A social network based approach for consensus achievement in multiperson decision making

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Abstract

Nowadays we are living the apogee of the Internet based technologies and consequently web 2.0 communities, where a large number of users interact in real time and share opinions and knowledge, is a generalized phenomenon. This type of social networks communities constitute a challenge scenario from the point of view of Group Decision Making approaches, because it involves a large number of agents coming from different backgrounds and/or with different level of knowledge and influence. In these type of scenarios there exists two main key issues that requires attention.

Firstly, the large number of agents and their diverse background may lead to uncertainty and or inconsistency and so, it makes difficult to assess the quality of the information provided as well as to merge this information. Secondly, it is desirable, or even indispensable depending on the situation, to obtain a solution accepted by the majority of the members or at least to asses the existing level of agreement. In this contribution we address these two main issues by bringing together both decision Making approaches and opinion dynamics to develop a similarity-confidence-consistency based Social network that enables the agents to provide their opinions with the possibility of allocating uncertainty by means of the Intuitionistic fuzzy preference relations and at the same time interact with like-minded agents in order to achieve an agreement.

Keywords: Group decision making, Uncertainty, Consensus, Intuitionistic fuzzy preference relations, Social network, e-democracy, Opinion Dynamics

1. Introduction

Traditionally Group Decision Making, GDM, has been regarded as a process in which a reduced group of agents interact in order to chose the best alternatives between all the available ones. Indeed

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this is still the case in many companies and administrations where important decisions are made by a reduced board of specialists. However, nowadays, societal demands and technological advancements are leading us to a global and interconnected society where thousands and even millions of users are sharing thoughts, opinions and preferences that can well lead to a global large scale decision making process [38, 63], in what is is denominated a social network,

A social network can be regarded as a set of people, or groups of people interacting with each other [48]. In politics some efforts are being taken in order to involve citizens in global decision in what is being called e-democracy, e-Governance and public deliberation [41, 32, 44]. For example, the European Commission has initiated the platform called *The European citizens' initiative*, [11] that allows one million EU citizens to participate directly in the development of EU policies, by calling on the European Commission to make a legislative proposal.

Social Networks are characterized for having a large and heterogeneous user base with low an intermittent participation rates and so a high degree of uncertainty related with them needs to be taken into consideration when modelling a large scale decision making process [27, 38, 63, 16]. In this particular, Intuitionistic fuzzy preference relations, based on Intuitionistic Fuzzy sets [4], suppose an interesting framework for the agents to express their judgements, since they allow them to allocate certain levels of uncertainty in their opinions [57, 54].

One of the main challenges in any GDM scenario consists in achieving a full and unanimous agreement among all the agents [43, 6]. However, in the majority of the occasions this is not reachable in practice, and so alternative approaches comes in to play such as the so called softer consensus measures, [7]. These approaches define the consensus process as a dynamic and iterative group negotiation with the purpose of bringing the agents opinions closer and so they better represent the human perception of the essence of consensus. In these approaches both the agents consistency, that is the coherence of the opinions, and the similarity among the agents, to asses the agreement existing between them, are used to guide the negotiations, providing suggestions to the agents [26, 40, 57].

In the majority of the negotiations processes to reach consensus in web communities the agents are not homogeneous [18, 63], that is, they present different agentise, participation rates and self-confidence levels [42, 37], however classical group decision making provides equal feedback mechanisms to all of them without taking into consideration their peculiarities with respect to self confidence and influence.With this regard, a recent survey in Social Network based consensus approaches [16] classifies these methodologies in two main paradigms, the ones based on trust relationships, and the ones based on opinion evolution, pointing out that the later are still in an early stage whereas the former lacks of tools to calculate dynamically the inter agents trust and influence.

When dealing with inter-agent influence we can keep in mind that according to Guha et al. in [24] in any real field decision making situation when agents give their responses to a particular alternative, their self confidence level regarding the opinions plays a key role. In fact, according to various researches the members of the groups composed of freely iterating individuals often decide to choose the positions of their most confident members as their group decisions [34, 49, 30]. Consequently confidence can be considered as a relevant indicator of influence and so it needs to be taken into account in the negotiation processes to reach consensus. Liu et al. in [37] propose an optimization approach to estimate the collective preference vector when using heterogeneous preference relations with self-confidence assessment. However, this model does not take into consideration the agents confidence nor to increase the consensus nor to carry out the aggregation of the information and it increase the complexity of the decision process forcing the agents to provide another value to asses their confidence. On the contrary Urena et al. in [54] propose an approach that directly computes the agents degree of confidence from their opinions expressed by means of Intuitionistic Fuzzy Preference Relations, IFPRs.

