State and output feedback control for Lipschitz nonlinear systems in reciprocal state space: synthesis and real-time validation

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ABSTRACT

In industrials process and many studies cases, state and output derivative variables can be easily modeled in reciprocal state space (RSS) form than the standard one. Formulation of stabilization problem with a guaranteed cost control using feedback principle for Lipschitz nonlinear systems (LNS) in RSS is presented in this brief. The asymptotic stability, using the proper Lyapunov functions of the closed-loop system, is guaranteed. The control design problem is guaranteed through a resolution of linear matrix inequalities (LMI) technique under certain lemmas and minimization of non-standard cost control. Experimental validation shows the good performances of the proposed method using real-time implementation (RTI) with a digital signal processing (DSP) device (Arduino MEGA 2560).

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NOMENCLATURE

- I_s is the identity matrix of dimension "s"
- Q^T is the transposed matrix of Q
- The notation $(*)$ is used for the matrix blocks induced by symmetry
- Q is a square matrix, then Q>0 (Q<0) defines that Q is positive definite (negative definite)

1. INTRODUCTION

The problems of monitoring and controlling the design of nonlinear systems have been exploited in many engineering areas using the state and output feedback principle [1]–[3] in standard state space (SSS) form. The synthesis problem of nonlinear control laws received remarkable attention with solid synthesis approaches: Tracking control with parametric uncertainties [4]; fuzzy and predictive control [5]; robust finite-time controller using sliding mode strategy [6]; backstepping controller using extended Kalman filter [7]; adaptive control using neural networks with input delay [8]; integral backstepping [9]; and feedback finite-time control for second-order systems [10].

In this context, the case of the Lipschitz nonlinear systems (LNS) in the SSS has been treated by many researchers considering their feasibility in real-time applications with different digital signal processors (DSP) cards: stabilization of one link flexible joint robot using sliding window of measurements with Arduino MEGA board [11]. Feedback stabilization using differential mean value theorem (DMVT) principle with Arduino UNO R3 board [12]. Tracking problem of the flux and the speed of induction motor using STM32 board [13]. Despite the important literature on approaches proposed in SSS (using state variables), many limitations exist. Effectively, many engineering systems and applications give measurements and data covering their basic physical phenomena or their operating principles according to derivatives state: photovoltaic systems [14]; mobile robots [15]; active suspension systems [16], [17]; and electronic impulse systems [18], [19].

This leads to the state derivatives being easily obtained in some engineering systems. Therefore, several sections have recently applied state derivative feedback controllers using the RSS reformulation. Effectively, derivative feedback principle has been addressed in many control problems for linear systems : static output feedback control [15]; robust control subject fault [16]; feedback control in linear polytypic systems [17]; pole placement for single-input single-output (SISO) linear systems [20]; sliding mode control [21]; linear optimal control [22], and linear quadratic regulator (LQR) [23]. Besides, many reliable methods and technics are used in the synthesis. For example, LQR design in the RSS framework is proposed by algebraic Riccati equations (ARE) in [24]. For descriptor systems, ARE. and linear matrix inequalities (LMI)-based solutions of LQR with state derivative feedback are developed in [25].

Similarly, for the case of nonlinear systems, recent works dealing with several control methods based on Lyapunov stability have been synthesized, such as H_∞ fuzzy state feedback [14]; adaptive stabilization and tracking control [19]; state feedback stabilization of one-sided Lipschitz systems [26]; and sliding mode control for Lipschitz systems[27].

Moreover, the studies on the derivative Lipschitz nonlinearity [18], [19] are few in control design problems. As well as for the case of control by output feedback in the RSS and treating the Lipschitz nonlinear systems is not treated to the authors' knowledge. For all these reasons and motivated by the latest results of [19], [22] and to overcome these limitations, this paper proposes the design of a unified approach using the feedback principle with output/state derivatives for LNS. The main contribution of this brief is based on a combination of the guaranteed cost control problem design with the use of the Lipschitz condition in RSS. The proposed design method (using RSS feedback principle with output/state derivatives) is based on Lyapunov theory to guarantee the closed-loop stability. The control design resolution problem is ensured under certain lemmas and through LMI technique with guaranteed cost control.

This brief is organized as follows: First, formulation of problems statement with useful preliminaries are presented. Second, the synthesis method of control law design with guaranteed cost control to NLS in RSS is detailed. Finally, the experimental validations are presented which demonstrate the performance of the proposed results tusing an RTI based on a DSP device (Arduino MEGA 2560) that is used as H.I.L.

