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Iterative Interpolated DFT-based Frequency Estimator for Electrical Waveform Affected by Decaying DC Offset

Daniel Belega[1], Dario Petri[2], Dominique Dallet[3]

[1]*Department of Measurements and Optical Electronics, University Politehnica Timisoara, Bv. V. Parvan, Nr. 2, 300223, Timisoara, Romania, email: daniel.belega@upt.ro*
[2]*Department of Industrial Engineering, University of Trento, Via Sommarive, 9, 38123, Trento, Italy, email: dario.petri@unitn.it*
[3]*IMS Laboratory, Bordeaux INP, University of Bordeaux, CNRS UMR5218, 351 Cours de la Libération, Bâtiment A31, 33405, Talence Cedex, France, email: dominique.dallet@ims-bordeaux.fr*

Abstract **– An iterative Interpolated Discrete Fourier Transform (IpDFT)-based frequency estimator of a sinewave affected by decaying dc offset is proposed in this paper. The accuracies of the proposed procedure and the classical IpDFT algorithm are compared with each other under different steady-state and dynamic testing conditions suggested in the IEC/IEEE 60255-118.1:2018 - Synchrophasors for power systems – Measurements. A decaying dc offset is added to the test signals suggested in the Standard and off-nominal frequency is considered. Procedure accuracy is analyzed through simulations by considering the maximum of the absolute value of the frequency error. Comparison with the frequency error thresholds proposed in the Standard is also performed.**

I. INTRODUCTION

When a fault occurs in power systems, current relaying signals often contain a decaying dc component. Since the spectrum of that component covers all frequencies, it may heavily affect the accuracy of phasor measurements returned by frequency-domain based algorithms. Various approaches have been proposed to overcome this issue [1]-[5]. Usually, the parameters of the decaying dc offset are firstly estimated and then used to remove that component from the analyzed electrical waveform. The proposed algorithms require only half or a full nominal cycle for decaying dc offset parameter estimation. Their main drawback is that they assume nominal waveform frequency, while this is often not the case when faults occur. Consequently, the estimated parameters are affected by the so-called picket-fence errors [6].

The Interpolated Discrete Fourier Transform (IpDFT) algorithm is capable to ensure accurate sinewave parameter estimates under off-nominal frequency condition [7]-[10]. Indeed, it compensates the picket-fence errors by estimating the inter-bin location of sinewave frequency. Classically, the estimation is performed by interpolating the two largest Discrete Fourier Transform (DFT) samples. Then sinewave amplitude and phase are estimated exploiting the knowledge of the estimated frequency. In addition, to reduce errors due to the spectral leakage from interfering tones, before performing frequency-domain interpolation, the acquired signal is weighted by means of a suitable window [11]. Unfortunately, frequency estimation accuracy of IpDFT algorithms can be heavily affected by a decaying dc offset due to its broad spectrum. In this paper an iterative IpDFT-based procedure that takes into account the contribution of that component is proposed. It is called iIpDFT-dc procedure and it exploits the decaying dc offset parameters estimation algorithm proposed in [12].

The accuracies of the iIpDFT-dc and the IpDFT procedures are then compared with each other under different steady-state and dynamic conditions considered in the Standard IEC/IEEE 60255-118.1:2018 – Synchrophasors for power systems – Measurements [13], called simply Standard in the following for the sake of brevity. Unlike the Standard, the test signals contain a decaying dc offset and off-nominal frequency occurs. The investigation is performed through computer simulations and the maximum of the Frequency Error (FE_{max}) absolute value is assumed to compare the algorithm accuracies.

II. THE iIPDFT-DC PROCEDURE

The acquired discrete-time electrical waveform is expressed as:

$$x(m+r) = x_s(m+r) + x_{dc}(m+r) = a\cos\left(2\pi \frac{f}{f_s}(m - M_h + \mathrm{r}) + \phi\right) + a_0 e^{-\frac{m-M_h+\mathrm{r}}{\tau \cdot f_s}},$$
$$m = 0, 1, \ldots, M-1\ ,\ r = m_0 + M_h, m_0 + M_h + 1, \ldots. \quad (1)$$

where f_s is the sampling rate, a, f, and ϕ are the amplitude, frequency, and phase of the sinewave $x_s(\cdot)$, a_0 and τ are the amplitude and the time constant of the decaying dc offset $x_{dc}(\cdot)$, and $M = 2M_h + 1$ is the acquisition length, which is assumed an odd number without loss of generality. It is worth noticing that the sample of index M_h corresponds to the middle of the observation interval. The fault causing the decaying dc offset is assumed to occur at the sampling instant m_0. The subsequent analyzed records of length M are updated sample by sample. They are identified by the index r.

