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# The role of mediators in compensation negotiations between gainers and losers 

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#### Abstract

This article aims at modelling compensation between gainers and losers as a system of bilateral bargaining games involving one or two mediators when unanimity is required for the implementation of an economic policy. Results show that there is no unanimity on the choice of negotiation protocol due to the conflicting interests between gainers and losers. But if we assume that the mediators have the choice of the protocol, they always prefer simultaneous rather than stackeberg Nash-in-Nash negotiations.


## 1 Introduction

When the implementation of an economic policy or a project implies gainers and losers, the Pareto criterion is by definition not applicable. However it is always possible to refer to the criteria of Kaldor (1939) or Hicks (1939) which recommend the use of compensation tests based on the idea that the gainers have to compensate the losers and still remain better off than without the policy or project (Hindriks and Myles, 2013). But it is well known that this compensation remains hypothetical and does not require any effective payment of compensation. Once compensations have to be effective, then there is no fundamental difference between the compensation and the Pareto principles. Hence, from the injunction of the gainers to compensate the losers, the question now becomes how to implement this compensation scheme?

This note aims at modelling these compensation tests as a bargaining game between gainers and losers when unanimity is required for the implementation of an economic policy. Although different bargaining structures and protocols can be considered, we first consider a negotiation process in which a gainer or a loser playing the role of "mediator" is involved in several bilateral and simultaneous negotiations with all the other players. This mediator can also acts as a leader of stackelberg negotiating with the members of one group after having negotiated with the members of the other group. Then we assume two mediators, one for the gainers and another one for the losers which negotiate together. We used the Nash-in-Nash bargaining protocol to characterise the outcomes of the simultaneous and stackelberg negotiation protocols (CollardWexler et al., 2019).

Section 2 presents the model. Section 3 gives the outcome of the simultaneous and stackelberg negotiation schemes in the presence of one mediator and Section 4 with two mediators. Section 5 concludes.

## 2 The model

Consider a project, if it is implemented, will change positively the utility function depending on the consumption level of goods $x$ for some players, called the gainers denoted by $g=1, . ., G$ and negatively for the remaining players, the losers denoted by $l=1, \ldots, L$. It means that for the gainers, their utility is higher with the project than the initial situation

$$
U_{g}\left(x_{g}+\Delta_{g}\right)>U_{g}\left(x_{g}\right),
$$

with $\Delta_{g}>0 \forall g$ but it is the opposite for the losers

$$
U_{l}\left(x_{l}\right)>U_{l}\left(x_{l}-\Delta_{l}\right),
$$

with $\Delta_{l}>0 \forall l$. The utility functions are increasing, concave and to simplify quasi-linear with respect to a numéraire good denoted by $y$ used for compensation, such that for $i=g, l$

$$
\begin{equation*}
U_{i}\left(x_{i}, y_{i}\right)=v_{i}\left(x_{i}\right)+y_{i} \tag{1}
\end{equation*}
$$

where $v_{i}\left(x_{i}\right)$ is concave.
We assume that to be implemented the project needs the agreement of all the players. Such a condition implies that the gainers have to compensate the losers. In our case this compensation scheme is the outcome of a negotiation process.

For each player, we determine the maximum amount of transfer $t_{g}^{\max }$ that a gainer can give as compensation for a loser and the minimum amount of transfer $\tau_{l}^{\min }$ that a loser has to receive for compensation. These transfers are given by the compensating variations defined as the maximum (minimum) amount of income reduction (increase) in the final state to get the same utility in the initial state. For a gainer $g=1, \ldots, G$, we have

$$
U_{g}\left(x_{g}+\Delta_{g}, y_{g}-t_{g}^{\max }\right)=U\left(x_{g}, y_{g}\right),
$$

and for a loser $l=1, \ldots, L$

$$
U_{l}\left(x_{l}-\Delta_{l}, y_{l}+\tau_{l}^{\min }\right)=U\left(x_{l}, y_{l}\right) .
$$