Apart form confidence, on opinion dynamics procedures, users can be either opinion leaders who are the agents who can exert influence on the opinions, and ordinary agents or followers, [2]. With this regard, in the propagation process of public opinion, the formers have a profound impact on the opinion formation of the followers even helping to the faster propagations of opinions in a network [2]. Bounded confidence based approaches construct opinion dynamics models [56, 35] to analyse the influence of opinion leaders in social networks revealing that, as long as the confidence levels of ordinary agents in a social group are sufficiently high, even if the initial opinions of the ordinary agents are dissimilar to those of the opinion leaders, the opinion leaders are eventually able to guide the ordinary agents to accept their desired opinions and so to reach a consensus solution.

Therefore, in decision making scenarios it is key to recognize which are the different profiles of the agents, influencers and followers, and provide selective recommendations in consequence. With this regard, in [42] it is presented a consensus model that deals with heterogeneous agents, and so it adjust the level of feedback depending on a pre-given value of agents importance. Whereas a trust based consensus approach where the recommendations are provided to the agents by means of a trust network is developed in [58]. However both of this approaches require extra information, either the confidence, either the degree of importance of the agents or the trust. However, for e-decision applications or, in general, web communities when carrying out decision making extra information apart from the agents opinion it is hard to get and so the challenge is to develop mechanisms to infer the agents influence and confidence directly from their opinions, and from that, develop a network of influence in which the agents presenting higher consistency in their opinions as well as higher levels of self confidence are placed in the most influential positions of the network.

Another challenge deals with the agents reluctance to accept the advice given during the negotiation , that implies that providing feedback does not mean that the agents are going to accept the suggestions given. With this regard in [18] it has been proposed a self-management mechanism to generate agents' weights dynamically in order to fuse all the agents opinions. This procedure relies in an a priori inter-agents evaluation to asses the agents influence and so it may not be suitable for large scale systems in which the agents may not know each others and so they are unable to provide pairwise agents evaluations. However bounded confidence opinion dynamics mechanisms [56, 25] state that the individuals rely on the opinions and social appraisal supported from people with similar interests [36] and so in order to ensure agents to take into account the advice, this recommendations should come from others like-minded agents. According to a recent survey in opinion dynamics procedures [17] it is interesting to develop mechanisms that brings the gap between opinion dynamics procedures and social network GDM approaches dealing at the same time with malicious and reluctant agents.

In summary, large scale group decision making constitute a complex scenario in which the uncertainty associated to the heterogeneity of the agents as well as the potential reluctance of some agents to change their view point make the opinions propagation with the objective of reaching consensus a challenging tasks. However the role of influencers and like-minded agents or agents in the network is key in to successfully propagate the opinions and to reach a solution accepted by the majority of the agents.

In this contribution, we model the consensus reaching problem as a social influence network in which each node constitutes one agent represented by its opinions and whose influence will be inferred from its consistency and confidence. Furthermore we present a networked feedback spreading mechanism inspired by the aforementioned bounded confidence opinion dynamics mechanisms [56, 25, 2, 59] to support agents to change some of their preference values in order to achieve a general consensus.

The main novelties presented in this contribution with respect to the ones in the literature that also uses agents dynamic weight calculation like the one in [18] are twofold: First of all in the proposed approach the inter-agents influence is calculated taking into consideration only the agents opinions in each iteration, without the necessity of other inter agents measure. Therefore this approach works in large scale decision making processes in which the agents do not have an apriori opinion of other agents. Secondly, the proposed approach is able to classify the agents in different profiles, pointing out the influencers and allocating them a preponderant position in the network, and isolating those that may present a malicious behaviour. Personalized advice is delivered to each agent according to its profile and its position in the network with the objective of both increasing the consistency of the information and the consensus degree.

The rest of the paper is set out as follows: Section 2 presents the main mathematical frameworks in both group decision making and opinion dynamics scenarios We present the new proximity based social network for feedback propagation in Section 3. In Section 4 we integrate the proposed network in a Group decision making scenario and we validate the model through extensive simulations. Finally in Section 5 we expose the conclusions of our work as well as pointing out future research challenges.

2. Background

This section is composed of two subsections, in the first one, the mathematical frameworks in Group Decision Making are described whereas the main opinion dynamics procedures are presented in the second one.

2.1. GDM frameworks

In decision making processes that involve several agents, they provide their preferences on the set of available alternatives (X). Different ways of carrying out this comparison when expressing preferences are thoroughly analyzed in [39], concluding that the most accurate approaches consist on pairwise comparison since they allow the decision makers to take into consideration only two alternatives at a time. This pairwise comparison can lead to three different preference states namely: preference of one alternative to the other, indifference between them or impossibility of expressing them.

There exist two main mathematical approaches based on the concept of preference relation. On the one hand, the first approach [21, 45] proposes to define a preference relation for each one of the three possible preference states. On the other hand, the second one integrates the three possible preference states into a single preference relation [5] proposing the following definition of preference relation.

