, or adaptive preference responses, systems strengthen their resilience

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From Resistance to Transformation: A Generic Metric of Resilience Through Viability

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Abstract In the last two decades resilience has emerged as a promising concept that can help societies and more generally social-ecological systems become less vulnerable to shocks and stressors. As such it has been adopted by a large number of disciplines—from psychology, physics, and ecology to disaster risk reduction, climate change adaption, and humanitarian and food security interventions. However, although numerous definitions or measures of resilience have been proposed, those were mainly discipline centered and, as such, failed to provide an adequate overarching framework. This paper explores the question of the formalization and measurement of resilience, with the objective to develop a generic metric that applies across the disciplines and to the different interpretations of resilience. Building on the definitions found in the literature, a continuum of five categories of resilience responses is identified: (i) resistance, (ii) coping strategies, (iii) adaptation, (iv) adaptive preference, and (v) transformation. Those categories are then reframed into a generic metric, using viability analysis—a mathematical formalism which builds on dynamic systems and control theory. Theoretical and empirical analyses are then conducted, looking in particular at how inertia and costs associated with the types of responses influence the level of resilience. To illustrate this new metric, we draw on two models widely discussed in the resilience literature: the exploitation of renewable resources and the case of lake eutrophication. Both theoretical and numerical analyses demonstrate the relevance of the typology as a generic framework for resilience but also highlight transformation as a particular case of resilience response.

1. Introduction

Since the 1960s, the concept of resilience has made its way to the forefront of both the academic and the international development agencies' lexicon. It has been used for more than 50 years in disciplines such as psychology (Glantz & Johnson, 1996), physics, and engineering sciences (Grimm & Wissel, 1997) and has now become ubiquitous in some part of ecology (Gunderson & Folke, 2005; Holling, 1973). In domains where issues of shocks, vulnerability, and risks are critical such as in humanitarian interventions and food security (von Grebmer et al., 2013), disaster risk reduction (Grafton & Little, 2017; Klein et al., 2003), climate change adaptation (Pelling & Manuel-Navarrete, 2011), or even social protection (Davies et al., 2013), the growing influence of the concept of resilience is particularly prominent. Not only do academics increasingly make reference to it, but practitioners and nongovernmental organizations are now exploring the modalities of its implementation in the field (see, e.g., the BOND resilience initiative; https://www.bond.org.uk/search?search_api_views_fulltext=resilience.). At the international level, many different institutions and development agencies such as the United Nations Food and Agriculture Organization (FAO), United Nations Development Programme (UNDP), or the World Food Programme (WFP) have now embraced the concept as a key objective in many of their programs. In this context, the appropriation of the concept by bilateral and multilateral organizations such as United States Agency for International Development (USAID), Australian Agency for International Development (AUSAID), United Kingdom Department of International Development (DfID) the World Bank, the European Union (EU), or the Organisation for Economic Co-operation and Development (OECD) is to be seen as additional evidence that resilience is now part of the post-2015 development discourse (Béné et al., 2014).

The fact that resilience is becoming a new paradigm in both academia and humanitarian and development domains is welcome to many. But it is also received with some concern by others for conceptual and empirical reasons (Béné et al., 2012; Davidson, 2010; Olsson et al., 2015). In particular, widely recognized is the fact that the relation to agency, conflict, knowledge, and power is not necessarily appropriately addressed through the resilience "lens" (Béné, Mehta, et al., 2017; Cannon & Muller-Mahn, 2010; Leach, 2008). Social scientists have therefore argued that the application of resilience to social systems requires more solid theoretical grounding (Davidson, 2010). Behind this important first limitation, there are also particular concerns about the relative "looseness" and malleability of the concept. Davoudi, (2012, p.299) talks about "a slippery concept," while Olsson et al. (2015, p.1) refer to "troubled dialogue internal to the sciences themselves," leading to the absence of consensual definitions or generic metrics. In effect an exponential number of definitions have been proposed in the literature in the past 10 to 15 years (Manyena, 2006), contributing to the current confusion that surrenders the concept—with the risk, some will argue, of blurring and diluting the meaning (Brand & Jax, 2007). Adding to these conceptual issues is the fact that the measurement of resilience remains methodologically and practically challenging in the field (Béné, Chowdhury, et al., 2017; Frankenberger & Nelson, 2013). While an increasing number of indicators and metrics are being proposed in the literature (Francis & Bekera, 2014; Kerner & Thomas, 2014), many are still derived from disconnected and sometimes contradicting approaches that lack replicability and breadth (Béné, 2013). This situation greatly diminishes the overall coherency of the whole initiative and precludes in particular the implementation of the comparative analyses that would be essential to reach a clearer understanding of what resilience is exactly and what interventions can strengthen it.

Recognizing this uncomfortable mix of fervent enthusiasm and more critical skepticism, this paper explores the question of the formalization of the concept of resilience in the social-ecological context, with the objective to offer some first element of mathematical rigor to the debate. We propose to do this through the development of a generalizable metric of resilience. For this we rely on a mathematical formalism—the viability analysis—which builds on both dynamic systems and control theory (Aubin, 1990; DeLara & Doyen, 2008; Frankowska et al., 1995).

To organize this research, the following general approach has been adopted (reflected in the structure of this paper): first we reviewed academic and gray literature on resilience, as an attempt to identify the latest progress made by both academics and practitioners in their understanding and conceptualization of resilience. These are presented in the next section of this paper. One of the emerging conclusions of this literature is the need to recognize and to integrate the "multiform" nature of resilience; that is, the fact that resilience results, or emerges, from a combination of different properties (or capacities), ranging from resistance to coping strategies, adaptive preference, adaptive capacity, and eventually transformability (Berkes et al., 2003; Béné et al., 2012; Enfors et al., 2011; Walker et al., 2004).

Building on this new understanding of resilience, the next step was to develop a generic framework that would allow us to capture and formalize rigorously these different resilience capacities/dimensions and explore their properties through a mathematical formalism. For this, we use the concept of *viability*. Viability is a very generic mathematical framework which derives directly from control theory and was developed specifically with the idea to identify the conditions on state and control variables (e.g., ecological and/or economic endowments) and the changes in controls (e.g., human behavior and/or public policy) that permit a system to remain viable within a set of predetermined thresholds. As such—and as we shall see below—viability is particularly well suited to explore some of the key questions around resilience. This viability framework is presented in section 3 of this paper.

Using this viability framework, a metric of resilience was then derived. The basic and intuitive idea that underpins this metric is that irrespective of the dimension of resilience considered (resistance, absorptability, adaptability, and transformability) the amplitude of the largest shock that a system can stand *without losing its long-term sustainability* is a good proxy for the level of resilience that characterizes that system. In other words, the larger the shock that the system can put up with and remain "viable" in the long run (that is, avoid irreversible damages), the more resilient the system is. The computation of those conditions of viability and the related resilience metric are presented below in section 3.5.

The next step in the analysis was then to test this generic metric through two models which were selected because of their wide and very frequent use in different parts of the resilience literature: the first one is the "lake eutrophication" model which has been one of the first models used to illustrate the concept of resilience in the domain of ecology (Carpenter et al., 2001; Cottingham & Carpenter, 1994); the second is the Gordon-Schaefer model used in the context of the management of renewable natural resources (e.g., fisheries and forestry) to model the ecological and economic interactions created by the exploitation of these natural resources (Clark, 1990). In this paper these two examples are not presented as evidence of the appropriateness of the metric proposed but instead as a first attempt to illustrate and ground elements of the framework in the reality of the field. Finally, we synthesized and discussed the key salient points that emerge from these analyses in the last section of the paper.

2. Resilience: From Resistance to Transformation

The origins of the concept of resilience are contested (Béné, Mehta, et al., 2017; Manyena, 2006; Olsson et al., 2015). While the concept was already in use in psychology as early as the 1940s (Egeland et al., 1993; Garmezy, 1971; Glantz & Johnson, 1996) in reference to the negative effect of adverse life events on vulnerable individuals and groups (Masten et al., 1990), other disciplines such as physics, material sciences, and engineering have also been using the concept since the 1960s and 1970s to characterize the response of material to physical stress such as pressure or deformation (Alexander, 2013). Soon after, ecologists picked up the concept and started to use it to describe properties of ecosystem dynamics around equilibria. One of the most quoted definitions (often — but wrongly — presented as the original definition of resilience) is that proposed by Holling in its seminal work on "Resilience and Stability of Ecological Systems," where resilience was defined as "a measure of the ability of these systems to absorb changes of state variables, driving variables, and parameters, and still persist" (Holling, 1973, p.17).