2. PROBLEM STATEMENT AND PRELIMINARIES

Based on the works of [15], [17], the NLS in RSS is described by:

$$\begin{cases} x = \bar{A}\dot{x} + \bar{B}u + \bar{f}(\dot{x}) \\ \dot{y} = \bar{C}\dot{x} \end{cases} \tag{1}$$

with:

$$\|\bar{f}(\dot{x})\| \leq \bar{\delta}\|\dot{x}\| \tag{2}$$

where $x \in \mathbb{R}^n$: state vector; $u \in \mathbb{R}^m$: input vector; $y \in \mathbb{R}^p$: output vector. $\bar{A}, \bar{B}, \bar{C}$ are constant matrices of appropriate dimensions. $\bar{f}(x) \in \mathbb{R}^{n\times m}$ is the nonlinear vector which satisfies the Lipschitz propriety (where $\bar{\delta}$ is the Lipschitz constant). Throughout this paper, the following proposals and lemmas will be considered:

Assumption 1 :[22]
Given two performance weighting matrices $\bar{Q} = \bar{Q}^T > 0$ and $\bar{R} = \bar{R}^T > 0$ of appropriate dimensions, then the non-standard quadratic cost function is as follows :

$$\bar{J} = \int_0^\infty (\dot{x}^T\bar{Q}\dot{x} + u^T(t)\bar{R})dt = \int_0^\infty \bar{\Omega}\,dt \tag{3}$$

Assumption 2:
Assume that $\bar{f}(0) = 0$.
Lemma 1: [28]

Given three real matrices, Y, Z, and F, of appropriate dimensions $F^T F \leq I$, then the following inequality holds for any constant $\bar{\epsilon} > 0$:

$$2x^T YFZy \leq \bar{\epsilon} x^T YY^T x + \bar{\epsilon}^{-1} y^T Z^T Zy \quad (4)$$

Lemma 2: [29]
Given two matrices X, Y, of appropriate dimensions, then the following inequality holds for any symmetric positive definite (SPD) matrix S of appropriate dimension :

$$X^T Y + Y^T X \leq \bar{\eta} X^T SX + \bar{\eta}^{-1} Y^T S^{-1} Y \quad (5)$$

This brief aims to design a unified state and output derivatives feedback control based on optimization of non-standard cost control.

3. DERIVATIVE FEEDBACK CONTROLLERS DESIGN

This part presents the synthesis conditions for the design of control laws based on the principle of feedback by state and output derivatives while guaranteeing a non-standard cost control for NLS (using LMI with semi-definite programming (SDP) formulation [30]).

3.1. State derivative feedback controller

The principle is to design a simple derivative-state feedback control law in the following form:

$$u(t) = \bar{K}_x \dot{x} \quad (6)$$

where $\bar{K}_x \in \mathbb{R}^{m\times n}$. Then, integrating (6) in (1), the closed-loop system becomes:

$$x = \underbrace{(\bar{A} + \bar{B}\bar{K}_x)}_{\tilde{A}} \dot{x} + \bar{f}(\dot{x}) \quad (7)$$

The state derivative feedback (6) controller with a guaranteed cost control laws are synthesized to minimize a non-standard quadratic cost function (3) for LNS (1) and in order to guarantee that (7) is asymptotically stable, the following state-derivative feedback control law is proposed:

Theorem 1
For LNS in RSS (1) with derivative-state feedback control (6), the closed-loop system (7) is asymptotically stable if there exists SPD matrices $\bar{X}$ and $\bar{M}$ of appropriate dimensions and scalar $\bar{\eta} > 0$, such that the following LMI is feasible

$$\begin{bmatrix} \bar{A}\bar{X} + \bar{X}\bar{A}^T + \bar{B}\bar{M} + \bar{M}^T\bar{B}^T + \bar{\eta} I_n & \bar{M}^T & \bar{X} \\ å & -\bar{R}^{-1} & 0 \\ å & å & -(\bar{Q} + \frac{\bar{\delta}}{\bar{\eta}} I_n)^{-1} \end{bmatrix} < 0 \quad (8)$$

where $\bar{K}_x = \bar{M}\bar{X}^{-1}$.