Using the individual utility function (1), these two conditions implies

$$
\begin{aligned}
t_{g}^{\max } & =v\left(x_{g}+\Delta_{g}\right)-v\left(x_{g}\right)>0, \\
\tau_{l}^{\min } & =v\left(x_{l}\right)-v\left(x_{l}-\Delta_{l}\right)>0 .
\end{aligned}
$$

These transfers $t_{g}^{\max }$ and $\tau_{l}^{\min }$ can also be interpreted as reservation prices. It implies that the transfer offer by a gainer $t_{g}$ will be lower or equal than $t_{g}^{\max }$ and the transfer received by a loser $\tau_{l}$ higher or equal than $\tau_{l}^{\min }$.

The project will increase social welfare if implemented if and only if the net surplus is positive

$$
\begin{equation*}
\Pi=\sum_{g=1}^{G} t_{g}^{\max }-\sum_{l=1}^{L} \tau_{l}^{\min }>0 \tag{2}
\end{equation*}
$$

which implies

$$
\sum_{g=1}^{G}\left(v\left(x_{g}+\Delta_{g}\right)-v\left(x_{g}\right)\right)>\sum_{l=1}^{L}\left(v\left(x_{l}\right)-v\left(x_{l}-\Delta_{l}\right)\right) .
$$

This paper aims at implementing decentralized negotiation procedures with one and two mediators. All negotiations are simultaneous and bilateral. Firstly we assume that one gainer (or one loser), say $k$ (or $p$ ), negotiate on the behalf of the other gainers (losers) with all the losers (gainers). A mediator gainer will negotiate with all the losers an amount of compensations denoted by $\tau$ and at the same time the transfers $t$ the gainers have to pay to him. A mediator loser will negotiate with the other losers the transfer $\tau$ he will pay to them and with the gainers the received transfers $t$. Secondly we consider two mediators, one gainer $k$ and one loser $p$, which negotiate over $t_{k p}$ together after having negotiated or not with the respective members of their groups.

When the project is implemented, the payoff of a gainer $g \neq k$ not involved in the negotiation with the losers is given by

$$
\begin{equation*}
U_{g}=v_{g}\left(x_{g}+\Delta_{g}\right)+y_{g}-t_{g} \tag{3}
\end{equation*}
$$

while the payoff of a loser $l \neq p$ is

$$
\begin{equation*}
U_{l}=v_{l}\left(x_{l}-\Delta_{l}\right)+y_{l}+\tau_{l} . \tag{4}
\end{equation*}
$$

When the negotiation involves one mediator gainer $k$ his payoff is

$$
\begin{equation*}
U_{k}=v_{k}\left(x_{k}+\Delta_{k}\right)+y_{k}-\sum_{l=1}^{L} \tau_{l}+\sum_{g=1}^{G-1} t_{g} \tag{5}
\end{equation*}
$$

and for a mediator loser $p$ his payoff is

$$
\begin{equation*}
U_{p}=v_{p}\left(x_{p}-\Delta_{p}\right)+y_{p}-\sum_{l=1}^{L-1} \tau_{l}+\sum_{g=1}^{G} t_{g} \tag{6}
\end{equation*}
$$

With two mediators their payoff functions are

$$
\begin{align*}
U_{k} & =v_{k}\left(x_{k}+\Delta_{k}\right)+y_{k}+\sum_{g=1}^{G-1} t_{g}-t_{k p}  \tag{7}\\
U_{p} & =v_{p}\left(x_{p}-\Delta_{p}\right)+y_{p}-\sum_{l=1}^{L-1} \tau_{l}+t_{k p} \tag{8}
\end{align*}
$$

The next section aims at determining how the compensating transfers are the result of negotiation process initiated by one mediator.

## 3 One mediator

### 3.1 Simultaneous negotiation

Consider a double simultaneous negotiation process between a mediator gainer $k$ with the $L$ losers and also with the remaining $G-1$ gainers. We use the Nash Bargaining Solution (NBS) to solve the negotiation process. The bargaining power of the different players are such that $\sum_{i=1}^{L} \alpha_{l}+\sum_{g=1}^{G-1} \alpha_{g}+\alpha_{k}=1$.