More recently, under the influence of social sciences, resilience evolved into a more elaborate concept where it "is no longer simply about resistance to change and conservation of existing structures [the engineering definition]" (Folke, 2006, p.7) or even about "buffer capacity and persistence to change while maintaining the same function" (the ecological definition) but instead interpreted as an emergent property that includes two other dimensions: adaptive capacity, that is, the "capacity to learn, combine experience and knowledge, adjust responses to changing external drivers and internal processes, and continue operating" (Berkes et al., 2003); and transformative capacity, that is, the "capacity to create a fundamentally new system when ecological, economic, or social structures make the existing system untenable" (Walker et al., 2004, p.5).

These four dimensions, namely, resistance, absorptive capacity, adaptive capacity, and transformative capacity, are the most widely accepted dimensions of resilience (Béné et al., 2014; Folke et al., 2010; Olsson et al., 2015; Walker et al., 2004). A careful review of sociopsychological literature reveals, however, the existence of another critical dimension of resilience, one that builds on the concept of adaptive preference (Clark, 2007; Nussbaum, 2001; Sen, 1999). Adaptive preference is the "deliberate or reflexive process by which people adjust their expectations and aspirations when trying to cope with deteriorating changes in their living conditions" (Béné et al., 2014, p.607). As such, adaptive preference adds another important dimension of resilience that refers to some more "subjective" element which needs to be considered when one intends to conceptualize resilience.

Altogether these five different types of resilience strategies (or responses) can be placed along a *continuum* that reflects the different levels/types of responses that the system will adopt in order to put up with the impacts of shocks or stressors. While we recognize that resistance to change and transformation represent the two extreme strategies to deal with changes, we also use the term "continuum" deliberately here — as opposed to "categories" — because we recognize that the boundaries between those strategies are disputable, case-specific, and subject to interpretation. While some would see, for instance, migrating as an adaptive response, others would present it as a transformation. Likewise, diversification is seen as a (positive) adaptation by some, while it is interpreted as a coping strategy by others. (This point refers to the blurry distinction between "diversification as a way to accumulate," as opposed to "diversification as a way to survive" often discussed in the development literature; see, e.g., Ellis, 1998.) Nevertheless, as this paper will demonstrate, these distinctions are conceptually useful to elaborate further our understanding of what resilience is really about. Those five resilience dimensions are represented in Table 1 along with the different mechanisms and the different outcomes which they lead to, in response to the initial shockservation is consistent w considered.

Another important point that emerges from this review is that these different types of responses are usually linked (at least theoretically) to different intensities/severities of shock or change, in a broadly linear manner. Cutter et al. (2008), for instance, describe this in the context of community resilience. The lower the intensity of the shock — relative to the ability/capacity to deal with that shock — the more likely the system will be able/willing to resist it effectively. This observation is consistent with the idea of inertia

Table 1
Continuum of Resilience Strategies (or Responses) and Their Characteristics

Types of responses	Outcome sought	Mechanisms at work	Costs of change
Resistance	Stability, constancy	No change in the dynamic and control of the system	Nil
Absorptive resilience (coping strategy)	Buffer, persistence	Temporary change in the parameters of the control of the system	Minimal
Adaptive resilience	Adaptation, flexibility	Adaptation—change in the parameters of the control of the system	Minimal to substantial
Adaptive preference	Adaptation, adjustment	Adjustment in the control and/or the expectations/constraints of the system	Minimal to substantial
Transformation	Transformability, changes	Change in the structure/identity (and therefore functioning) of the system	Generally substantial

and "costs of change" which we will discuss later: as much as possible people/societies avoid engaging in actions that induce changes, because these changes also induce some form of costs (financial, psychological, relational, etc.).

Alternatively, the system/people may have to engage in some form of coping strategies and to rely on their buffer capacity to absorb impacts and maintain structures and functions. In the ecological literature a frequent example is the capacity of a lake to buffer the impact of increased eutrophication, at least up to a certain point (Cottingham & Carpenter, 1994). In a humanitarian context, this could be the decision by the head of a household affected by a temporary fall in income to reduce the family expenses until the new harvest has been secured (Corbett, 1988).

When the shock or stressor exceeds the system's absorptive capacity however, the latter will need to change "something," that is, they will engage in some form of adaptive strategies. The term "adaptive strategies" refers to the various incremental changes and adaptations that systems undergo in order to continue functioning in response to a shock or a growing stress, without undertaking any major *qualitative* changes in the way they operate (Nelson et al., 2007). These adjustments can take many forms, involving change in the functions of the system. A good example here would be a farmer who decides to try out a new variety of heat-resistant seed as a response to the increasing number of droughts that hit the region where he lives. Alternatively, those adjustments could be related to the expectations that people have about their future or their aspirations. In that particular case where people (individuals, communities, or society as a whole) adjust their aspirations—for instance, in relation to their standard of living or their quality of life—we talk about adaptive preference as a way to deal with shock. A concrete example here would be the case of a middle-class family that decides to relocate in a less secure and poorer neighborhood and rent out a smaller apartment to adjust for the father's loss of job.

Finally, when the changes required in response to shocks or stressors are so large that they overwhelm the adaptive capacities of the system, transformation will have to take place if irreversible consequences are to be averted. These changes (sometimes deliberate, sometimes imposed) are transformative (and not adaptive any longer) because they alter permanently and drastically the system's functioning or its structure (Hughes et al., 2003). Examples here could include the case of a country, whose economy depends for a large part on a single export-oriented crop (e.g., cotton or coffee), the government of which decides to encourage farmers to shift to more diversified livelihood systems through various incentives (taxes, subsidies, policies, etc.) as an attempt to reduce its economy's vulnerability to primary-product world price decline (Brigulio et al., 2005).

In sum the literature confirms that resilience can be conceptualized as the combination of various types of responses that vary greatly in nature and intensity and lead to different outcomes (Walker et al., 2004). Folke et al. (2010) underline this idea when they conclude "resilience thinking incorporates the dynamic interplay of persistence, adaptability and transformability." In line with these authors an increasing number of academics now stress the necessity to conceptualize "resilience as resulting from the *tension between persistence and change*" (Enfors et al., 2011, our emphasis). This interpretation of resilience as the result of dynamic synergies and tensions between different (and sometimes contradicting) strategies/responses—stretching from resistance to transformation—is the conceptualization adopted in the rest of this paper. The important point is that in all the examples above, the responses that were made as an attempt to deal with a particular adverse event (be it a series of droughts, the loss of job, or the volatility of the world economy), are all associated with some transaction "costs": the risk and costs of trying a new variety of seed, the psychological and emotional costs of giving up a secure and friendly neighborhood for a cheaper rent, or the costs of transitioning a country economy from a single-product export-oriented system to a more diversified economy.

3. Generic Model

3.1. Why the Viability Approach?

Formalizing resilience within a rigorous framework requires accounting for a certain number of important elements. First and foremost, resilience is about dynamic systems, including the possible existence of feedbacks, nonlinear trajectories, and thresholds. In the context of dynamic systems, viability approach (Aubin, 1990) provides a generic formalism for modeling and evaluating these types of issues. Derived from control theory (Bellman, 1964), the aim of viability approach is to analyze the compatibility between the (possibly uncertain) dynamics of a system and a series of constraints and to determine a set of controls, actions, or decisions that would allow the system to stay within the "sustainable zone" defined by the limits of the various constraints. This sustainable zone is the multidimensional space within which the system is viable, that is, does not violate its viability constraints today and in the future (Baumgärtner & Quaas, 2009; Cissé et al., 2013; Doyen et al., 2017; Doyen & Martinet, 2012; Mouysset et al., 2013; Schuhbauer & Sumaila, 2016). In mathematical terms, this sustainable zone is called the "viability kernel."

Second, resilience analysis requires the consideration of some form of dynamic control or action (management) — in particular in the case of socioecological systems where the dynamics of the system should account for, and include, possible anthropological responses to shocks or stressors. Here again viability modeling can accommodate for this need in the sense that the approach is built around control and decision variables incorporated in the set of variables that constitutes the dynamic model.