Proof:

First, the Lyapunov function is chosen as $V = x^T Px$. Then, using the Bellman-Lyapunov inequality, the following condition must be satisfied by the designed controller with a guaranteed cost control:

$$\bar{\Lambda} = \dot{V} + \bar{\Omega} < 0 \Leftrightarrow \dot{x}^T Px + x^T P\dot{x} < -(\dot{x}^T \bar{Q}\dot{x} + u^T(t)\bar{R}u(t)) \quad (9)$$

the derivative of V with (9) gives (for simplicity $\bar{f}(\dot{x}) = \bar{f}$):

$$\dot{x}^T P\tilde{A}\dot{x} + \dot{x}^T \tilde{A}^T P\dot{x} + \underbrace{\dot{x}^T P\bar{f} + \bar{f}^T P\dot{x}}_{\bar{L}} < -\dot{x}^T(\bar{Q} + \bar{K}_x^T \bar{R}\bar{K}_x)\dot{x} \quad (10)$$

by applying *Lemma 2* on the term $\bar{L}$ (where $X^T = \dot{x}^T P$, $Y = \bar{f}$ and $S = I$) and using the Lipschitz condition (2) with *Assumption 2*, (9) becomes:

$$\bar{\Lambda} = \dot{x}^T[P\tilde{A} + \tilde{A}^T P + \bar{Q} + \bar{K}_x^T \bar{R}\bar{K}_x + \bar{\eta}PP + \frac{\bar{\delta}}{\bar{\eta}} I_n]\dot{x} = \dot{x}^T \bar{\Theta}_x \dot{x} < 0 \tag{11}$$

Second, the use Schur complements on $\bar{\Theta}_x$ leads to:

$$\bar{\Theta}_x = \begin{bmatrix} P\tilde{A} + \tilde{A}^T P + \bar{Q} + \bar{\eta}PP + \frac{\bar{\delta}}{\bar{\eta}} I_n & \bar{K}_x^T \\ \text{å} & -\bar{R}^{-1} \end{bmatrix} \tag{12}$$

introduce a new variable $\bar{X} = \bar{X}^T = P^{-1}$and pre-and post-multiply (12) by $\begin{bmatrix} \bar{X} & 0 \\ 0 & I \end{bmatrix}$ Then, (12) is congruent to:

$$\bar{\Theta}_x = \begin{bmatrix} \tilde{A}\bar{X} + \bar{X}\tilde{A}^T + \bar{X}(\bar{Q} + \frac{\bar{\delta}}{\bar{\eta}} I_n)\bar{X} + \bar{\eta} I_n & \bar{X}\bar{K}_x^T \\ \text{å} & -\bar{R}^{-1} \end{bmatrix} \tag{12a}$$

then, applying the Schur complement [25] and considering the expression of $\tilde{A}$ leads to:

$$\bar{\Theta}_x = \begin{bmatrix} \bar{A}\bar{X} + \bar{B}\bar{K}_x\bar{X} + \bar{X}\bar{A}^T + (\bar{B}\bar{K}_x\bar{X})^T + \bar{\eta} I_n & \bar{X}\bar{K}_x^T & \bar{X} \\ \text{å} & -\bar{R}^{-1} & 0 \\ \text{å} & \text{å} & -(\bar{Q} + \frac{\bar{\delta}}{\bar{\eta}} I_n)^{-1} \end{bmatrix} \tag{12b}$$

now, let us consider the variable change $\bar{M} = \bar{K}_x\bar{X}$. The expression of (12b) can be rewritten as:

$$\bar{\Theta}_x = \begin{bmatrix} \bar{A}\bar{X} + \bar{B}\bar{M} + \bar{X}\bar{A}^T + (\bar{B}\bar{M})^T + \bar{\eta} I_n & \bar{M}^T & \bar{X} \\ \text{å} & -\bar{R}^{-1} & 0 \\ \text{å} & \text{å} & -(\bar{Q} + \frac{\bar{\delta}}{\bar{\eta}} I_n)^{-1} \end{bmatrix} \tag{12c}$$

finally, it is easy to find the inequality (8).