The NBS between $k$ and $l$ is given by the Nash Bargaining Product (NBP) of the net payoffs

$$
\arg \max _{\tau_{l}} N B P_{k, l}=\left(t_{k}^{\max }-\sum_{l=1}^{L} \tau_{l}+\sum_{g=1}^{G-1} t_{g}\right)^{\alpha_{k}}\left(\tau_{l}-\tau_{l}^{\min }\right)^{\alpha_{l}}
$$

The net payoffs are obtained as the difference of the payoffs in case of agreement, (5) for $k$ and (4) for $l$, and their payoffs in case of disagreement. Since we
assume that the agreement of all the players is necessary for the projet to be implemented, the disagreement payoffs are simply the initial payoffs without the project given by (1).

First-order condition yields $\forall l$

$$
\tau_{l}=\tau_{l}^{\min }+\frac{\alpha_{l}}{\alpha_{k}}\left(t_{k}^{\max }-\sum_{l=1}^{L} \tau_{l}+\sum_{g=1}^{G-1} t_{g}\right)
$$

Taking the sum over the $L$ losers gives

$$
\begin{equation*}
\sum_{l=1}^{L} \tau_{l}=\frac{\alpha_{k}}{\alpha_{k}+\sum_{i=1}^{L} \alpha_{l}} \sum_{l=1}^{L} \tau_{l}^{\min }+\frac{\sum_{i=1}^{L} \alpha_{l}}{\alpha_{k}+\sum_{i=1}^{L} \alpha_{l}}\left(t_{k}^{\max }+\sum_{g=1}^{G-1} t_{g}\right) \tag{9}
\end{equation*}
$$

Substitute in the above expression of $\tau_{l}$ gives

$$
\begin{equation*}
\tau_{l}=\tau_{l}^{\min }+\frac{\alpha_{l}}{\alpha_{k}+\sum_{i=1}^{L} \alpha_{l}}\left(t_{k}^{\max }+\sum_{g=1}^{G-1} t_{g}-\sum_{i=1}^{L} \tau_{l}^{\min }\right) . \tag{10}
\end{equation*}
$$

Now when gainer $k$ negotiates with the other $G-1$ gainers, $k$ takes as given the transfers paid to the losers $\sum_{l=1}^{L} \tau_{l}$.

The NBS between $k$ and $g$ is solution of

$$
\arg \max _{t_{g}} N B P_{k, g}=\left(t_{k}^{\max }-\sum_{l=1}^{L} \tau_{l}+\sum_{g=1}^{G-1} t_{g}\right)^{\alpha_{k}}\left(t_{g}^{\max }-t_{g}\right)^{\alpha_{g}} .
$$

First order conditions are $\forall g$

$$
t_{g}=t_{g}^{\max }-\frac{\alpha_{g}}{\alpha_{k}}\left(t_{k}^{\max }-\sum_{l=1}^{L} \tau_{l}+\sum_{g=1}^{G-1} t_{g}\right)
$$

Taking the sum gives over the $G-1$ gainers gives

$$
\begin{equation*}
\sum_{g=1}^{G-1} t_{g}=\frac{\alpha_{k}}{1-\sum_{l=1}^{L} \alpha_{l}} \sum_{g=1}^{G-1} t_{g}^{\max }-\frac{\sum_{g=1}^{G-1} \alpha_{g}}{1-\sum_{l=1}^{L} \alpha_{l}}\left(t_{k}^{\max }-\sum_{l=1}^{L} \tau_{l}\right) . \tag{11}
\end{equation*}
$$

Substitute in the expression of $t_{g}$ gives

$$
\begin{equation*}
t_{g}=t_{g}^{\max }-\frac{\alpha_{g}}{1-\sum_{l=1}^{L} \alpha_{l}}\left(\sum_{g=1}^{G} t_{g}^{\max }-\sum_{l=1}^{L} \tau_{l}\right) \tag{12}
\end{equation*}
$$