Finally, a large number of interpretations of resilience — especially those derived from the engineering but also ecological sciences — put great emphasis on the question of the system's recovery (the "bouncing back" element), which is often mentioned explicitly or implicitly in many definitions of resilience (Grimm & Wissel, 1997; Liao, 2012; Macgillivray & Grime, 1995). Some of the tools developed around the viability approach are specifically designed to explore those recovery processes. We refer here to the concept of "minimal time of crisis" which corresponds to the time it takes for the system to come back into its viability space, once it has been "pushed out" of that viability space, often under the impact of a shock (Béné et al., 2001; Doyen & Saint-Pierre, 1997). Some of the initial works which attempt to link more formally resilience and viability have built their analyses around this time of crisis, proposing in particular that the computation of this minimal time of crisis can be used as a proxy for (the inverse of) resilience: the longer the time of crisis, (i.e., the longer it takes for a system to come back into its viability space), the less resilient the system is (Deffuant & Gilbert, 2011; Hardy et al., 2016, 2017; Martin, 2004; Rougé et al., 2013).

Viability approach appears therefore particularly well suited to provide a rigorous and sound basis for the formalization of resilience. What we propose in the rest of this paper is to expand further those initial attempts by developing a generic formalization of resilience that goes beyond the simple consideration of this time of crisis and explores more systematically the different types of responses (resistance, absorption, adaptation, and transformation) that are put in place by socioecological systems in responses to shocks and stressors.

3.2. Dynamic Controlled Model

We start with a dynamic modeling framework inspired by control theory. We use discrete time — although the whole approach can be applied under continuous dynamic. In doing so, we tried to keep the mathematical formalism to a minimum level. We consider the following control dynamic system in discrete time,

$$\begin{cases} x(t+1) = F\left(x(t), c(t)\right), \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where t is the time index ranging from the initial time t_0 to horizon T. The state $x(t) = (x_1(t), \ldots, x_n(t))$ is a vector whose components represent the level over time of different stocks; each stock $x_i(t)$ can be renewable resource (typically an abundance or biomass of species), a man-made reproducible capital, or even a pollution level; the vector x_0 corresponds to the state at the initial time t_0 of these stocks. The vector of controls $c(t) = (c_1(t), \ldots, c_p(t))$ may include extraction rate, harvesting effort, investment, consumption, or polluting emissions. The mapping F stands for the dynamics capturing the processes affecting the trajectories of the system through time. This formulation is therefore generic enough to be used to represent a wide set of cases that are relevant from a resilience perspective: species or population dynamics, ecological or ecosystem changes, economic processes involving capital and labor, or pollution accumulation-absorption processes.

3.3. Marginal Costs of Change and Inertia of Decisions

To account for inertia and costs of change in the decision (control) process, we consider the following dynamic constraint

$$c(t+1) = c(t) + u(t)\,, \qquad \text{with } \|u(t)\| \leq \frac{1}{\theta}, \tag{2}$$

where θ is the inertia (or alternatively the inverse of the costs of change), while the value $\|u(t)\|$ stands for the norm (typically for Euclidean norm, this reads $\|u(t)\| = \sqrt{\sum_{j=1}^{p} u_j^2}$) of the vector $u(t)$. When $\theta = 0$, the inertia is nil, which means that the control $c(t)$ is very flexible. On the contrary, when $\theta = +\infty$ the inertia is maximal. In that case, the control is inflexible, that is, $c(t+1) = c(t)$ for any period t and the marginal cost of change is 0. Between these two extreme values $[0, +\infty[$, a gradient of "rigidity" exists for the control constraint (2).

3.4. Sustainability and Viability Kernel

Viability space. The main outputs of any controlled dynamic system as formalized in (1) can be measured by k different indicators $I_k\,(x(t), c(t))$ that potentially depend on both the states and the controls involved in the system. From a sustainability point of view, we can think of these indicators as instantaneous measurement of quantities that characterize different aspects/dimensions of sustainability including biodiversity, catches, rents, CO_2 emissions, consumption, etc. From a development perspective, we can think of these as welfare indicators including income, nutritional or food security indicators, or assets. Suppose now that the decision-maker's goal is to ensure food security, profitability, biodiversity, or more generally to maintain the system's indicators above some viability thresholds, so that

$$I_k\,(x(t), c(t)) \geq I_k^{\lim}\,, \qquad k = 1, \ldots, K, \tag{3}$$

where $I_k^{\lim}$ is the value which stands for some sustainable—normative/socially defined, or biophysically determined—boundaries recognized as critical limits or thresholds. (Without loss of generality a "bad" indicator, such as pollution, can be represented by its negative value, so that the direction of the inequality in (3) holds.) Those thresholds could be for instance the poverty line US$ 1.25 per day, or a minimum biomass under which a species is threatened of extinction such as the International Council for the Exploration of the Sea (ICES) precautionary threshold $B_{\lim}$ for fisheries management. Another pertinent example would be some form of planet boundaries (Rockstrom et al., 2009) beyond which the system is no longer viable. Using these indicators and associated thresholds as sustainable boundaries (or constraints), a path/trajectory of the system is said to be viable when it meets all the constraints (3) at all times t.

Viability kernel. A viability analysis is an analysis that aims at identifying all the system's states (biological, economic, or social-ecological endowments) and controls (decisions) for which the resulting path is viable at the present time and remains viable in the future, given the dynamics of the system. A key mathematical tool of viability analysis is the viability kernel (Aubin, 1990) noted $\mathbb{V}\text{iab}$. It is composed of all initial states for which viable trajectories exist, that is, states for which intertemporal decisions can be found that result in trajectories satisfying the constraints from the initial time t_0 until horizon T. In our case, in mathematical terms, this would read

$$\mathbb{V}\text{iab}(T, F, I^{\lim}, \theta) = \left\{ (x_0, c_0) \;\middle|\; \begin{array}{l} \text{there exist controls } \big(c(t_0), \ldots, c(T-1)\big) \\ \text{there exist states } \big(x(t_0), \ldots, x(T)\big) \\ \text{starting from } (x_0, c_0) \\ \text{such that dynamics (1), (2) and constraints (3)} \\ \text{hold true for all time } t = t_0, \ldots, T \end{array} \right\}. \tag{4}$$

Conducting a viability analysis consists therefore in identifying the set of conditions in (4). (Viability kernels can be computed using dynamic programming equations—a process which we term viable dynamic programming; DeLara & Doyen, 2008. This viable dynamic programming is not totally "new" in the sense that it builds on classical dynamic programming equations as proposed conventionally in dynamic optimization and optimal control; Bellman, 1964.) Favorable situations occur when the viability kernel is not empty as this indicates the existence of possible "sustainable solutions" which fulfill the system constraints (3). The case where the viability kernel (4) is empty is also informative, however, as it indicates the inconsistency and/or irreconcilable nature over time of at least two of the constraints (3).

3.5. Linking Viability Analysis With Resilience

Viability can be used to formalize more rigorously the typology of resilience strategies/responses that have been presented in section 2. In a viability context, the different resilience strategies (resistance, absorptive response, adaptive response, adaptive preference, and transformation) correspond to different types of responses, that is, change of controls $u(t) = c(t+1) - c(t)$ as in equation (2), characterized by different costs of change $\frac{1}{\theta}$ and different control rigidity $|u(t)|$. In the rest of this section, we present this typology of resilience responses and how they can be formalized under a viability framework.