The ratio between the nominal frequency f_n and the sampling rate f_s is chosen in such a way that:

$$\frac{f_n}{f_s} = \frac{J}{M-1}, \qquad (2)$$

where J is the number of analyzed cycles of a sinewave at nominal frequency. Due to possible off-nominal frequency, the ratio between the waveform frequency f and the sampling rate f_s can be written as in the following expression:

$$\frac{f}{f_s} = \frac{\nu}{M} = \frac{l+\delta}{M}, \qquad (3)$$

where $\nu = l + \delta$ is the number of acquired sinewave cycles, expressed as the summation of an integer part l and a fractional part δ representing the inter-bin frequency location.

The time constant τ is expressed as a fraction k of the waveform period $T = 1/f$, that is $\tau = T/k$.

The proposed iIpDFT-dc procedure weights the acquired signal (1) by using a suitable window $w(\cdot)$. Then the Discrete-Time Fourier Transform (DTFT) of the windowed signal $x_{ws}(m + r) = x_s(m + r)\cdot w(m)$ is evaluated:

$$X_{sw}(\lambda) \triangleq \sum_{m=0}^{M-1} x_{sw}(m+r) e^{-j2\pi\frac{\lambda}{M}(m-M_h+r)} \qquad (4)$$

The parameters of the sinewave are estimated by the classical IpDFT algorithm based on the Hann window [7]-[10], defined as:

$$w(m) = 0.5 + 0.5\cos\left(\frac{2\pi}{M}(m - M_h)\right), \qquad (5)$$

$$m = 0, 1, \dots M-1$$

The parameter estimators are given by the following expressions:

$$\hat{\nu} = l + \hat{\delta} = l + \frac{(1+i)\alpha - 2 + i}{\alpha + 1}, \qquad (6)$$

in which $\alpha = \frac{|X_{sw}(l+i)|}{|X_{sw}(l+i-1)|}$ with $i = 0$ if $|X_{sw}(l-1)| > |X_{sw}(l+1)|$ or $i = 1$ if $|X_{sw}(l-1)| < |X_{sw}(l+1)|$

$$\hat{a} = \frac{2|X_{sw}(l)|}{|W(-\hat{\delta})|}, \qquad (7)$$

where $W(\lambda) \cong \frac{M\cdot\sin(\pi\lambda)}{2\pi\lambda(1-\lambda^2)}$ is the DTFT of the Hann window, and

$$\hat{\phi} = \arg\{X_{sw}(l)\}. \qquad (8)$$

The DTFT of the decaying dc offset is equal to:

$$X_{dc}(\lambda) \triangleq \sum_{m=0}^{M-1} x_{dc}(m+r) e^{-j2\pi\frac{\lambda}{M}(m-M_h+r)} \qquad (9)$$

After some algebraic calculations, it results:

$$X_{dc}(\lambda) = a_0 e^{-k\frac{r-M_h}{M}\nu} \frac{1-e^{-j2\pi\lambda}\beta^M}{1-e^{-j\frac{2\pi}{M}\lambda}\beta} e^{-j\pi\frac{\lambda}{M}(r-M_h)}, \qquad (10)$$

where $\beta \triangleq e^{-\frac{k\nu}{M}}$.

From (10) the parameters of the decaying dc offset can be determined as follows [12]:

$$\beta = \frac{|X_{dc}(0) - X_{dc}(1)|}{\left|X_{dc}(0) - e^{-j\frac{2\pi}{M}} X_{dc}(1)\right|}, \qquad (11)$$

and

$$a_0 = \frac{1-\beta}{1-\beta^M} \beta^{-(r-M_h)} |X_{dc}(0)|. \qquad (12)$$

The proposed iIpDFT-dc procedure is based on the following steps:

step 1: initialize $\hat{x}_s(m+r) = x(m+r)$, $m = 0, 1, \dots, M-1$, $r = m_0 + M_h, m_0 + M_h + 1, \dots$

step 2: estimate the parameters of the sinewave $\hat{x}_s(\cdot)$ by means of the IpDFT algorithm based on the Hann window, using (6) – (8);

step 3: estimate the decaying dc offset contribution:

$$\hat{x}_{dc}(m+r) = x(m+r) - \hat{a}\cos\left(2\pi\frac{\hat{\nu}}{M}(m - M_h + r) + \hat{\phi}\right)$$

step 4: calculate the parameters $\hat{\beta}$ and $\hat{a}_0$ of the estimated decaying dc offset $\hat{x}_{dc}(\cdot)$, using (11) and (12)

step 5: estimate the sinewave contribution:

$$\hat{x}_s(m+r) = x(m+r) - \hat{a}_0\hat{\beta}^{(m+r-M_h)}$$

repeat steps 2-5 Q times

step 6: estimate the sinewave frequency $\hat{\nu}$ by applying the IpDFT algorithm based on the Hann window to the last estimated sinewave $\hat{x}_s(\cdot)$.

III. COMPUTER SIMULATIONS

In this Section the accuracies of the proposed iIpDFT-dc procedure and the IpDFT algorithm based on the Hann window are compared with each other considering the off-nominal frequency, harmonics, and modulation testing conditions suggested in the Standard. A decaying dc offset is added to the analyzed signal and off-nominal frequency is considered also in harmonics and modulation testing. The testing signals correspond to either *P-class* or *M-class* performance. The amplitudes of both sinewave and decaying dc offset are equal to 1 p.u. The nominal frequency is $f_n = 50$ Hz, and the sampling rate is $f_s = 6$ kHz. According to (2), 120 samples/nominal cycle are acquired. The reporting rate is $RR = 50$ readings/s. The fault occurs at $m_0 = 0$, *i.e.,* at the beginning of the $r = M_h$ analyzed record. $J = 4$ or 5 nominal cycles are observed, so that the record length is $M = 481$ or $M = 601$ samples, respectively. $Q = 2$ iterations are performed since no significant frequency estimation accuracy improvement was observed using a higher number of iterations. The related FE_{max} Standard thresholds are also shown in the presented figures.

A. Off-nominal frequency testing condition

Fig. 1 shows FE_{max} of both the IpDFT algorithm and the iIpDFT-dc procedure as a function of the waveform frequency for $J = 4$ and 5 nominal cycles when $\tau = T$. In addition, Fig. 2 shows the frequency errors of the considered methods when $r = 1000$ waveforms with $f = 48$ Hz and $J = 4$ nominal cycles are considered.

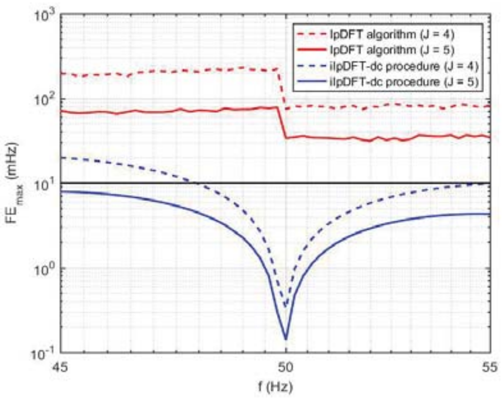


Fig. 1. Off-nominal frequency condition: FE_{max} of the IpDFT algorithm and the iIpDFT-dc procedure versus the waveform frequency when $J = 4$ and 5 nominal cycles and τ equal to the sinewave period T.

The proposed iIpDFT-dc procedure clearly outperforms the IpDFT algorithm since it is capable to significantly compensate the contribution of the decaying dc offset on the estimated frequency. Moreover, the results returned by the iIpDFT-dc procedure comply with the *P-class* or the *M-class* performance when $J = 4$ or $J = 5$ nominal cycles, respectively.

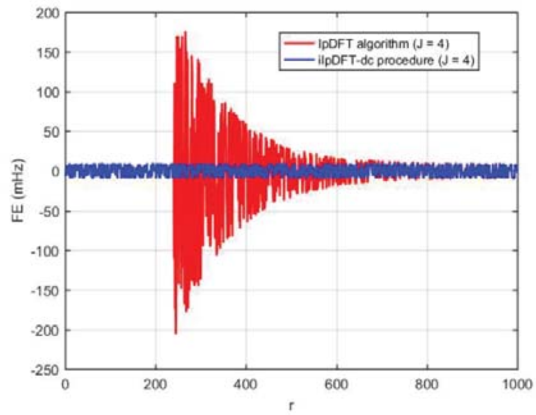


Fig. 2. Off-nominal frequency condition: frequency errors of the IpDFT algorithm and the iIpDFT-dc procedure versus the analyzed record index r when $f = 48$ Hz, $J = 4$ nominal cycles, and τ equal to the sinewave period T.