After solving the system (9)-(11) and after substituting in (10) and (12), we obtain the individual transfers

$$
\begin{align*}
t_{g}^{*} & =t_{g}^{\max }-\alpha_{g} \Pi>0  \tag{13}\\
\tau_{l}^{*} & =\tau_{l}^{\min }+\alpha_{l} \Pi
\end{align*}
$$

### 3.2 Stackelberg negotiation

We assume that $k$ acts as a leader of stackelberg when he negotiates with the gainers after having negotiated with the losers. It implies that $k$ takes into account the total amount of transfers (9) he has given to them in his negotiation with each gainer $g$ characterized by his payoff function (3). Substitute (9) in the payoff (5) of $k$ yields
$U_{k}=v_{k}\left(x_{k}+\Delta_{k}\right)+y_{k}+\frac{\alpha_{k}}{1-\sum_{g=1}^{G-1} \alpha_{g}}\left(\sum_{g=1}^{G-1} t_{g}-\sum_{l=1}^{L} \tau_{l}^{\min }\right)-\frac{\sum_{i=1}^{L} \alpha_{l}}{1-\sum_{g=1}^{G-1} \alpha_{g}} t_{k}^{\max }$.
The NBS when $k$ negotiates with $g$ is solution of
$\arg \max _{t_{g}} N B P_{k, g}=\left(\frac{\alpha_{k}}{1-\sum_{g=1}^{G-1} \alpha_{g}}\right)^{\alpha_{k}}\left(t_{k}^{\max }-\sum_{l=1}^{L} \tau_{l}^{\min }+\sum_{g=1}^{G-1} t_{g}\right)^{\alpha_{k}}\left(t_{g}^{\max }-t_{g}\right)^{\alpha_{g}}$.
We obtain the individual transfers

$$
\begin{align*}
\tau_{l}^{*} & =\tau_{l}^{\min }+\frac{\alpha_{l} \alpha_{k}}{\left(1-\sum_{l=1}^{L} \alpha_{l}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi \\
t_{g}^{*} & =t_{g}^{\max }-\frac{\alpha_{g}}{1-\sum_{l=1}^{L} \alpha_{l}} \Pi>0 \tag{14}
\end{align*}
$$

We now assume that $k$ acts as a leader of stackelberg when he negotiates with the losers after having negotiated with the gainers. In this symmetric configuration, $k$ takes into account the transfers (11) he received from the gainers when he negotiates with the losers. The payoff of $k$ is
$U_{k}=v_{k}\left(x_{k}+\Delta_{k}\right)+y_{k}+\frac{\alpha_{k}}{1-\sum_{l=1}^{L} \alpha_{l}} \sum_{g=1}^{G-1} t_{g}^{\max }-\frac{\sum_{g=1}^{G-1} \alpha_{g}}{1-\sum_{l=1}^{L} \alpha_{l}}\left(t_{k}^{\max }-\sum_{l=1}^{L} \tau_{l}\right)$,
The NBS when $k$ negotiates with $l$ is solution of

$$
\arg \max _{\tau_{l}} N B P_{k, l}=\left(\frac{\alpha_{k}}{1-\sum_{l=1}^{L} \alpha_{l}}\right)^{\alpha_{k}}\left(\sum_{g=1}^{G} t_{g}^{\max }-\sum_{l=1}^{L} \tau_{l}\right)^{\alpha_{k}}\left(\tau_{l}-\tau_{l}^{\min }\right)^{\alpha_{l}} .
$$

We obtain the individual transfers

$$
\begin{align*}
\tau_{l}^{*} & =\tau_{l}^{\min }+\frac{\alpha_{l}}{\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi \\
t_{g}^{*} & =t_{g}^{\max }-\frac{\alpha_{k} \alpha_{g}}{\left(1-\sum_{l=1}^{L} \alpha_{l}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi>0 \tag{15}
\end{align*}
$$

The net payoffs in these different configurations with a mediator gainer $k$ are given in Table (1). In a symmetric way, the net payoffs with a mediator loser $p$ are given in Table (2).