Resistance: Constancy and infinite inertia. Resistance corresponds to a situation where the system's dynamic F, its set of threshold/critical limits I_k^{lim}, and its control/response $c(t)$ remain unchanged for all $t = t_0, \dots, T$. This corresponds to the stability or constancy condition as described in the engineering literature mentioned in Table 1. In the formalism proposed above, it means that the system is characterized by an extreme rigidity $\theta = +\infty$, while the costs associated with eventual changes in the control (resilience response) are minimal (in fact nil) as there is no change in control:

$$c(t+1) = c(t) = c_0.$$

In a viability context, these different conditions rely on the study of the following viability kernel

$$\mathbb{V}\text{iab}(T, F, I^{\text{lim}}, +\infty). \tag{5}$$

Absorptive resilience: Persistence and high inertia. Absorptive responses (such as coping strategies) correspond to situations where the system's overall dynamic F and its set of threshold/critical limits I_k^{lim} remain unchanged, but the control can be slightly altered (at least temporarily) in the sense that $\|c(t+1) - c(t)\| = \|u(t)\| \le \varepsilon$. This change corresponds to the adoption of buffer/absorptive responses as described in the ecological or development literature. In the formalism proposed above it indicates situations where the system is characterized by some level of rigidity, reflected by a relatively high inertia in the control $\theta = \varepsilon^{-1}$. In contrast, the costs of change are reduced (cf. Table 1). In a viability formalism, this means we consider the following viability kernel:

$$\mathbb{V}\text{iab}\left(T, F, I^{\text{lim}}, \frac{1}{\varepsilon}\right) \tag{6}$$

Adaptive resilience: Adaptation and low inertia. Adaptive response refers to situations where the system's overall dynamic F and its set of threshold/critical limits I_k^{lim} remain unchanged, but the controls $c(t)$ can be significantly modified and adjusted throughout time. This corresponds to situations described in the social-ecological system literature where systems display ability to learn and to adapt to respond to a specific shock/stressor. In the formalism proposed above, it indicates situations where the system is characterized by lower level of rigidity but potentially higher costs of change (which are the costs of adaptation). This is equivalent to considering the following kernel:

$$\mathbb{V}\text{iab}(T, F, I^{\text{lim}}, 0). \tag{7}$$

Adaptive preference: Adjustments in the constraints of the system. Systems or individuals who undergo an adaptive preference adjustment are those who, in the face of a crisis or a shock, decide (unconsciously or deliberately) to modify their expectations (usually downward) as a defensive mechanism to be able to cope with the consequences of that crisis or shock. By modifying their expectations, they essentially adjust the constraints of the system I_k^{lim} (which usually comes with some costs), without necessarily changing the system's functioning. Using our mathematical formalism, this situation corresponds to the following kernel:

$$\mathbb{V}\text{iab}\left(T, F, \widetilde{I^{\text{lim}}}, 0\right), \tag{8}$$

where $\widetilde{I_k^{\text{lim}}}$ represents the new (adjusted) thresholds underpinning the constraints and the sustainability zone.

Transformative response: Changes in dynamics and in controls. Transformability corresponds to situations where the shock or the impact of the stressor are so important that only drastic changes in both the system dynamics

F and the controls and/or constraints can avoid the system to collapse. Those structural changes however come at high costs. In that case, we consider the following kernel:

$$\mathbb{V}\text{iab}\left(T, \tilde{F}, \widetilde{I^{\lim}}, 0\right), \tag{9}$$

where $\widetilde{I^{\lim}}$ represents the new (adjusted) constrains and $\tilde{F}$ represents the new (transformed) system dynamics.

3.6. Metrics of Resilience

Using the continuum of resilience responses introduced above, we can now derive a generic resilience metric with respect to a given state control (x_0, c_0). The metric is based on the basic and intuitive idea that a good proxy for the level of resilience of a system is the amplitude of the largest shock that the system can stand without violating its viability constraints. In other words, for every type of resilience responses put in place (resistance, absorptability, adaptability, adaptive preference, or transformability), the larger the shock that a system can put up with and remain viable in the long run, the more resilient that system is. The first part of the last sentence ("for every type of resilience responses put in place") is important as it points out at why this metric of resilience is *not* equivalent to a "narrow" definition of resilience where resilience would be interpreted as the "ability of the system to resist shocks." Instead, the metric is *generic*, in the sense that it can be applied to all the different types of responses, from resistance to transformation, that have been recognized in the literature. In a more mathematical form, this approach is equivalent to computing the distance between the current state x_0 and the no sustainable zone, namely, the complementary of the viability kernel, and can be formalized as follows:

$$\textsc{resi}(x_0, c_0, T, F, I^{\lim}, \theta) = \min_{(x, c_0) \notin \mathbb{V}\text{iab}(T, F, I^{\lim}, \theta)} \|x_0 - x\|. \tag{10}$$

Applying this generic metric to the different resilience responses that have been identified above, we obtain the following typology:

- $\textsc{Resist}(x_0, c_0) = \textsc{resi}(x_0, c_0, T, F, I^{\lim}, +\infty)$ in the case of resistance as defined in (5) where the inertia of controls is set to $\theta = +\infty$ and the costs associated with change are nil.
- $\textsc{AbsorR}(x_0, c_0) = \textsc{resi}(x_0, c_0, T, F, I^{\lim}, \frac{1}{\epsilon})$ in the case of absorptive resilience as defined in (6) where the controls of the system are characterized by a relatively high inertia θ and the costs of change are low.
- $\textsc{AdapR}(x_0, c_0) = \textsc{resi}(x_0, c_0, T, F, I^{\lim}, 0)$ in the case of adaptive resilience as defined in (7) where the inertia of controls is low ($\theta = 0$), but the costs of change can be substantial.
- $\textsc{AdapP}(x_0, c_0) = \textsc{resi}(x_0, c_0, T, F, \widetilde{I^{\lim}}, 0)$ in the case of adaptive preference as defined in (8) where preferences or goals can be adjusted but with some costs.
- $\textsc{Transf}(x_0, c_0) = \textsc{resi}(x_0, c_0, T, \tilde{F}, \widetilde{I^{\lim}}, 0)$ in the case of transformation as defined in (9) where there is no inertia any longer ($\theta = 0$) and the whole system's dynamics and constraints can be modified but usually at a substantial cost.

3.7. Resilience Strategies as Matryoshka (Russian) Dolls

Before we proceed with the two empirical models (lake eutrophication and natural resource exploitation), we propose to push the analytical part of this exercise one step further and explore more formally the relationship that exists between the five resilience responses as identified above. We initially said that those five responses should be considered as a continuum of strategies adopted in the face of shocks or stressors with the objective to allow the system to remain viable (i.e., sustainable in the long run) at the lowest costs of change as possible. It is however possible to prove analytically that those different strategies are not just independently located along that continuum but instead are organized in a nested relationship, like a set of Matryoshka (or Russian) dolls. In particular, it is possible to demonstrate mathematically the following relationship

$$\textsc{Resist}(x_0, c_0) \leq \textsc{AbsorR}(x_0, c_0) \leq \textsc{AdapR}(x_0, c_0) \leq \textsc{AdapP}(x_0, c_0) \leq \textsc{Transf}(x_0, c_0) \tag{11}$$

under the following two conditions:

$$\widetilde{I^{\lim}} \leq I^{\lim} \text{ and graph}\,(F) \;\subset\; \text{graph}\left(\tilde{F}\right). \tag{12}$$

What this relationship (11) means is that for the same severity or nature of shock (or stressor), being able to adopt an adaptive response creates a "larger" viability kernel—and therefore offers a higher likelihood to remain viable—than adopting an absorptive response. Likewise, adopting an absorptive strategy is associated with a larger viability kernel than a resistance strategy, other things being equal . The last part of the relationship means that for the same intensity or nature of shock (or stressor), adopting an adaptive preference strategy creates a larger viability kernel by relaxing the viability constraints with $\widetilde{I^{\lim}} \leq I^{\lim}$, thus increasing further the resilience of the system, than would be achieved by adopting an adaptive resilience strategy. Likewise, adopting a transformative strategy creates a larger viability kernel, compared to what could be achieved by adopting an adaptive preference strategy. For this last scenario to happen, however, we need the condition graph $(F) \subset$ graph $(\tilde{F})$ to hold. This condition captures the idea that the dynamics $\tilde{F}$ provides new possibilities as compared to initial dynamics F. Such an assumption is relatively stringent, however, and may not be satisfied in every case, as will be exemplified in the first numerical example below. The mathematical proof of the relationship (11) is provided in Appendix A.

4. Stylized Bioeconomic Examples

We now illustrate the previous analytical findings and definitions through two stylized bioeconomic models: one related to renewable resource management and another related to lake eutrophication.