Definition 1 (Preference Relation)**.** *A preference relation P on a set X is a binary relation $\mu_P : X \times X \to D$, where D is the domain of representation of preference degrees provided by the decision maker.*

Therefore, a preference relation P constitutes a matrix $P = (p_{ij})$ of dimension $\#X$, in which $p_{ij} = \mu_P(x_i, x_j)$ is the degree or intensity of preference of alternative x_i over x_j. The elements of P could be numeric or linguistic depending on the type of decision making process that is being carried out. Indeed, for the case of numeric preference relations the main types used in decision making approaches are: crisp preference relations, additive preference relations, multiplicative preference relations, interval-valued preference relations and intuitionistic preference relations [60]. Among them, in this contribution we will focus on intuitionistic fuzzy preference relations since they allow the agents to express their uncertainty.

2.1.1. *Fuzzy Set and Intuitionistic Fuzzy Preference Relation*

Definition 2 (Fuzzy Set)**.** *Let U be a universal set defined in a specific problem, with a generic element denoted by x. A fuzzy set X in U is a set of ordered pairs:*

$$X = \left\{(x, \mu_X(x)) | x \in U\right\}$$

where $\mu_X \colon U \to [0,1]$ is called the membership function of A and $\mu_X(x)$ represents the degree of membership of the element x in X.

The degree of non-membership of the element x in X is here defined as $\nu_X(x) = 1 - \mu_X(x)$. Thus, $\mu_X(x) + \nu_X(x) = 1$.

Definition 3 (Fuzzy Preference Relation)**.** *A fuzzy preference relation $R = (r_{ij})$ on a finite set of alternatives X is a fuzzy relation in $X \times X$ that is characterized by a membership function $\mu_R : X \times X \longrightarrow [0,1]$ with the following interpretation:*

- $r_{ij} = 1$ *indicates the maximum degree of preference for x_i over x_j*
- $r_{ij} \in]0.5, 1[$ *indicates a definite preference for x_i over x_j*
- $r_{ij} = 1/2$ *indicates indifference between x_i and x_j*

When

$$r_{ij} + r_{ji} = 1 \ \forall i, j \in \{1, \ldots, n\}$$

is imposed the fuzzy preference relation is called reciprocal.

2.1.2. *Consistency of fuzzy preference relations*

Consistency of fuzzy preference relations is related with the transitivity in the pairwise comparison among any three alternatives, that is, if alternative x_i is preferred to alternative x_j ($x_i \succ x_j$) and this one is preferred to x_k ($x_j \succ x_k$) then alternative x_i should be preferred to x_k ($x_i \succ x_k$), which is normally referred to as *weak stochastic transitivity* [9]. Any property that ensures the transitivity of the preferences can be denominated as a consistency property. Obviously, the lack of consistency in decision making can lead to not coherent information, therefore it is necessary to develop measures to asses the consistency levels in the opinions of the agents. [46]. Several properties have been suggested as rational conditions to be verified by a consistent fuzzy preference relation [9, 29], among them we can point out the followings: triangle condition, weak transitivity, max-min transitivity, max-max transitivity, restricted max-min transitivity, restricted max-max transitivity, additive transitivity, and multiplicative transitivity. Particularly in our approach we will focus on Tanino's Multiplicative transitivity property to model consistency.

Definition 4 (Multiplicative transitivity [52])**.** *A fuzzy preference relation $R = (r_{ij})$ on a finite set of alternatives X is multiplicative transitive if and only if*

$$r_{ij} \cdot r_{jk} \cdot r_{ki} = r_{ik} \cdot r_{kj} \cdot r_{ji} \;\; \forall i,k,j \in \{1,2,\ldots n\} \tag{1}$$

The preference value between a pair of alternatives (x_i,x_j) with $(i<j)$ can be estimated, using another different intermediate alternative x_k $(k \neq i,j)$ by means of the multiplicative consistency property (1) as follows:

$$mr_{ij}^k = \frac{r_{ik} \cdot r_{kj} \cdot r_{ji}}{r_{jk} \cdot r_{ki}} \tag{2}$$

Given a non zero denominator, mr_{ij}^k is one of the partially multiplicative transitivity based estimated fuzzy preference values of the pair of alternatives (x_i,x_j) given the intermediate alternative x_k. The average of all possible estimated values of the pair of alternatives (x_i,x_j) is considered as the global multiplicative transitivity based fuzzy preference relation value:

$$mr_{ij} = \frac{\sum\limits_{k \in R_{ij}^{01}} mr_{ij}^k}{\# R_{ij}^{01}};$$

where $R_{ij}^{01} = \{k \neq i,j | (r_{ik}, r_{kj}) \notin R^{01}\}$, $R^{01} = \{(1,0),(0,1)\}$, and $\# R_{ij}^{01}$ is the cardinality of R_{ij}^{01}. Consequently, for every fuzzy preference relation, $R = (r_{ij})$, we can estimate the multiplicative transitivity based fuzzy preference relation, $MR = (mr_{ij})$. Notice that it has been proved in [8] that when a fuzzy preference relation $R = (r_{ij})$ is multiplicative transitive then $R = MR$. What it is more, if R is multiplicative then (1) it is verified $\forall i,j,k$.