We obtain the following propositions

|  | SIM | $k$ leader with losers | $k$ leader with gainers |
| :---: | :---: | :--- | :--- |
| $U_{g}^{*}$ | $\alpha_{g} \Pi$ | $\frac{\alpha_{g}}{1-\sum_{l=1}^{L} \alpha_{l}} \Pi$ | $\frac{\alpha_{k} \alpha_{g}}{\left(1-\sum_{l=1}^{L} \alpha_{l}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi$ |
| $U_{k}^{*}$ | $\alpha_{k} \Pi$ | $\frac{\alpha_{k}^{2}}{\left(1-\sum_{l=1}^{L} \alpha_{l}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi$ | $\frac{\alpha_{k} \alpha_{l}}{\left(1-\sum_{l=1}^{L} \alpha_{l}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi$ |
| $U_{l}^{*}$ | $\alpha_{l} \Pi$ | $\frac{\alpha_{l}}{\left(1-\sum_{l=1}^{L} \alpha_{l}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi$ | $\frac{1-\sum_{g=1}^{G-1} \alpha_{g}}{1}{ }^{2}$ |

Table 1: Net payoffs with a gainer mediator.

|  | SIM | $p$ leader with losers | $p$ leader with gainers |
| :--- | :--- | :--- | :--- |
| $U_{g}^{*}$ | $\alpha_{g} \Pi$ | $\frac{\alpha_{g}}{1-\sum_{l=1}^{L-1} \alpha_{l}} \Pi$ | $\frac{\alpha_{p} \alpha_{g}}{\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)\left(1-\sum_{g=1}^{G} \alpha_{g}\right)}$ |
| $U_{p}^{*}$ | $\alpha_{p} \Pi$ | $\frac{\alpha_{p}^{2}}{\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)\left(1-\sum_{g=1}^{G} \alpha_{g}\right)} \Pi$ | $\frac{\alpha_{p} \alpha_{l}}{\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)\left(1-\sum_{g=1}^{G} \alpha_{g}\right)} \Pi$ |
| $U_{l}^{*}$ | $\alpha_{l} \Pi$ | $\frac{\alpha_{l}}{\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)\left(1-\sum_{g=1}^{G} \alpha_{g}\right)} \Pi$ | $\frac{\alpha_{l}}{1-\sum_{g=1}^{G} \alpha_{g}} \Pi$ |

Table 2: Net payoffs with a loser mediator.

Proposition 1 The comparison of the payoffs in the bargaining procedures shows that

- The gainers always prefer that the mediator, either gainer or loser, negotiates as a leader of stackelberg with the losers,
- The losers always prefer that the mediator, either gainer or loser, negotiates as a leader of stackelberg with the gainers,
- The mediator, either gainer or loser, always prefers the simultaneous Nash-in-Nash bargaining.

Gainers are in a better bargaining position when the mediator has to take into account in his utility function the transfers he has to give to losers since gainers will have to pay lower compensations. Positivity condition $t_{g}>0$ in (13), (14) and (15) implies different values of $t_{g}^{\max }$ which can be ranked as follows:

$$
t_{g}^{\max }>\frac{\alpha_{g}}{1-\sum_{l=1}^{L} \alpha_{l}} \Pi>\alpha_{g} \Pi>\frac{\alpha_{k} \alpha_{g}}{\left(1-\sum_{l=1}^{L} \alpha_{l}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi
$$

Compensations given by (14) are the lowest but the highest with (15).
Corollary 1 If we assume that the mediator has the choice of the bargaining protocol, he then will choose the simultaneous Nash-in-Nash bargaining.

## 4 Two mediators

We now consider two mediators, one gainer $k$ with payoff function (7) and one loser $p$ with payoff (8), which will negotiate together after having negotiated or not with the respective members of their groups. The bargaining power of the different players are such that $\alpha_{k}+\alpha_{p}+\sum_{g=1}^{G-1} \alpha_{g}+\sum_{l=1}^{L-1} \alpha_{l}=1$.