4.1. Renewable Resource Management

We consider first a renewable resource dynamical model with one species harvested by a group of agents as described in Clark (1990). The discrete-time control dynamical system relies on a Beverton-Holt population renewal as follows

$$x(t+1) = f(x(t) - qe(t)x(t)) \text{ with } f(x) = (1+r)x\left(1 + \frac{r}{K}x\right)^{-1}, \tag{13}$$

where the renewable stock $x(t)$ is a state of the system, while the intensity of catch $e(t)$ (or exploitation rate) stands for the decision variable. Biological parameters include both the maximal growth of the stock r and the carrying capacity K of the ecosystem, while q corresponds to a technological parameter usually termed catchability in the fisheries science sector (determining the efficiency of the exploitation rate). The potential change in the decision $e(t)$ variable is captured by the following dynamics

$$e(t+1) = e(t) + u(t) \text{ with } |u(t)| \leq \frac{1}{\theta}. \tag{14}$$

To ensure sustainability, the following ecological and economic constraints are taken into account:

- Resource conservation threshold: for all $t = t_0, \dots, T$,

$$x(t) \geq x^{\lim}, \tag{15}$$

  where $x^{\lim}$ is a viable stock threshold under which the biological survival of the resource is threatened.
- Profitability constraint threshold: for all $t = t_0, \dots, T$

$$\pi(x(t), e(t)) = pqe(t)x(t) - ce(t) \geq \pi^{\lim}, \tag{16}$$

  where p is the selling price, c is the unit cost of effort, and $\pi^{\lim}$ is a guaranteed profit level which can coincide with fixed costs. (Hereafter, for the sake of simplicity, we assume that the viable stock threshold is large enough in the sense that $x^{\lim} \leq x_{oa} = \frac{c}{pq}$. It can be shown that in that case the viability thresholds are reduced to the profitability requirement $\pi^{\lim}$; (see, e.g., Béné et al., 2001). Moreover, to avoid cases of empty viability kernels, we also assume that the fixed costs are low enough to guarantee $\pi^{\lim} \leq \pi^{MEY}$, where π^{MEY} is the profit at maximum sustainable yield (MEY) (Clark, 1990) namely, $\pi^{MEY} = \max_{(x,e) \text{ at equilibrium}} \pi(x, e)$.)

Using dynamic viability programming (DeLara & Doyen, 2008), we can now compute the viability kernels for the five different types of resilience responses discussed above and the two constraints (conservation and profitability). The results of those computations are displayed in Figure 1 for specific numerical values. The more generic and analytic formulations with an infinite time horizon $T = +\infty$ are as follows:

- Resistance: extreme rigidity $\theta = +\infty$ and no transaction costs

$$\mathbb{V}\text{iab}(+\infty, F, \pi^{\lim}, +\infty) = \left\{ (x, e) \,\middle|\, \pi(x, e) \geq \pi^{\lim},\ x \geq x_{pa},\ e_{pa} \geq e \right\},$$

  where (x_{pa}, e_{pa}) is the equilibrium point binding the profit constraint (namely, the solution of system of two equations $\pi(x, e) = \pi^{\lim},\ e = \frac{1}{q}(1 - \frac{1}{1+r-\frac{r}{K}x}))$ and depending on the dynamics F (in particular through catchability q) as well as viability threshold $\pi^{\lim}$.

- Adaptive resilience: no inertia $\theta = 0$ but high costs of change (costs of adaptation).

$$\mathbb{V}\mathrm{iab}(+\infty, F, \pi^{\lim}, 0) = \left\{(x,e)|\ \pi(x,e) \geq \pi^{\lim},\ x \geq x_{pa},\ e_{pa}(x) \geq e\right\},$$

where the exploitation level (fishing effort) $e_{pa}(x)$ is a function (induced by the viability condition $f(x-qex) \geq x_{pa}$) of stock x

$$e_{pa}(x) = \frac{1}{q}\left(1 - \frac{x_{pa}}{x}(1 - qe_{pa})\right). \tag{17}$$

- Coping: strong rigidity $0 << \theta < +\infty$ and low transaction costs

$$\mathbb{V}\mathrm{iab}(+\infty, F, \pi^{\lim}, \theta) = \left\{(x,e)|\ \pi(x,e) \geq \pi^{\lim},\ x \geq x_{pa},\ e \geq e_{\theta}(x)\right\},$$

where the exploitation level (fishing effort) $e_{\theta}(x)$ is a function as in Figure 1 such that

$$e_{pa} \leq e_{\theta}(x) < e_{pa}(x).$$

As inertia θ has a high value, the curve $e_{\theta}(x)$ is close to effort threshold e_{pa}.

- Adaptive preference: profitability constraints relaxed by reducing the fixed costs $\pi^{\lim}$. The new "adapted" viability kernel $\mathbb{V}\mathrm{iab}\left(\infty, F, \widetilde{\pi^{\lim}}, 0\right)$ is then computed with this new constraint $\widetilde{\pi^{\lim}}$ used in the computation of x_{pa} and $e_{pa}(x)$.

$$\mathbb{V}\mathrm{iab}\left(+\infty, F, \widetilde{\pi^{\lim}}, 0\right) = \left\{(x,e)|\ \pi(x,e) \geq \widetilde{\pi^{\lim}},\ x \geq x_{pa},\ e_{pa}(x) \geq e\right\}.$$

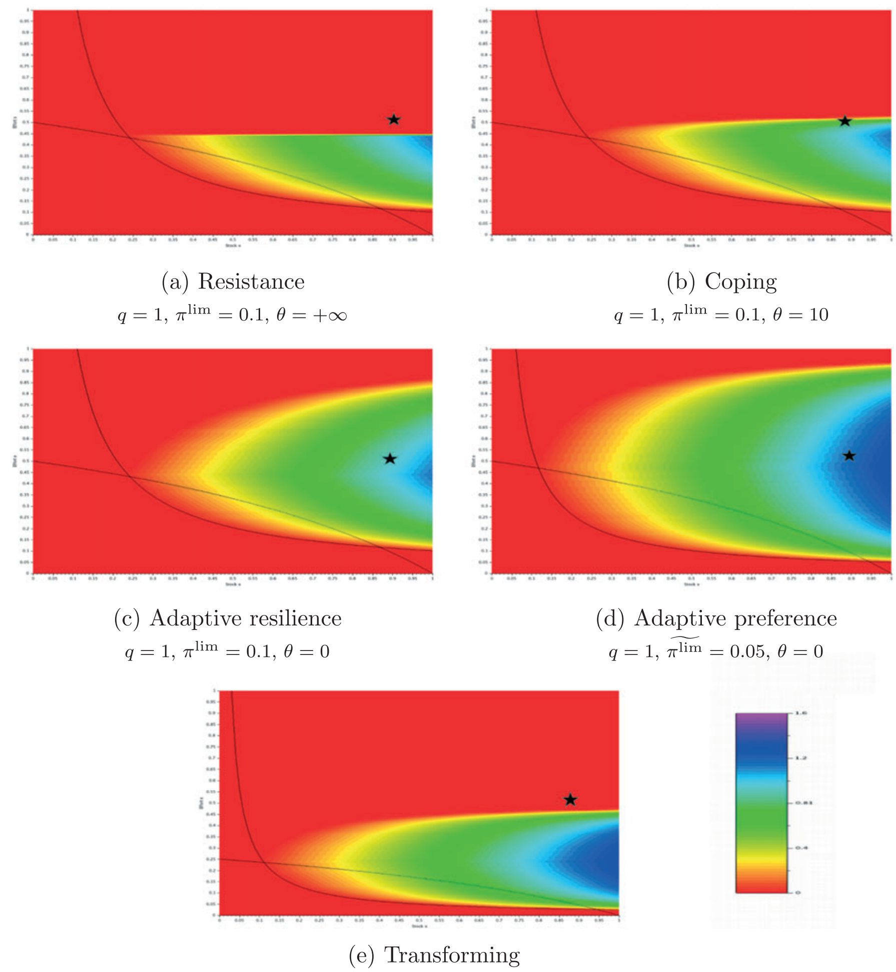
- Transformation: change within the dynamic processes (13) is achieved by modifying the technology and in particular increasing the catchability q.

$$\mathbb{V}\mathrm{iab}\left(+\infty, \tilde{F}, \widetilde{\pi^{\lim}}, 0\right) = \left\{(x,e)|\ \pi(x,e) \geq \widetilde{\pi^{\lim}},\ x \geq x_{pa},\ \widetilde{e_{pa}}(x) \geq e\right\},$$

where viable upper effort $\widetilde{e_{pa}}(x)$ is derived from equation (17) with $\tilde{q} > q$.