Definition 5 (Multiplicative Consistency)**.** *A fuzzy preference relation $R = (r_{ij})$ is multiplicative consistent if and only if $R = MR$.*

The degree of similarity existing between the values r_{ij} and mr_{ij} has been proposed in [28] as a measure of the level of consistency existing on a given fuzzy preference relation at three different levels namely pair of alternatives, alternatives and relation. In this contribution we assume that the preference relations are reciprocal, and so we consider only the upper diagonal of the matrix, moreover we define the consistency in only one level as follows:

Definition 6 (Consistency index on the fuzzy preference relation)**.**

$$C_T = \frac{\sum\limits_{i=1;\, i \neq j; j>i}^{n} 1 - d(r_{ij}, mr_{ij})}{n(n-1)} \tag{3}$$

Where $d(r_{ij}, mr_{ij})$ is the distance between the values r_{ij} and mr_{ij}.

The following results characterize the multiplicative consistency of a fuzzy preference relation using its corresponding consistency level.

Proposition 1. *A fuzzy preference relation R is multiplicative consistent if and only if $Cs = 1$.*

2.1.3. Intuitionistic Fuzzy Set and Intuitionistic Fuzzy Preference Relation

The concept of an *Intuitionistic Fuzzy Set* (IFS) was introduced by Atanassov in [4]:

Definition 7 (Intuitionistic Fuzzy Set)**.** *An intuitionistic fuzzy set X over a universe of discourse U is given by*

$$X = \Big\{ \big(x, \langle \mu_X(x), \nu_X(x) \rangle\big) \big| x \in U \Big\}$$

where $\mu_X\colon U \to [0,1]$, and $\nu_X\colon U \to [0,1]$ verify

$$0 \leq \mu_X(x) + \nu_X(x) \leq 1 \quad \forall x \in U.$$

$\mu_X(x)$ and $\nu_X(x)$ represent the degree of membership and degree of non-membership of x in X, respectively.

Notice that an intuitionistic fuzzy set it is a fuzzy set when $\mu_X(x) = 1 - \nu_X(x)$ $\forall x \in U$. On the contrary whenever there is at least one value $x \in U$ such that $\mu_X(x) < 1 - \nu_X(x)$, an extra parameter needs to be given when working with intuitionistic fuzzy sets, that is the hesitancy degree, $\tau_X(x) = 1 - \mu_X(x) - \nu_X(x)$, that represents the amount of lacking information in determining the membership of x to X.

The intuitionistic fuzzy preference relation is defined as a generalisation of the concept of fuzzy preference relation [51].

Definition 8 (Intuitionistic Fuzzy Preference Relation)**.** *Give a finite set of alternatives $X = \{x_1, \ldots, x_n\}$, an intuitionistic fuzzy preference relation B is composed of a membership function $\mu_B\colon X \times X \to [0,1]$ a non-membership function $\nu_B\colon X \times X \to [0,1]$ and a hesitancy function $\tau_B\colon X \times X \to [0,1]$ such that*

$$0 \leq \mu_B(x_i,x_j) + \nu_B(x_i,x_j) \leq 1 \;\; \forall (x_i,x_j) \in X \times X.$$

with $\mu_B(x_i,x_j) = \mu_{ij}$ interpreted as the certainty degree up to which x_i is preferred to x_j; and $\nu_B(x_i,x_j) = \nu_{ij}$ interpreted as the certainty degree up to which x_i is non-preferred to x_j and $\tau_B(x_i,x_j) = \tau_{ij} = 1 - \mu_{ij} - \nu_{ij}$ interpreted as the degree of hesitation with the opinions provided.

2.1.4. Expert's degree of confidence

When working with intuitionistic fuzzy preference relations, $B = (b_{ij}) = (\langle \mu_{ij}, \nu_{ij}, \tau_{ij} \rangle)$, we can take advantage of the hesitancy degree to directly asses the confidence level of the agent with the preference provided at three different levels as follows :

- The **confidence level associated to the intuitionistic preference value** b_{ij} is measured as

  $$CFL_{ij} = 1 - \tau_{ij},$$

  being τ_{ij} the hesitancy degree associated to b_{ij}.

  Notice the higher the value of τ_{ij} the more hesitation is present in the intuitionistic value b_{ij} and so the lower the value of the confidence CFL_{ij}, .

- The **confidence level associated to the alternative** x_i

  $$CFL_i = \frac{\sum_{\substack{j=1 \\ i \neq j}}^{n} \left(CFL_{ij} + CFL_{ji} \right)}{2(n-1)}.$$

  Due to the fact that B is reciprocal then $CFL_{ij} = CFL_{ji}$ $(\forall i, j)$ and so

  $$CFL_i = \frac{\sum_{\substack{j=1 \\ i \neq j}}^{n} CFL_{ij}}{n-1}.$$

- The **confidence level** C_F **associated to the preference relation**

  $$C_F = \frac{\sum_{i=1}^{n} CFL_i}{n}. \tag{4}$$

In [54] it has been proved the existence of a one-to-one correspondence between the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations. In this contribution we take advantage of this isomorphism to use some concepts for an intuitionistic fuzzy preference relation via the equivalent known ones associated to the asymmetric fuzzy preference relation, as it is the case of the Multiplicative transitivity.