### 4.1 Simultaneous negotiation

We assume that $k$ bargains with the other gainers and $p$ with the other losers but when $k$ and $p$ bargains they take as given what they have to take or give to their members. In the negotiation between $k$ and $g$, the NBS is solution of

$$
\arg \max _{t_{g}} N B P_{k, g}=\left(t_{k}^{\max }+\sum_{g=1}^{G-1} t_{g}-t_{k p}\right)^{\alpha_{k}}\left(t_{g}^{\max }-t_{g}\right)^{\alpha_{g}}
$$

The individual and total amount of transfers are

$$
\begin{align*}
t_{g} & =t_{g}^{\max }-\frac{\alpha_{g}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}}\left(\sum_{g=1}^{G} t_{g}^{\max }-t_{k p}\right), \\
\sum_{g=1}^{G-1} t_{g} & =\frac{\alpha_{k}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}} \sum_{g=1}^{G-1} t_{g}^{\max }-\frac{\sum_{g=1}^{G-1} \alpha_{g}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}}\left(t_{k}^{\max }-t_{k p}\right) . \tag{16}
\end{align*}
$$

In the negotiation between $p$ and $l$, the NBS is solution of

$$
\arg \max _{\tau_{l}} N B P_{p, l}=\left(t_{k p}-\tau_{p}^{\min }-\sum_{l=1}^{L-1} \tau_{l}\right)^{\alpha_{p}}\left(\tau_{l}-\tau_{l}^{\min }\right)^{\alpha_{l}}
$$

The individual and total amount of transfers are

$$
\begin{align*}
\tau_{l} & =\tau_{l}^{\min }+\frac{\alpha_{l}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}}\left(t_{k p}-\sum_{l=1}^{L} \tau_{l}^{\min }\right) \\
\sum_{l=1}^{L-1} \tau_{l} & =\frac{\alpha_{p}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}} \sum_{l=1}^{L-1} \tau_{l}^{\min }+\frac{\sum_{l=1}^{L-1} \alpha_{l}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}}\left(t_{k p}-\tau_{p}^{\min }\right) . \tag{17}
\end{align*}
$$

In the bilateral negotiation between $k$ and $p$, the NBS is solution of

$$
\arg \max _{t_{k p}} N B P_{k, p}=\left(t_{k}^{\max }+\sum_{g=1}^{G-1} t_{g}-t_{k p}\right)^{\alpha_{k}}\left(t_{k p}-\tau_{p}^{\min }-\sum_{l=1}^{L-1} \tau_{l}\right)^{\alpha_{p}}
$$

The negotiated transfer $t_{k p}$ is

$$
t_{k p}^{*}=\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right) \sum_{g=1}^{G} t_{g}^{\max }+\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right) \sum_{l=1}^{L} \tau_{l}^{\min }
$$

and the individual transfers are for $g=1, \ldots, G$ and $l=1, \ldots, L$

$$
\begin{align*}
t_{g}^{*} & =t_{g}^{\max }-\alpha_{g} \Pi>0  \tag{18}\\
\tau_{l}^{*} & =\tau_{l}^{\min }+\alpha_{l} \Pi
\end{align*}
$$

### 4.2 Stackleberg negotiation for the mediator gainer

We assume now that the mediator gainer take into account the total amount of transfers received by the remaining gainers in his utility function. Substitution of (16) in the utility function of $k(7)$ yields
$U_{k}=v_{k}\left(x_{k}+\Delta_{k}\right)+y_{k}+\frac{\alpha_{k}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}} \sum_{g=1}^{G-1} t_{g}^{\max }-\frac{\sum_{g=1}^{G-1} \alpha_{g}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}} t_{k}^{\max }-\frac{\alpha_{k}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}} t_{k p}$.
The NBS between $k$ and $p$ is solution of $\arg \max _{t_{k p}} N B P_{k, p}=\left(\frac{\alpha_{k}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}}\right)^{\alpha_{k}}\left(\sum_{g=1}^{G} t_{g}^{\max }-t_{k p}\right)^{\alpha_{k}}\left(t_{k p}-\sum_{l=1}^{L-1} \tau_{l}-\tau_{p}^{\min }\right)^{\alpha_{p}}$.