In Figure 1 the horizontal axis represents the level of biomass and the vertical axis represents the intensity of fishing effort (exploitation rate). The decreasing convex black curve corresponds to the profitability threshold. Below this curve the fisheries is economically not profitable and therefore not sustainable in the long run. The decreasing concave black curve corresponds to the sustainable yield equilibrium points as defined in the classical approach under Beverton and Holt model (namely, $e(x) = \frac{1}{q}\left(1 - \frac{1}{1+r-\frac{r}{K}x}\right)$). The levels of resilience proxied through the viability status of the system are indicated by the different colors: no-viable condition (i.e., no resilience) is shown in red, while maximum resilience corresponds to blue-purple zones. Lower level of resilience (less than optimal) are indicated by the other colors (blue-green-yellow). The red area under the profitability curve corresponds to conditions where the profitability constraint is not satisfied, while the red area on the top part of each figure corresponds to a zone of overexploitation (too high levels of effort $e(t)$ with respect of the level of biomass $x(t)$).

In line with the theoretical discussion above related to the continuum of responses, the figure illustrates very clearly the "Russian doll" relation (11) between the different types of responses. We can observe in particular that the viability kernel is progressively expanding when resilience strategies shift from resistance to adaptive preferences. For illustration, the point $x = 0.9$, $e = 0.5$ indicated by a black star in the figures, which is not viable (and not resilient) under a resistance response, becomes progressively resilient under the absorptive, adaptive, and preference adaption response—passing from a red to a green, blue, and finally purple color—indicating the higher level of shock intensity it can withstand without losing its long-term viability. This means that a system adopting an absorptive or coping response—Figure 1b—will be able to deal with more severe events (shocks or stressors) as it displays a higher level of resilience (other things being equal) than a system which is only resistant—Figure 1a; likewise, a system able to adopt an adaptive response—Figure 1c—is characterized by a higher level of resilience than a system which adopts a response based on some form of absorptive response—Figure 1b; and finally, an adaptive preference



(a) Resistance
$q = 1,\ \pi^{\mathrm{lim}} = 0.1,\ \theta = +\infty$

(b) Coping
$q = 1,\ \pi^{\mathrm{lim}} = 0.1,\ \theta = 10$

(c) Adaptive resilience
$q = 1,\ \pi^{\mathrm{lim}} = 0.1,\ \theta = 0$

(d) Adaptive preference
$q = 1,\ \widetilde{\pi^{\mathrm{lim}}} = 0.05,\ \theta = 0$

(e) Transforming
$\widetilde{q} = 2,\ \pi^{\mathrm{lim}} = 0.05,\ \theta = 0$

Figure 1. Resilience metrics $\mathrm{RESI}(x, e)$ for the different resilience strategies of renewable resource management: (a) resistance, (b) coping, (c) adaptive resilience, (d) adaptive preferences, and (e) transforming. No resilience ($\mathrm{RESI}(x, e) = 0$) in red. Weak resilience in yellow. High resilience in blue. Fixed parameters are $r = 1$, $K = 1$, $p = 1$, $c = 0.01$, and time horizon $T = 30$. Inertia θ varies in (a)–(c). Preferences vary in (d) with guaranteed profit $\widetilde{\pi^{\mathrm{lim}}} = 0.05$ instead of $\pi^{\mathrm{lim}} = 0.01$ as in (a)–(c). Dynamics varies in (d) with catchability $\tilde{q} = 2$ instead of $q = 1$ as in (a)–(d). The black star stands for point $x = 0.9$, $e = 0.5$.

response—Figure 1d—will create a larger viable kernel and thus be associated with a higher level of resilience than an adaptive resilience response.

In contrast, although transforming strategy—Figure 1e—provides viability shapes similar to adaptive preferences—Figure 1d—no clear inclusion exists between (d) and (e). The "transformed" kernel is indeed expanded leftward along the horizontal axis (stock side x) meaning that additional resilience was gained at lower levels of stock; but it shrinks along the vertical axis (effort), indicating lower level of resilience along this axis compared to the situation before the transformation took place (in this particular case this outcome results from the fact that, on one hand, improving the catchability of the fleet through technical innovation enhances the efficiency of the fisheries but, on the other hand, also increases the pressure on the stock). This is visible using again the specific example of the point $x = 0.9$, $e = 0.5$. While those system's conditions had become progressively more resilient under the other responses (absorptive strategies, adaptation, adaptive preference), it turns back to red under the transformative response. Such an outcome illustrates that the nonlinearities of both the dynamics and constraints of the system as included when we transform the system do not allow for straightforward inclusive relation from adaptive preference to transformation and that one has therefore to pay attention to unexpected/unpredictable outcomes when dealing with such transformative strategy.

4.2. Lake Eutrophication

Lakes and their potential eutrophication have been intensively studied in relation to resilience, in particular because this problem can be tackled through stylized nonlinear dynamics characterized by a regime shift at equilibrium (Carpenter et al., 2001; Ludwig et al., 2003). In such a stylized framework, the evolution of phosphorus concentration $x(t)$ depending on the phosphorus input rate $c(t)$ reads in discrete time as follows:

$$x(t+1) = (1-s)x(t) + c(t) + r\frac{x(t)^q}{m^q + x(t)^q}, \tag{18}$$

where the parameter s is the phosphorus sink rate (i.e., the rate of phosphorus that flows out of the lake), r is the maximal recycling rate by the lake algae, m is the value of phosphorus concentration $x(t)$ for which the recycling term is half its maximal value, while q is a dimensionless parameter. The decision (or control) $c(t)$ related to the level of phosphorus input needed (as fertilizer) for farming activities is captured by the following dynamics

$$c(t+1) = c(t) + u(t) \text{ with } |u(t)| \leq \frac{1}{\theta}. \tag{19}$$

A lake can have two regimes, and phosphorus concentration has been found to trigger such a regime shift. Namely, the switch is from the oligotrophic or clear water regime, in which both ecologic and economic benefits from the lake are high, to the eutrophic or turbid water regime in which algae blooms feeding on the phosphorus high concentration causes oxygen depletion, leading in turn to a so-called dead lake. We therefore consider the two viability constraints:

- Eutrophication threshold: for all $t = t_0, \dots, T$,

$$x(t) \leq x^{\text{lim}}, \tag{20}$$

  where x^{lim} is a threshold above which the lake turns eutrophic.
- Farming input requirement threshold: for all $t = t_0, \dots, T$

$$c(t) \geq c^{\text{lim}}, \tag{21}$$

  where c^{lim} corresponds to the minimum quantity of phosphorus needed as fertilizer for farming activities to remain economically viable.

In Figure 2, the horizontal axis represents the concentration of phosphorus in the lake $x(t)$, while the vertical axis represents the level of phosphorus discharge arriving in that lake $c(t)$ as a consequence of farming activities. The nonlinear black curve corresponds to the equilibrium curve (the equilibrium curve corresponds to $c(x) = sx - r\frac{x^q}{m^q+x^q}$) of phosphorus dynamics (18). The black horizontal straight line represents the farming input requirement threshold c^{lim}, and the black vertical line represents the eutrophication threshold x^{lim}. Like in the case of the Beverton-Holt model, the levels of resilience as well as the viability status of the system are indicated by the gradient of colors: no-viable condition is shown in red, and maximum resilience conditions correspond to blue-purple areas. In line with our expectations, Figure 2 indicates that any level of phosphorus input $c(t)$ below c^{lim} is not viable from a farming perspective (as it leads to too low productivity), while any level of phosphorus concentration $x(t)$ exceeding x^{lim} is not viable from an ecological perspective (as it leads to eutrophication). Those zones appear therefore in red in Figure 2.

The figure also reveals the existence of another red area. This corresponds to a zone of input overintensity located above the viability kernel, where the farming inputs $c(t)$ are too high with respect to the state of phosphorus $x(t)$. Such a situation entails a rise in the phosphorus concentration in the lake leading to the violation of the eutrophication threshold in the long run. In that zone, the dynamics of the system makes it impossible to avoid the eutrophication even when the input is reduced.