2.2. Opinion Dynamics

Opinion Dynamics can be regarded, on the context of influence networks of individuals and their interpersonal relations, as the mechanisms of network formation and transformation by which individuals' attitudes, opinions and behaviours toward particular objects are modified by the displayed attitudes, opinions, and behaviours of other individuals toward the same object [12, 17]. Thus, interpersonal influence networks can be defined as social cognition structures assembled by individuals who are dealing with a common issue [33, 22, 23]. These networks can be mathematically formulated as a matrix W as follows:

Definition 9. Social influence network

$$W = [w_{ij}],\ w_{ij} \in [0,1] \forall i,j,\ \sum_j w_{ij} = 1\ \forall i \tag{5}$$

Each edge of this network correspond to a value of the matrix W representing the influence and weight accorded by agent i to agent j.

In the literature we can find two main mathematical model for opinion dynamics, the DeGroot model [12] and its generalization proposed by Friedkin and Johnsen [22, 23].

The DeGroot model [12] considers the opinion evolution of the individuals as a weighted average of the opinions of the individuals in his/her environment.

This can be formalized mathematically as follows:

$$y(t+1) = Wy(t),\ t = 0,1,2,\cdots \tag{6}$$

where y is a real-valued vector representing the individuals' opinions at time t.

The generalization of DeGroot model proposed by Friedkin and Johnsen [22, 23],FJ Model, considers as well how an individual can evolve with respect to their own opinions. Thus this approach introduces a positive diagonal matrix I_n that models the own individuals opinions as follows:

$$y(t+1) = \delta W y(t) - (l_n - \delta) y(0), t = 0,1,2,\cdots \tag{7}$$

According to the FJ Model, when opinion formation reaches equilibrium that is, Consensus or a deadlock, the final opinions can be predicted by mean of combining three pieces of information:

1. y is an N by 1 column vector consisting of actors' opinions on an issue, $y(0)$ corresnpond to the actors' initial opinions.

2. δ is an N by N diagonal matrix of actors' openness to influence on the issue under consideration.

3. W is an N by N matrix of direct interpersonal influences, w_{ij}, each of which represents the direct weight (or significance) that actor i accords to the opinion held by Actor j, including the self-weight that Actor i accords to his or her own opinion. Influence weights range from 0 to 1, and the set of weights held by any actor for all group members, including himself or herself, sums to 1. In the standard model, actors' susceptibilities to influence are coupled with the weights they accord to their own opinions (w_{ii});

In the FJ Model the actors modify their opinions on an issue by forming a weighted average of group members opinions, including their own, as the micro-level influence process plays out over time. This is supported by the work in [33] that empirically proves the assumption that individuals update their opinions as convex combinations of their own and others' displayed opinions, based on weights that are automatically generated by individuals in their responses to the displayed opinions of other individuals.

Particular cases of the FJ Model are the so called bounded confidence models. According to these models, each agent solely communicates with the agents who hold similar opinions and ignores the ones whose opinions are far. This similarity of opinions can be seen as a confidence level between agents. Therefore in these models, the similarity between agents as well as their initial opinions determine the opinion neighbourhood in which the agent is likely to interact at every instant. The two main bounded confidence-based models are the Hegselmann-Krause (HK) model [25], and the DeffuantWeisbuch (DW) model [56].

These models are composed of three main steps: 1. Determine for each agent the confidence set, that is the agents that are in the vicinity. 2. Asses each agents degree of influence with respect to a given one, 3. Determine the updated opinion for each agent.

The HK model can be mathematically formalize as follows:

$$y_i(t+1) = \frac{\sum_{j:|y_i(t)-y_j(t)|\leq\varepsilon} w_{ij}x_j(t)}{\sum_{j:|y_i(t)-y_j(t)|\leq\varepsilon} w_{ij}} \tag{8}$$

Where $y_i(t)$ is the opinion of agent i , ε is the confidence level and w_{ij} is the interaction weight of agent j on agent i .

On the other hand the DW model can be described in the following way:

$$y_i(t+1) = y_i(t) + \lambda[y_j(t) - y_i(t)] \tag{9}$$

$$y_j(t+1) = y_j(t) + \lambda[y_i(t) - y_j(t)] \tag{10}$$

Where $y_i(t)$ is the opinion of agent i, ε is the confidence level, and λ is the convergence parameter.

As we can observe, the main difference between these two models is that the DW model adopts an asynchronous opinion updating process, whereas the HK model adopts a synchronous updating process.