3.2. Output derivative feedback controller

The principle is to design a simple derivative-output feedback control law in the following form:

$$u(t) = \bar{K}_y\dot{y} = \bar{K}_y\bar{C}\dot{x} \tag{13}$$

where $\bar{K}_y \in \mathbb{R}^{m\times p}$. Then, integrating (13) in (1), the closed-loop system becomes:

$$x = (\bar{A} + \bar{B}\bar{K}_y\bar{C})\dot{x} + \bar{f}(\dot{x}) \tag{14}$$

The problem formulation now is to design an output derivative feedback (13) which ensures the asymptotic stability of (14) as well as the minimization of the criterion (3). Then, the following output-derivative feedback control law is proposed:

Theorem 2

For the nonlinear RSS system (1) with derivative-output feedback control (13), the closed-loop system (14) is asymptotically stable if there exists SPD Matrices $\bar{X}$, $\bar{U}$ and $\bar{T}$ of appropriate dimensions and positives scalars$\bar{\eta}$; $\bar{\alpha}$, such that the following LMIs are feasible:

$$\begin{bmatrix} \bar{A}\bar{X} + \bar{X}\bar{A}^T + \bar{B}\bar{U}\bar{C} + \bar{C}^T\bar{U}^T\bar{B}^T + \bar{\eta} I_n & \bar{C}^T\bar{U}^T & \bar{X} \\ \text{å} & -\bar{R}^{-1} & 0 \\ \text{å} & \text{å} & -(\bar{Q} + \frac{\bar{\delta}}{\bar{\eta}} I_n)^{-1} \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} \bar{\alpha} I_n & (\bar{C}\bar{X} - \bar{T}\bar{C})^T \\ \text{å} & \bar{\alpha} I_n \end{bmatrix} > 0 \tag{16}$$

where $\bar{K}_y = \bar{U}\bar{T}^{-1}$.

Proof:

First, the same Lyapunov function given in the previous section is chosen. Then using the same analogy and to synthesis a controller with guaranteed cost control law, the following inequalityn must be satisfied:

$$\bar{\Lambda} = \dot{V} + \bar{\Omega} = \dot{x}^T \bar{\Theta}_y \dot{x} < 0$$
$$\Leftrightarrow \tag{17}$$
$$\dot{x}^T P x + x^T P \dot{x} < -\dot{x}^T (\bar{Q} + \bar{C}^T \bar{K}_y^T \bar{R} \bar{K}_y \bar{C}) \dot{x}$$

according to (13)- (14) and applying the Schur complement [25] on (17), $\bar{\Theta}_y$it becomes:

$$\bar{\Theta}_y = \begin{bmatrix} P(\bar{A} + \bar{B}\bar{K}_y\bar{C}) + (\bar{A} + \bar{B}\bar{K}_y\bar{C})P + \bar{Q} + \bar{\eta}PP + \frac{\bar{\delta}}{\bar{\eta}} I_n & \bar{C}^T \bar{K}_y^T \\ å & -\bar{R}^{-1} \end{bmatrix} < 0 \tag{18}$$

Next, according to the same steps presented in the previous section: i) use a slack variable $\bar{X} = \bar{X}^T = P^{-1}$ and pre and post multiply (18) by $\begin{bmatrix} \bar{X} & 0 \\ 0 & I \end{bmatrix}$; ii) apply the Schur complement [25]. Moreover, by applying them to (18), $\bar{\Theta}_y$it becomes:

$$\begin{bmatrix} \bar{A}\bar{X} + \bar{X}\bar{A}^T + \bar{B}\bar{K}_y\bar{C}\bar{X} + (\bar{B}\bar{K}_y\bar{C}\bar{X})^T + \bar{\eta} I_n & \bar{X}\bar{C}^T\bar{K}_y^T & \bar{X} \\ * & -\bar{R}^{-1} & 0 \\ * & * & -(\bar{Q} + \frac{\bar{\delta}}{\bar{\eta}} I_n)^{-1} \end{bmatrix} < 0 \tag{19}$$

since the two variables $\bar{K}_y$ $\bar{X}$,are multiplied directly, (19) is a BMI. For the linearization, we consider the following new variables:

$$\begin{cases} \bar{C}\bar{X} \triangleq \bar{T}\bar{C} \\ \bar{U} \triangleq \bar{K}_y \bar{T} \end{cases} \tag{20}$$

it is easy to find the inequality (15). Then, the equality constraint is approximated with the proposed choice of a new variable [26] which must satisfy :

$$(\bar{C}\bar{X} - \bar{T}\bar{C})(\bar{C}\bar{X} - \bar{T}\bar{C})^T < \bar{\alpha}^2 I_n \tag{21}$$

finally, this leads to inequality (16).