Like in the Beverton and Holt model above, Figure 2 shows that in case of lake eutrophication, the viability kernel of the system expands progressively when resilience responses change from resistance to adaptive preference. In that sense the lake eutrophication case also confirms the Russian doll inclusion relationship (11) discussed earlier. In contrast to the previous renewable resource example, however, the inclusion also seems to hold true between adaptive preference (Figure 2d) and transformative resilience (Figure 2e) in the sense that the viability kernel generated under the adaptive preference appears smaller than the viability kernel created by a transformative response. The reason for this is that increasing the phosphorus sink rate s

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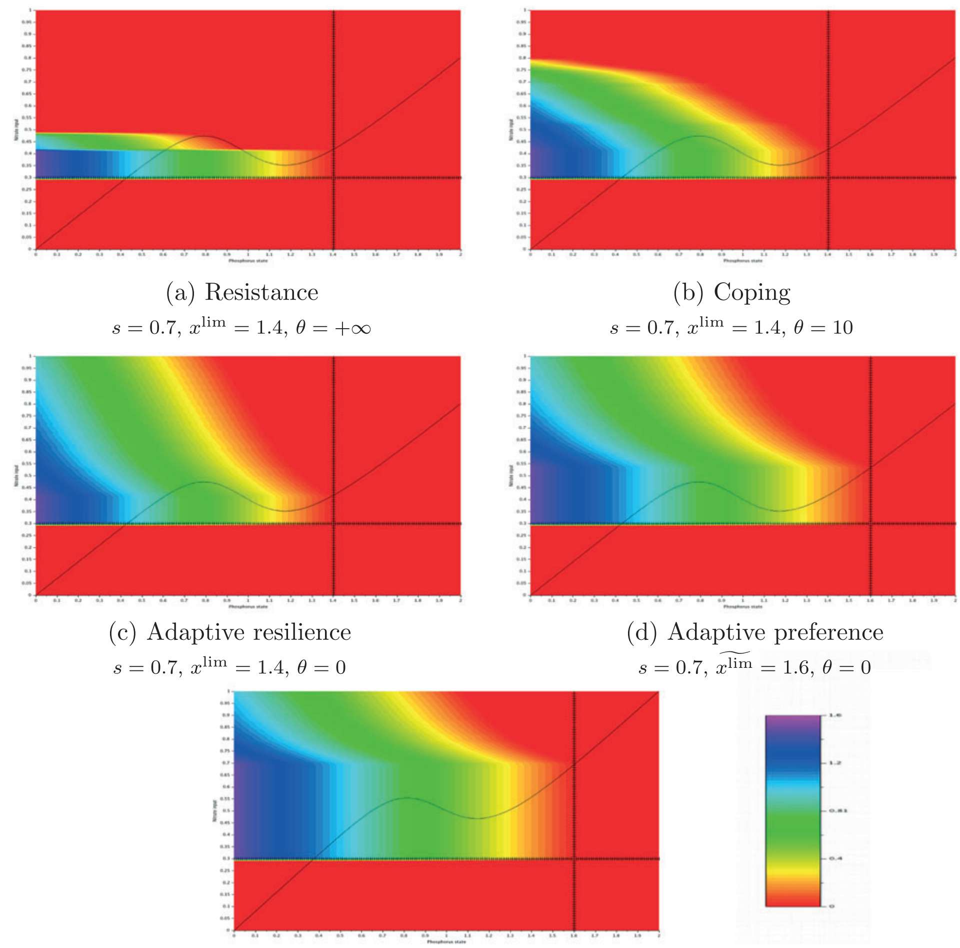

(a) Resistance
$s = 0.7,\ x^{\lim} = 1.4,\ \theta = +\infty$

(b) Coping
$s = 0.7,\ x^{\lim} = 1.4,\ \theta = 10$

(c) Adaptive resilience
$s = 0.7,\ x^{\lim} = 1.4,\ \theta = 0$

(d) Adaptive preference
$s = 0.7,\ \widetilde{x^{\lim}} = 1.6,\ \theta = 0$

(e) Transforming
$\tilde{s} = 0.8,\ \widetilde{x^{\lim}} = 1.6,\ \theta = 0$

Figure 2. Resilience metrics RESI(x, e) for the different management strategies of lake euthrophication: (a) resistance, (b) coping, (c) adaptive resilience, (d) adaptive preferences, and (e) transforming. No resilience (RESI$(x, e) = 0$) in red. Weak resilience in yellow. High resilience in blue. Fixed parameters include $r = 0.6$, $m = 1$, $q = 8$, and $c^{\lim} = 0.3$ and time horizon $T = 30$. Inertia θ varies in (a)–(c). Preferences varies in (d) and (e) with a safe level for phosphorus set to $\widetilde{x^{\lim}} = 1.6$ instead of $x^{\lim} = 1.4$ in (a)–(c). Phosphorus dynamics only varies in (e) with a larger sink rate $\tilde{s} = 80\%$ instead of $s = 70\%$ for (a)–(d).

(chosen as the "transformative" change in the system dynamics) leads the equilibrium curve of phosphorus dynamics to move upward, allowing a higher level of initial phosphorus to be absorbed by the lake without triggering the irreversible regime shift toward eutrophication.

5. Discussion and Conclusion

In the last 10 years or so the rapid increase in the use of the concept of resilience across a wide and growing spectrum of disciplines has been accompanied by an equally growing level of concern expressed by many within the scientific community as well as among practitioners about the potential misuse of the concept (Béné et al., 2012; Cannon & Muller-Mahn, 2010; Davidson, 2010; Leach, 2008; Olsson et al., 2015). Part of this concern derives from the fact that resilience is difficult to reduce to one single dimension, may materialize under several different forms, seems to exist across several scales, and, as such, is hard to define and to measure (Béné, 2013). This lack of universal measure of resilience—reflecting the intrinsic latent nature of this concept—has slowed down progress in the operationalization of the concept in the field and is also one of the main reasons for the absence of any consensual definitions or generic metrics of resilience in the more theoretical/academic literature.

Recognizing both the potential of the concept and yet the difficulty to operationalize it, this paper aims at exploring further the question of the formalization of resilience in the context of dynamic systems, with the objective to offer some elements of mathematical rigor to the discussion. We propose to do this through the development of a generalizable metric of resilience, relying on the mathematical tools of viability analysis. In the recent past a series of papers have already explored the possibility of linking more formally resilience and viability through the concept of time of crisis (Deffuant & Gilbert, 2011; Hardy et al., 2016, 2017; Martin, 2004; Rougé et al., 2013). The concept of minimal time of crisis corresponds to the time it takes for a system to come back into its viability space once it has been pushed out of that viability space (Béné et al., 2001; Doyen & Saint-Pierre, 1997). As such the concept of time of crisis offers an interesting initial theoretical bridge between viability and resilience. It also constitutes a first clear step toward the measurement of resilience in that one can consider using it as a measurable proxy for the inverse of resilience: the longer it takes for a system to come back into its viability space after a shock, the less resilient the system is (Deffuant & Gilbert, 2011; Hardy et al., 2016; Rougé et al., 2013).

This particular use of the concept of time of crisis to "quantify" resilience is useful and allowed the authors of those studies to explore some interesting questions, for instance, around the role of collective action and cooperation or technological innovation in creating or enhancing resilience (Hardy et al., 2016, 2017). This particular approach is, however, associated with a specific interpretation of resilience, one that puts emphasis on the question of recovery (the bouncing back element of resilience) often found in the engineering and the initial ecological literature on resilience (Grimm & Wissel, 1997; Holling, 1973; Liao, 2012).

In the present paper we expanded this thinking one step further, acknowledging that resilience is now increasingly recognized to be more than just a bouncing-back property. In particular, the review of the literature across several disciplines (psychology, ecology, physics, social-ecological, disaster and humanitarian interventions, etc.) reveals that many different types of responses can be interpreted as forms of resilience strategies adopted by individuals, households, communities, or higher-level components of systems. Five generic types of responses were thus identified across the literature: resistance, coping strategies, adaptation (including both adaptive responses and adaptive preferences), and transformation.

Putting those different types of responses into distinct categories may be subject to critic as it could lead to overlooking and discounting some important nuances and/or subtleties. Using this typology was, however, conceptually very useful as it provided us with an overall coherent framework that allows to "map" and contrast those different resilience responses in a comprehensive but also consistent manner. This represented therefore an important first step toward the construction of a generic metric of resilience measurement.