The challenge in this model consists on the way on assessing the influence between the agents that takes par in the decision process, that is to calculate the matrix W in which each element w_{ij} consist on the influence of agent i over agent j. In the following section we present an approach in which the influence will be assessed by means of the agents self confidence and the consistency of the information provided.

3. Proximity-Confidence-Consistency Influence Model to reach consensus

As already stated, in many human interactions more importance is given to those who present more confidence with their statements. Obviously, only using confidence could lead to serious bias or even manipulation in the decision, since it could be the case of an agent reporting high degree of confidence but presenting very incoherent answers motivated by the lack of in depth knowledge or even with a malicious intention. Therefore, apart from confidence, other measures should be taken into account, such as the consistency of the opinions that provides a measure of the coherence, or the quality of the information provided by the agents.

Apart from that, we should bear in mind that providing suggestions does not mean that the agents will take them into account. In this sense, as aforementioned, it has been proved that agents tend to provide more attention to those who have opinions closer to them [13, 16, 35]. In this section we present a new similarity- consistency-confidence social network in which the agents opinion propagation is designed to provide recommendations in order to bring closer the agents opinions and at the same time increase the coherence and the quality of the provided opinions. The proposed approach is composed of a number of steps, as depicted in figure 1, explained in detail in the subsequent subsections.

3.1. Agents classification

During any negotiation the decision maker's point of view may evolve. In this particular sense, it has been observed that those individuals who are more confident with their opinions are the ones leading the group, or, in other words they are the ones who have more influence in the rest of the members of the team. Whereas those agents presenting lower levels of confidence, maybe due to lack of in depth knowledge of the subject under consideration, are the ones that will adopt with more easiness the advice provided by those agents who they presents opinions closely enough to them [2].

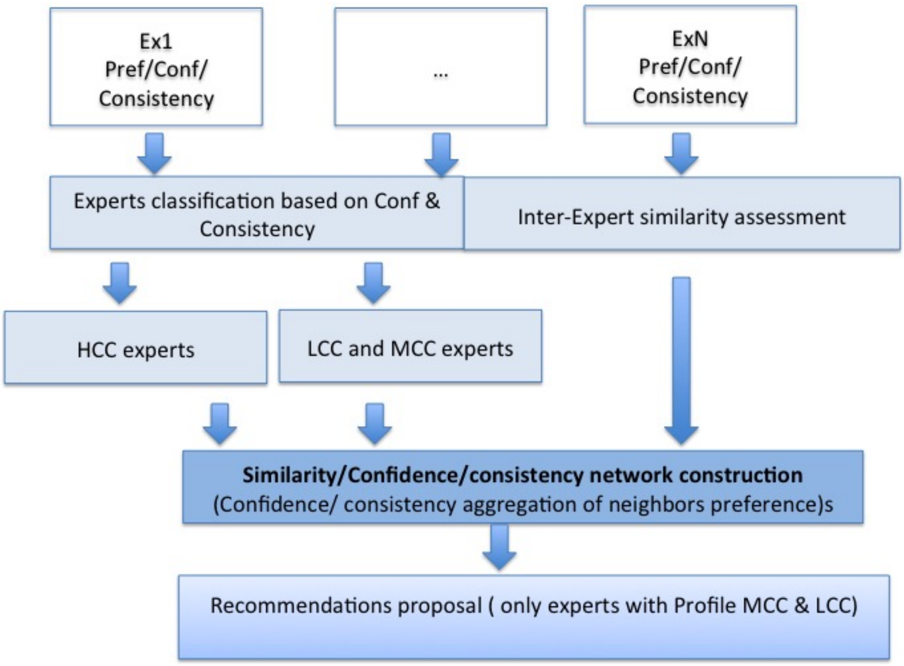


Figure 1: Confidence/Consistency/Similarity network construction approach.

The proposed approach classifies the agents in three different profiles and based on that it calculates their influences when spreading their opinions and receiving the others opinion feedback.

To do so, firstly both agent's confidence and consistency are aggregated in an unique value called agent Knowledge Degree, *KD*, by means of the following T-NORM operator:

Definition 10. *A t-norm is a function* $T : [0,1] \times [0,1] \to [0,1]$ *which satisfies the following properties:*

- *Commutativity:* $T(a,b) = T(b,a)$
- *Monotonicity:* $T(a,b) \leq T(c,d)$ *if* $a \leq c$ *and* $b \leq d$
- *Associativity:* $T(a,T(b,c)) = T(T(a,b),c)$
- *The number 1 acts as identity element: T(a, 1) = a*

In this case we are going to use the a particular case of strict Archimedean t-norm called Hamacher product.

$$T(a,b) = \begin{cases} 0 & \text{if } a = b = 0 \\ \frac{ab}{a+b-ab} & \text{otherwise} \end{cases} \tag{11}$$

Definition 11. *Expert Knowledge Degree*

$$KD^h = T(C_T^h, C_F^h) \tag{12}$$

We have chosen the Hamacher T-norm operator in (11) since it penalizes the low values. Therefore if an agent presents a very high confidence but low consistency the KD value will be low, and vice versa The only way to have a high KD value is by having both values high, as it is the purpose of our classification and influence assessment process as we explain in the following.