The negotiated transfer $t_{k p}$ is

$$
t_{k p}^{*}=\frac{\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right)}{\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \sum_{g=1}^{G} t_{g}^{\max }+\frac{\alpha_{k}}{\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \sum_{l=1}^{L} \tau_{l}^{\min }
$$

with the individual transfers

$$
\begin{align*}
t_{g}^{*} & =t_{g}^{\max }-\frac{\alpha_{g} \alpha_{k}}{\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi>0  \tag{20}\\
\tau_{l}^{*} & =\tau_{l}^{\min }+\frac{\alpha_{l}}{1-\sum_{g=1}^{G-1} \alpha_{g}} \Pi .
\end{align*}
$$

### 4.3 Stackleberg negotiation for the mediator loser

Substituting (17) in the utility function of $p$ given by (8) gives
$U_{p}=v_{p}\left(x_{p}-\Delta_{p}\right)+y_{p}+\frac{\alpha_{p}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}} t_{k p}-\frac{\alpha_{p}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}} \sum_{l=1}^{L-1} \tau_{l}^{\min }+\frac{\sum_{l=1}^{L-1} \alpha_{l}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}} \tau_{p}^{\min }$.
The NBS is solution of
$\arg \max _{t_{k p}} N B P_{k, p}=\left(\frac{\alpha_{p}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}}\right)^{\alpha_{p}}\left(\sum_{g=1}^{G-1} t_{g}+t_{k}^{\max }-t_{k p}\right)^{\alpha_{k}}\left(t_{k p}-\sum_{l=1}^{L} \tau_{l}^{\min }\right)^{\alpha_{p}}$.

The negotiated transfer $t_{k p}$ is

$$
t_{k p}^{*}=\frac{\alpha_{p}}{\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)} \sum_{g=1}^{G} t_{g}^{\max }+\frac{\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)}{\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)} \sum_{l=1}^{L} \tau_{l}^{\min }
$$

with the individual transfers

$$
\begin{align*}
t_{g}^{*} & =t_{g}^{\max }-\frac{\alpha_{g}}{1-\sum_{l=1}^{L-1} \alpha_{l}} \Pi>0  \tag{22}\\
\tau_{l}^{*} & =\tau_{l}^{\min }+\frac{\alpha_{l} \alpha_{p}}{\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right)\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)} \Pi
\end{align*}
$$

### 4.4 Stackleberg negotiation for both mediators

In that last case the utility functions of the two mediators are given by (19) and (21). The NBS is solution of
$\arg \max _{t_{k p}} N B P_{k, p}=\left(\frac{\alpha_{p}}{\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}}\right)^{\alpha_{p}}\left(\frac{\alpha_{k}}{\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}}\right)^{\alpha_{k}}\left(\sum_{g=1}^{G} t_{g}^{\max }-t_{k p}\right)^{\alpha_{k}}\left(t_{k p}-\sum_{l=1}^{L} \tau_{l}^{\min }\right)^{\alpha_{p}}$.
The negotiated transfer $t_{k p}$ is

$$
t_{k p}^{*}=\frac{\alpha_{p}}{\alpha_{k}+\alpha_{p}} \sum_{g=1}^{G} t_{g}^{\max }+\frac{\alpha_{k}}{\alpha_{k}+\alpha_{p}} \sum_{l=1}^{L} \tau_{l}^{\min }
$$

with the individual payoffs

$$
\begin{align*}
t_{g}^{*} & =t_{g}^{\max }-\frac{\alpha_{g} \alpha_{k}}{\left(\alpha_{k}+\alpha_{p}\right)\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi>0  \tag{23}\\
\tau_{l}^{*} & =\tau_{l}^{\min }+\frac{\alpha_{l} \alpha_{p}}{\left(\alpha_{k}+\alpha_{p}\right)\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right)} \Pi
\end{align*}
$$

The net payoffs in the different configurations with two mediators are given in Table (3)

We obtain the following proposition.
Proposition 2 The comparison of the payoffs in the bargaining procedures shows that