Fig. 3 shows the FE_{max} obtained with both the IpDFT algorithm and the iIpDFT-dc procedure as a function of the ratio τ/T when $f = 48$ Hz. As it can be seen, the accuracy of the estimates provided by the proposed iIpDFT-dc procedure are almost independent of the time constant τ, unlike the accuracy of the IpDFT algorithm.

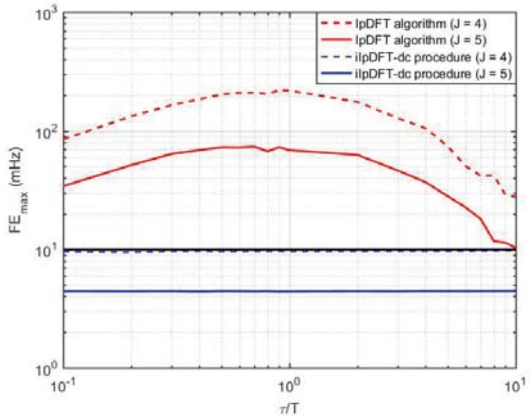


Fig. 3. Off-nominal frequency condition: FE_{max} of the IpDFT algorithm and the iIpDFT-dc procedure versus the ratio τ/T when $f = 48$ Hz and $J = 4$ and 5 nominal cycles.

B. Harmonics testing condition

When frequency-domain based estimation algorithms are used, the lower order harmonics, and in particular the 2nd order harmonic, are responsible for the most detrimental contributions to the estimated frequency [7], [10]. Thus, the effect of a 2nd harmonic on the results returned by the iIPDFT-dc procedure is analyzed in the following.

Fig. 4 shows the FE_{max} of both the IpDFT and the iIpDFT-dc procedures as a function of the waveform frequency for both considered values of the number of observed nominal cycles J when the electrical waveform is affected by a 2nd harmonic and $\tau = T$.

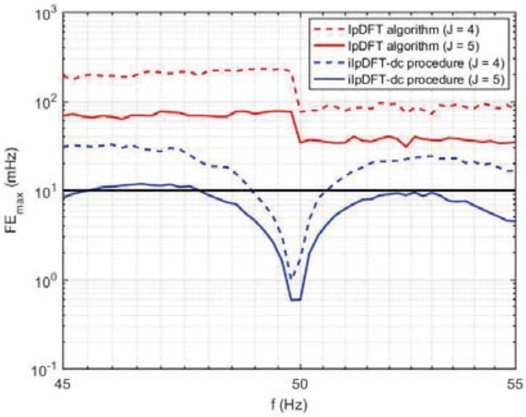


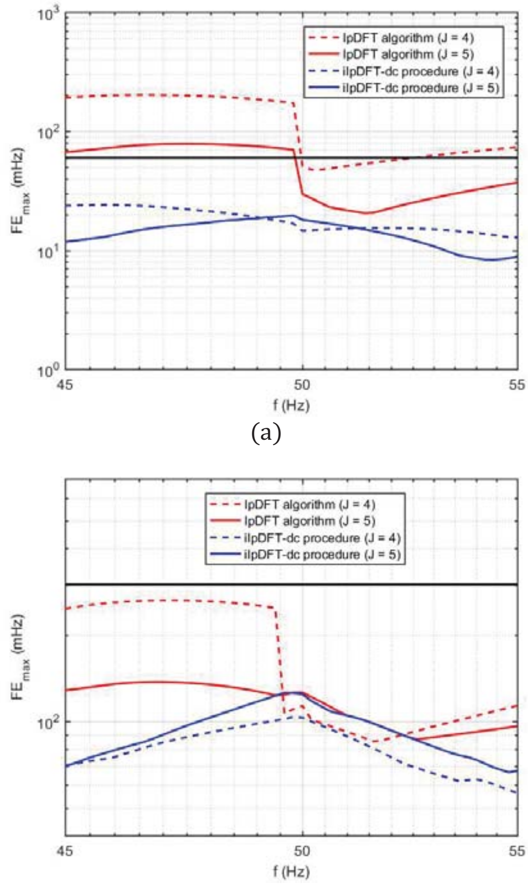
Fig. 4. Harmonics condition: FE_{max} of the IpDFT algorithm and the iIpDFT-dc procedure versus the waveform frequency when the electrical waveform is affected by a 2nd harmonic, $J = 4$ and 5 nominal cycles, and τ equal to the sinewave period T.