Viability was then used to formalize those different types of resilience responses. As part of this formalization, and in line with the literature, we characterized those different categories of resilience strategies with regard to the degree/intensity of changes in the dynamics of the systems (Berkes et al., 2003; Cutter et al., 2008; Folke, 2006; Folke et al., 2010; Walker et al., 2004). We, however, also proposed to account for the transactional costs of changes and the inertia associated with those changes (Béné et al., 2012, 2014).

Using those categories of resilience responses, we were then able to derive a generic resilience metric. This metric is based on the intuitive idea that a good proxy for the level of resilience of a system is the amplitude of the largest shock that the system can stand without violating its viability constraints. More concretely, the larger the shock that the system can handle and remain viable in the long run, the more resilient the system is. For this, the metric identifies the states and controls (x_0, c_0) of the system that allow that system to remain functional (viable) now and in the future, despite being affected by a shock. As such the metric captures the essence of what resilience is about: being able to deal with shocks in a way that mitigates the risks of long-term negative implications (Constas et al., 2013).

It is important to stress that those different levels of resilience are defined with respect to *all the different types of resilience responses that can be adopted by a system* and not simply the bouncing back strategy. This difference is the reason why this approach is fully generic and offers in particular a metric that goes beyond "resilience as the capacity of a system to come back to its initial state" underpinning many interpretations of resilience in the literature.

The metric—and within it, the consideration of those different types of responses—offers a second important contribution to the literature on resilience measurement. It confirms first theoretically—cf. Appendix A—and then empirically through the two case studies that the resilience of a system does not simply depend on the conditions characterizing that system at the time it is affected by a shock/stressor combined with the amplitude (or severity) of that event but also on the type of responses put in place. This result appears clearly in Figures 1 and 2 where we observe that for a given set of conditions (x_0, c_0) the intensity of the shock that can be withstood by the system without losing its long-term (intertemporal) viability depends on the type of response adopted (we recall here the case of the point (0.9; 0.5) in the Beverton-Holt model discussed above). In that regard, both our theoretical and numerical analyses converge toward the same conclusions: by shifting from resistance to absorptive, adaptive or even adaptive preference responses, systems are able to expand their viability kernel, which consequently means that they are able to strengthen or increase their resilience. Those results which appear consistently across the two cases (natural resources exploitation and lake eutrophication) are also in line with some of the main conclusions reached in the literature on resilience measurement in relation to humanitarian and food security interventions, where it is emphasized that resilience results from the combinations of the direct impacts of the shocks and the longer-term effects of the responses put in place by the households (Béné et al., 2015; Constas et al., 2014).

Interestingly, both our theoretical and empirical results also converge to show that the case of transformation stands aside in this overall analysis and is not as straightforward as the other responses. While the decision of shifting from resistance to adaptive preference leads to a progressive increase in the level of resilience of the system, the outcome of adopting a transformative response is not so predictable. In some cases the transformation seems to result in a further strengthening of the system resilience—as it was the case in Figure 2e; but in other cases, adopting a transformative response strategy may result in losing resilience—as observed in Figure 1e. As mentioned earlier such result may be explained by the fact that the nonlinearities of both the dynamics and constraints of the systems do not lead to straightforward (linear) outcomes when one adopts a transformative strategy. This last result can be related to the current discussion on transformation found in the more general literature—especially in the context of climate change—where scholars recognize both the importance and the complexity of the concept of transformation in relation to the need for societal changes (Berman et al., 2012; Folke et al., 2010; Gunderson & Holling, 2001; Leach et al., 2012; Pelling & Manuel-Navarrete, 2011).

There are several directions in which this work can be extended. One of these is the better integration of uncertainty into the analysis. The current approach and its formulations (as represented by equations (1), (2), and (4) is essentially deterministic. Yet resilience is also acknowledged in the literature for its consideration for uncertainty—especially unexpected shocks that can cause abrupt changes (Chu et al., 2015). Some would in fact argue that resilience can become a tool to manage (or navigate) uncertainty (Olsson et al., 2006). A more explicit integration of uncertainty in the framework presented in this paper is, however, possible through some recently developed mathematical tools, such as the stochastic viability kernel (Doyen & DeLara, 2010).

Appendix A

A1. Proof of the Relationship (11)

To demonstrate inequalities (11), we first prove that the viability kernel $\mathbb{V}\mathrm{iab}\left(T, F, I^{\lim}, \theta\right)$ is increasing with respect to inertia θ as follows:

$$\theta_1 \geq \theta_2 \Longrightarrow \mathbb{V}\mathrm{iab}\left(T, F, I^{\lim}, \theta_1\right) \subset \mathbb{V}\mathrm{iab}\left(T, F, I^{\lim}, \theta_2\right). \tag{A1}$$

Consider indeed $(x_0, c_0) \in \mathbb{V}\mathrm{iab}\left(T, F, I^{\lim}, \theta_1\right)$. By the very definition of the viability kernel described in (4), there exists sequences $c(t_0), \dots, c(T-1)$ and $x(t_0), \dots, x(T)$ starting from (x_0, c_0) and satisfying constraints (3), dynamics (1), and rigidity constraint

$$|c(t+1) - c(t)| = |u(t)| \leq \frac{1}{\theta_1}.$$

As $\theta_1 \geq \theta_2$, the sequences $c(t_0), \dots, c(T-1)$ also comply with the constraint $|c(t+1) - c(t)| \leq \frac{1}{\theta_2}$. Therefore, $(x_0, c_0) \in \mathbb{V}\mathrm{iab}\left(T, F, I^{\lim}, \theta_2, T\right)$. By virtue of property (A1) and the very definition of resilience metrics from (10), we deduce that

$$\begin{aligned} Resist(x_0, c_0) = \mathrm{RESI}(x_0, c_0, T, F, I^{\lim}, +\infty) &= \min_{(x, c_0) \notin \mathbb{V}\mathrm{iab}(T, F, I^{\lim}, +\infty)} \|x_0 - x\| \\ &\leq \min_{(x, c_0) \notin \mathbb{V}\mathrm{iab}(T, F, I^{\lim}, \varepsilon^{-1})} \|x_0 - x\| = \mathrm{RESI}(x_0, c_0, T, F, I^{\lim}, \varepsilon^{-1}) = AbsorR(x_0, c_0). \end{aligned}$$

We proceed similarly for the last inequality involving $AdapR(x_0, c_0)$.

We need then to demonstrate the fact that the viability kernel is enlarged when the constraints are relaxed. Thus, we deduce that

$$\widetilde{I^{\lim}} \leq I^{\lim} \implies \mathbb{V}\text{iab}\left(T, F, I^{\lim}, \theta\right) \subset \mathbb{V}\text{iab}\left(T, F, \widetilde{I^{\lim}}, \theta, T\right). \tag{A2}$$

Pick up indeed $(x_0, c_0) \in \mathbb{V}\text{iab}\left(T, F, I^{\lim}, \theta, T\right)$, then there exists sequences $c(t_0), \dots, c(T-1)$ and $x(t_0), \dots, x(T)$ starting from (x_0, c_0) and satisfying dynamics (1), rigidity constraint $|c(t+1) - c(t)| = |u(t)| \leq \frac{1}{\theta}$, and

$$I_k\left(x(t), c(t)\right) \geq I_k^{\lim}.$$

Assumption $I^{\lim} \geq \widetilde{I^{\lim}}$ makes possible to conclude as

$$I_k\left(x(t), c(t)\right) \geq \widetilde{I_k^{\lim}}.$$

Regarding the second assertion, let us define the set-valued map

$$\mathcal{F}(x) = \left\{F(x, c),\ c \in \mathbb{R}^p\right\}$$

associated with the dynamic F. The condition graph(F) $\subset$ graph($\tilde{F}$) means mathematically that for any state x, we have the inclusion

$$\mathcal{F}(x) \subset \tilde{\mathcal{F}}(x).$$

Consequently, every trajectory $(x(.), c(.))$ solution of dynamics (1) is also a solution of the transformed dynamics

$$x(t+1) = \tilde{F}\left(x(t), c(t)\right), \quad t = t_0, \dots, T-1.$$

We easily derive the required property.

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