Remark

A condition has been proposed by [27] in order to ensure the optimization of the criterion $\bar{J}$ in the case where the system has non-zero initial conditions as follows:
Introduce a decision variable $\bar{Y} \in \mathbb{R}^{n \times p}$ $\bar{Y} - I_n \bar{X}^{-1} I_n > 0$: Then, minimize $trace$ $(\bar{Y})$ the subject to minimize the cost function$\bar{J}$.

4. ANALYSIS AND INTERPRETATION OF THE EXPERIMENTAL RESULTS

The following section is interested in the practical validation throughout a RTI by using the DSP device Arduino MEGA 2560 in HIL mode as well as the demonstration of the good performances offered by the proposed approaches with two examples.

In these examples, Arduino I/O mode is chosen where all technical implementation details are given in [12]. In a similar way to the implementation conditions given in [27], the same noise signals are considered whose amplitudes and frequencies are variable from 7 to 80 Hz and ($\pm 0.35V$) respectively. In this section, we present the variation of the different variables in the closed-loop mode to show the stability offered by the proposed approach.

Note: *All magnitudes are in Volts [V].*

4.1. State feedback controller

The dynamic equation for the considered system is given by: $\bar{A} = \begin{bmatrix} -2.5 & 2 \\ 1.5 & -2 \end{bmatrix}$; $\bar{B} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$; $\bar{f}(\dot{x}) = \begin{bmatrix} 0.25 sin(\dot{x}_1(t)) \\ 0 \end{bmatrix}$. The different parameters are as follows: $\bar{\delta} = 0.25$;$\bar{Q} = 100 * I_2$ and $\bar{R} = 0.01$. Subsequently, by applying the **Theorem 1**, the resolution of the LMI (8) with YALMIP® gives the following solution of a guaranteed cost state feedback controller:

$$\bar{X} = \begin{bmatrix} 0.4738 & 0.1391 \\ 0.1391 & 0.1733 \end{bmatrix}; \bar{K}_x = \begin{bmatrix} -33.169 & -31.0135 \end{bmatrix}$$

Now, Figures 1 and 2 plot the evolution of control and state of the closed-loop systems (with $x_0 = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$). Based on the findings of Figure 1 and Figure 2, it is clear that the states are asymptotically stable with the state derivative feedback law under the minimization of cost control.

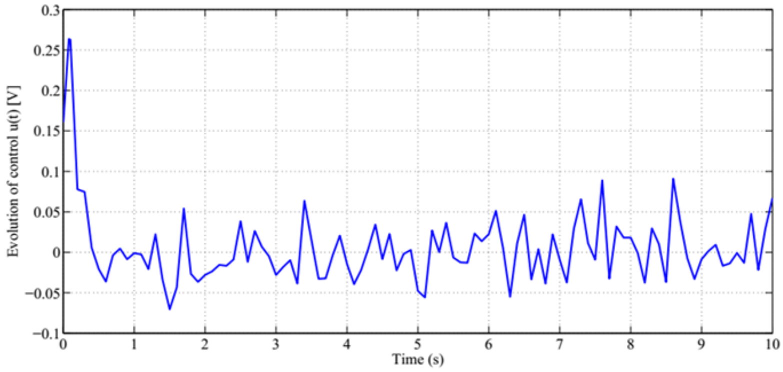

Figure 1. Variation of u(t) with state derivative feedback control

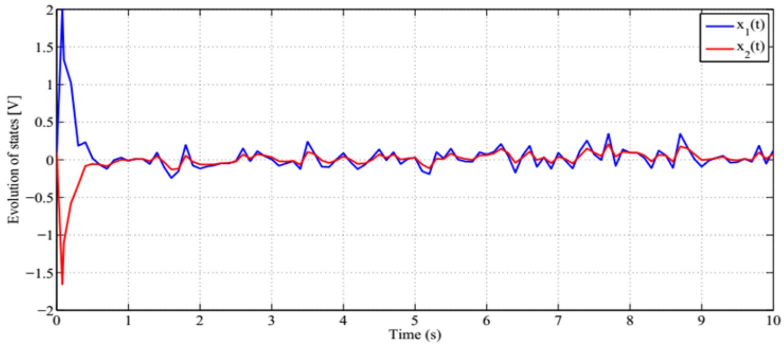

Figure 2. Evolution of *x(t)* with state derivative feedback control

4.2. Output feedback controller

The considered system is described by:

$$\bar{A} = \begin{bmatrix} -0.1 & 0 & -0.0202 & -0.0375 \\ 0 & -0.1 & -0.0022 & -0.0375 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \bar{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0111 & 0 \\ -0.0007 & 0.0007 \end{bmatrix}; \bar{f}(\dot{x}) = \begin{bmatrix} 0 \\ 0.33sin(\dot{x}_2(t)) \\ 0 \\ 0 \end{bmatrix}$$

and

$$\bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The different parameters are as follows: $\bar{\delta} = 0.33; \bar{Q} = 10^3 * I_4$ and $\bar{R} = 0.01 * I_2$. In the same way, by applying the **Theorem 2.**, the resolution of the LMIs (15)-(16) with *YALMIP®* gives the following solution of a guaranteed cost output feedback controller:

$$\bar{X} = \begin{bmatrix} 1.404 & 0.014 & 3.586 & 0.239 \\ 0.014 & 1.385 & 0.005 & 3.48 \\ 3.586 & 0.005 & 4.009 & 0.042 \\ 0.239 & 3.48 & 0.042 & 3.967 \end{bmatrix}; \bar{K}_y = \begin{bmatrix} 1.452 & -0.06 \\ 0.07 & 0.793 \end{bmatrix}$$

Next, Figure 3 and Figure 4 show the evolution of control and state of the closed-loop systems with an output derivative feedback control (with $x_0 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.25 \\ 0.25 \end{bmatrix}$).

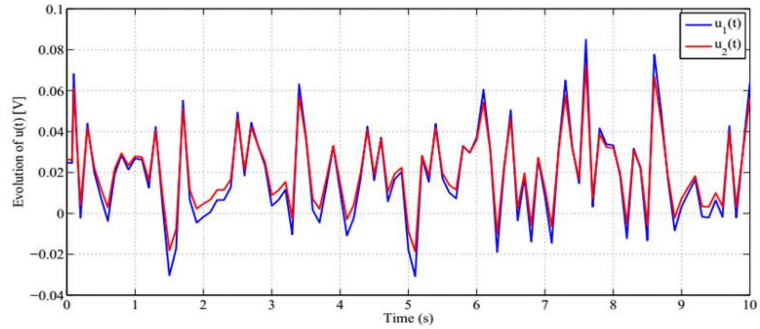

Figure 3. Evolution of *u(t)* with output derivative feedback control

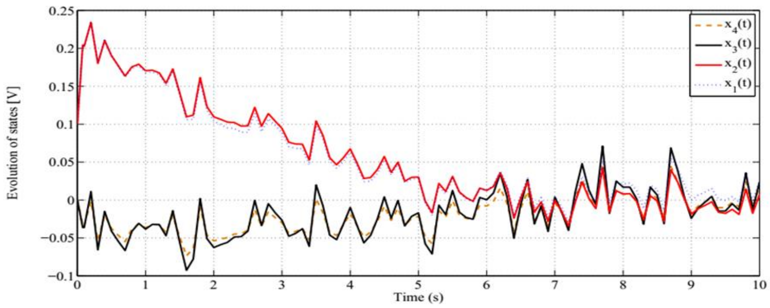

Figure 4. Evolution of x(t) with output derivative feedback control

It is clear from Figure 3 and Figure 4 that the designed controller with output derivative feedback reduce the effect of destabilization caused by the amplitudes and perturbations with a guarantee of closed loop stability of the system. This results in minimization of nonstandard cost control where the evolution of the performance index $\bar{J}_0$ [22] is described by:

$$\bar{J}_0 = E\left\{\int_0^{\infty}(\dot{x}^T\bar{Q}\dot{x} + u^T(t)\bar{R})dt\right\} + E\{x_0^T\bar{X}x_0\}$$

The value of $\bar{J}_0$ decreases from $3{,}58791.10^5$ to $3{,}49662.10^5$ (as given in **Remark 1**). As can be observed from all Figures 1-4, the proposed unified controllers with state or output feedback in RSS form are less sensitive to externals perturbations (noises).

5. CONCLUSION

This paper presents a state and output feedback design methodology for a class of LNS in RSS. The proposed control law ensures the minimization of non-standard cost control unsing the Lyapunov theory to guarentee the asymptotic stability of the closed-loop system. The proposed controllers are deduced from a convex resolution problem with the use of LMIs and some lemmas. RTI with ARDUINO MEGA 2560 board used as a real-time emulator in HIL mode further demonstrates the performances of stabilization offered by the proposed approach. Expanding proposed robust synthesis methods with uncertain parameters and the generalization of the proposed method for one-sided Lipschitz systems might be a direction for further studies.

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