- The gainers including the mediator gainer always prefer when the mediator loser acts as a leader of stackelberg,
- The losers including the mediator loser always prefer when the mediator gainer acts as a leader of stackelberg.

|  | SIM | $k$ leader | $p$ leader | $k$ and $p$ leaders |
| :---: | :---: | :--- | :--- | :--- |
| $U_{g}^{*}$ | $\alpha_{g} \Pi$ | $\frac{\alpha_{g} \alpha_{k}}{\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi$ | $\frac{\alpha_{g}}{1-\sum_{l=1}^{L-1} \alpha_{l}} \Pi$ | $\frac{\alpha_{g} \alpha_{k}}{\left(\alpha_{k}+\alpha_{p}\right)\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi$ |
| $U_{k}^{*}$ | $\alpha_{k} \Pi$ | $\frac{\alpha_{k}^{2}}{\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi$ | $\frac{\alpha_{k}}{1-\sum_{l=1}^{L-1} \alpha_{l}} \Pi$ | $\frac{\alpha_{k}^{2}}{\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)\left(\alpha_{k}+\alpha_{p}\right)} \Pi$ |
| $U_{p}^{*}$ | $\alpha_{p} \Pi$ | $\frac{\alpha_{p}}{1-\sum_{g=1}^{G-1} \alpha_{g}} \Pi$ | $\frac{\alpha_{p}^{2}}{\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right)\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)} \Pi$ | $\frac{\alpha_{l} \alpha_{p}}{\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right)\left(\alpha_{k}+\alpha_{p}\right)} \Pi$ |
| $U_{l}^{*}$ | $\alpha_{l} \Pi$ | $\frac{\alpha_{l}}{1-\sum_{g=1}^{G-1} \alpha_{g}} \Pi$ | $\frac{\alpha_{l} \alpha_{p}}{\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right)\left(1-\sum_{l=1}^{L-1} \alpha_{l}\right)} \Pi$ | $\frac{\left(\alpha_{p}+\sum_{l=1}^{L-1} \alpha_{l}\right)\left(\alpha_{k}+\alpha_{p}\right)}{1}$ |

Table 3: Net payoffs with two mediators.

Gainers are in a better bargaining position when the mediator loser acts as a leader of stackelberg since the transfers given by (22) they will have to pay are the lowest and the highest for transfers (23). Positivity condition $t_{g}>0$ in (18), (20), (22) and (23) shows

$$
t_{g}^{\max }>\frac{\alpha_{g}}{1-\sum_{l=1}^{L-1} \alpha_{l}} \Pi>\alpha_{g} \Pi>\frac{\alpha_{g} \alpha_{k}}{\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)\left(1-\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi>\frac{\alpha_{g} \alpha_{k}}{\left(\alpha_{k}+\alpha_{p}\right)\left(\alpha_{k}+\sum_{g=1}^{G-1} \alpha_{g}\right)} \Pi .
$$

Corollary 2 The outcome of the game when the two mediators negotiate simultaneously is the unique Nash equilibrium.

It can be shown that for both mediators, negotiating simultaneously is a dominant strategy rather than negotiating as a leader of stackelberg.

## 5 Conclusion

This article shows how a compensation mechanism can be negotiated by mediators coming from the gainer or loser groups when unanimity is required to implement a project or an economic policy. With one mediator our results show that there is no unanimity on the choice of negotiation procedure but when the mediator, either or loser, has the choice of the protocol, he will prefer simultaneous negotiation. With two mediators, we show that negotiating simultaneously is the unique Nash equilibrium of the bilateral bargaining game between the two mediators.

Assuming a strict compensation for all the losers will change the nature of the game even with unanimity, allowing some gainers to free ride to the compensation scheme and becoming pivotal negotiators (Raskovich, 2003). Analysing the consequences of a majority rule instead of unanimity would require more complex bargaining structures assuming random selection of the players in legislature multilateral bargaining game (Baron and Ferejohn, 1989) or in coalitional Nash Bargaining (Okada, 1996; Compte and Jehiel, 2010).

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