Given a set $E = 1, 2\cdots, n$ agents and an agent $h \in E$ having a Knowledge Degree KD^h and given a Minimum KD Threshold $KD_{THmin} \in \{0,1\}$ and a superior KD threshold $KD_{THsup} \in \{0,1\}$, the agents can be classified in the following profiles:

- **Profile 1: Agents with both high degree of confidence and consistency, HCC Agents, Influencers** An agent h is considered as a HCC agent if and only if

  $$KD^h > KD_{THsup}$$

  These agents con be regarded as influencers and so their opinions should be highly taken into consideration. Therefore, these agents will not receive any type of advice but their opinions will be recommended to other agents and so high importance will be associated to them in the aggregation in order to obtain the global solution. Among this type of agents we can recognize a subtype whose opinions are far from the global opinion, they could be regarded as outliers. These agents will be disconnected from the rest of the nodes in the proposed similarity network and consequently they will have none or few influence in the global negotiation.

- **Profile 2: Agents with medium level of consistency and medium level of confidence, MCC agents**

  An agent h is considered as a MCC agent if and only if

  $$KD_{THmin} \leq KD^h \leq KD_{THsup}$$

  These type of agents, which constitute the majority in a negotiation process, are the ones, that are quite knowledgeable presenting good levels of consistency in the opinions provided and good levels of confidence. In general these type of agents are likely to change their minds in other to reach high levels of agreement and as well their opinions should be taken into consideration in the negotiation process.

- **Profile 3: Agents with low consistency and/or low confidence level, LCC agents**

  An agent h is considered as a LCC agent if and only if

$$KD^h < KD_{THmin}$$

Within this category we can find three different profiles:

- **Agents with both low consistency and low confidence level** This type of agent is not confident with the opinions provided and the degree of coherence of his/her answers is not high. That means that the agent is not very knowledgeable about the topic that is being taken into consideration and so, in other to provide interesting opinions he/she requires of some suggestions.
- **Agents with high level of consistency and low levels of confidence:** This type of agents, even though they present good levels of consistency in the opinions provided, they are not able to report high levels of confidence with these opinions. In general these type of agents are likely to change their minds in other to reach high levels of agreement if the good recommendations are provided to them.
- **Agents with low level of consistency and high level of confidence** This is the common profile of over confident agents that are not very knowledgeable with the topic taken into consideration and so their opinions are not very reliable. It is worthy to recognize these type of agents and provide them with recommendations to increase the level of general consensus, as well as observe their general behavior since they are likely to present a malicious intention.

The agents within this category receive feedback from their HCC and MCC neighbours in the network but they will not have any influence in the opinions of their neighbours, since in the given iteration their opinions are not considered as very worth it and they could even present a malicious intention.

3.2. Inter-agents similarity assessment

To asses the similarities between the agents it is necessary to use a convenient distance measure [50]. With this regard in [10] it has been carried a study that evaluates the impact of the different distance measures in the consensus process concluding that the Manhattan and the Euclidean distances help to increase the consensus level when the number of agents is high. Whereas the Cosine and the Dice distances produce the same results regardless of the number of agents. These measures asses the numerical distance existing between the given preferences without considering if the user prefers or not one alternative. However, it makes sense to believe that the users that prefer the same alternatives are closer between each other than those that even if their alternatives in value are close they prefer different alternatives. In this contribution we propose a new way of assessing the similarity between agents at a global level, taking advantage of the the Jaccard similarity index. This index, is described as the size of the intersection between two sample sets divided by the size of the union between the same sample sets [31]. In our case we consider the intersection between two agents' preference relation as the number of same preferences that both agents prefers. Therefore the Jaccard similarity between an agent, e_k with a matrix of preferences P^k and an agent e_l with a matrix of preferences P^l is defined as follows:

Definition 12. Jaccard similarity between agents

$$Sim(P^k, P^l) = \frac{\#I^{kl}}{n(n-1)} \tag{13}$$

where n is the total number of alternatives and # is the cardinality of the following set

$$I^{kl} = \{p^k_{ij} > 0.5 \wedge p^l_{ij} > 0.5\} \; \forall ij \in \{1, n\} \wedge j > i \tag{14}$$

Definition 13. Similarity index on the relation.

The similarity of an agent's preference relation P^k, to an agent's preference relation P^l denoted as S^{kl}, is defined as:

$$S^{kl} = Sim(P^k, P^l) \tag{15}$$

The similarity of an agent, e_h, preference relation P^h to the preference relation obtained by the aggregation of all agents preference relations in one only matrix of preference G is defined as:

$$S^h_G = Sim(P^h, G) \tag{16}$$

This global matrix G is computed using the KD-IOWA operator defined in (18) and (19). Taking into consideration all the agents pondered by their influence.