As it can be seen, the proposed iIpDFT-dc procedure provides more accurate frequency estimates than the IpDFT algorithm for both the considered values of J. When $J = 4$ nominal cycles, FE_{max} smaller than the 10 mHz Standard threshold is achieved when $f \in [49, 50.5]$ Hz. Conversely, when $J = 5$ nominal cycles, the Standard requirements are fulfilled in almost the whole considered frequency range. Conversely, the FE_{max} of the IpDFT algorithm is always worse than the Standard threshold.

Moreover, simulations showed that in this testing condition the FE_{max} values obtained by the iIpDFT-dc procedure are almost independent of the time constant τ when it takes values between 0.1 and 10 times the sinewave period.

C. Modulation testing condition

Simulations with electrical waveforms affected by either amplitude or phase modulations have been also performed. The amplitude and phase modulation indices were $k_a = 0.1$ and $k_p = 0.1$. Fig. 5 shows the FE_{max} of both the IpDFT algorithm and iIpDFT-dc procedure as a function of the waveform frequency when the waveforms are affected by both amplitude and phase modulation with frequency f_m equal to 2 Hz (Fig. 5(a)) and 5 Hz (Fig. 5(b)). The time constant of the decaying dc component τ is equal to the sinewave period T. Moreover, Fig. 6 shows the frequency errors returned by the considered procedures when $r = 1000$ waveforms with $f = 50$ Hz and $J = 4$ nominal cycles are considered.



(a)

(b)

Fig. 5. Modulation condition: FE_{max} of the IpDFT algorithm and the iIpDFT-dc procedure versus the waveform frequency when the modulation frequency is $f_m = 2$ Hz (a) or $f_m = 5$ Hz (b), $J = 4$ and 5 nominal cycles, and τ equal to the sinewave period T. Results obtained by considerring both amplitude and phase modulation are presented.

Fig. 5 shows that the proposed iIpDFT-dc procedure outperforms the IpDFT algorithm in most of the considered conditions. Moreover, it provides FE_{max} values smaller than the related thresholds (i.e., 60 mHz for the *P-class* performance and 300 mHz for the *M-class* performance) in the whole considered frequency range, even if only nominal frequency is considered in the Standard. In addition, for a given modulation frequency, the FE_{max} values of the iIpDFT-dc procedure

obtained by observing $J = 4$ or 5 nominal cycles are quite close to each other. Observe also that, the classical IpDFT algorithm complies only with the *M*-class Standard requirements when the whole frequency range is considered.
As in the previous testing conditions, the FE_{max} values obtained by the iIpDFT-dc procedure are almost independent of the time constant τ when it takes values between 0.1 and 10 times the sinewave period.

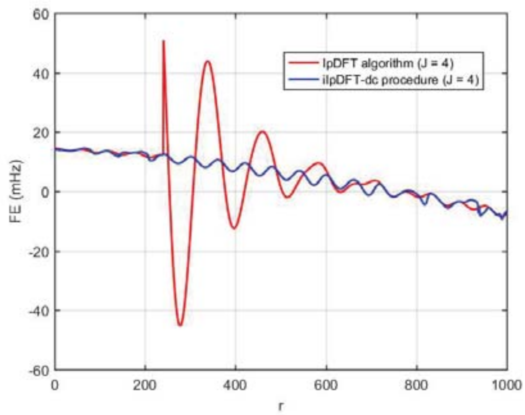


Fig. 6. Modulation condition: frequency errors of the IpDFT algorithm and the iIpDFT-dc procedure versus *r*. Modulation frequency $f_m = 2$ Hz, sinewave frequency $f = 50$ Hz, $J = 4$ nominal cycles, and τ equal to the sinewave period *T*.

Finally, Fig. 6 shows that, in the considered testing conditions, the contribution of the decaying dc component on the frequency estimates returned by the iIpDFT-dc procedure is almost negligible.

IV. CONCLUSIONS

The iIpDFT-dc procedure for frequency estimation of sinewave affected by a decaying dc offset has been proposed. It is based on the classical IpDFT algorithm, and it can be simply implemented. The accuracy of iIpDFT-dc procedure is almost independent of the decaying dc offset time constant. Under off-nominal frequency, harmonics, and modulation conditions the iIpDFT-dc frequency estimator complies with the Standard requirements for *P-class* or *M-class* performance when four or five nominal cycles are analyzed, respectively. The proposed procedure outperforms the classical IpDFT algorithm in all considered testing conditions.

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