3.3. Network construction

In the following we propose a new feedback model based on the HK [35] opinion dynamic model that assumes that in a social network the agents that are close will communicate between each other. With this in mind, in [47] it has been stated that people in social networks are most likely to interact with similar people where similarity is based on the context and the domain. First of all a similarity based influence network to spread agents opinions with the purpose of reaching a consensus solution will be developed. To do so every agent will be consider as a node of a directed graph.

Definition 14. *A directed graph is an ordered pair G = (N, W) where N is a set of nodes; and W is a set of directed edges that interconnect the nodes. In the proposed approach there are H nodes, each of one correspond to an agent e^h that it is characterized by its matrix of preferences P^h.*

Definition 15. *The adjacency matrix $M = (m_{kl})_{HxH}$ of the graph $G = (N, M)$. The value of each edge, m_{kl} is calculated as the Similarity between the preferences for the agent k with a matrix of preferences P^k and the agent l with preferences P^l as described in (15).*

$$m_{kl} = \begin{cases} S^{kl} & if\ S^{kl} > \alpha_{sim} \wedge (Profile_l = HCC \vee Profile_l = MCC)) \\ 0 & if\ S^{kl} < \alpha_{sim} \vee Profile_l = HCC \vee Profile_l = LCC) \end{cases} \tag{17}$$

Where $k, l \in [1, H] \wedge l > k$ and α_{sim} is a minimum similarity threshold, so if the similarity between two agents is less than the threshold these two agents will not be connected. This measure allows to automatically isolate the agents that even though they present profile HCC their opinions far from the other ones for different reasons including those with malicious intentions.

In each iteration, each agent receives as recommendation the weighted fusion of the opinions of the agents connected to him that presents profiles HCC and MCC. The agents with profile LCC only will receive recommendations, and the agents with profile HCC only will provide recommendations. The opinion spreading mechanism between the different agents profiles is depicted in figure 2. The opinions of each agent in the vicinity will be weighted based on each agent's influence. This influence is calculated based on both the agents confidence and consistency.

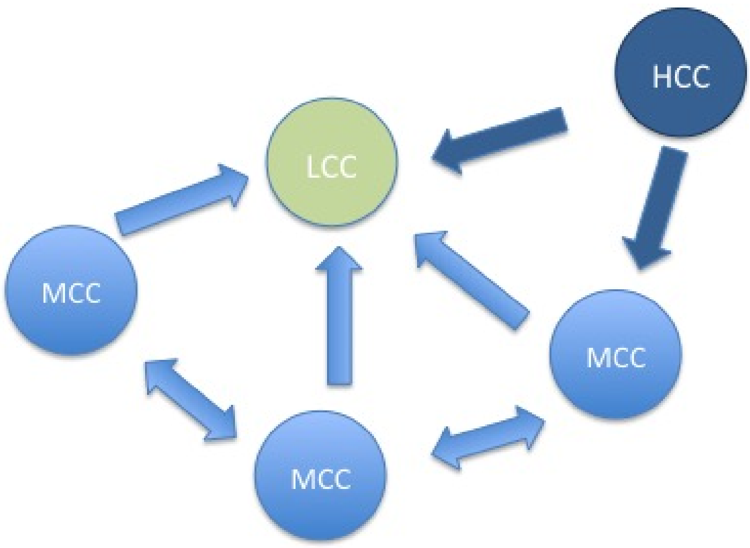


Figure 2: Feedback spreading scheme.

Note that the agents that could present a malicious behaviour, (MCC agents whose opinions are very far from the rest of the agents, or LCC agents) are automatically detected, and their influence in the network is limited.

3.4. Feedback aggregation

Every agent in the network receive a recommendation the preferences of the nodes connected to him/her fused by means of a Knowledge Degree Induced Ordering Weighting Averaging KD-IOWA Operator, that allocates more importance to those agents in the vicinity that presents higher KD. We have chosen to use the IOWA operator [61] instead of a pondered average as it is used in others opinion dynamics model [35, 14] since in group decision making scenarios it has been proved its effectiveness for both trading of the information depending on certain criteria and dealing against malicious behaviour or manipulation [15]. Some examples of IOWA are the consistency based IOWA operator, [28], in which the reordering of arguments to aggregate as well as the computation of the aggregation weights are obtained using consistency degrees or the Consitency-Confidence IOWA, CC-IOWA [54] that strikes the balance between agents'consistency and confidence for the case of IFPRs.

The general definition of an IOWA operator is as follows:

Definition 16. *An IOWA operator of dimension m is a function $\Phi_W : (\mathbb{R} \times \mathbb{R})^m \to \mathbb{R}$, to which a set of weights or weighting vector is associated, $W = (w_1, \ldots, w_m)$, such that $w_i \in [0,1]$ and $\Sigma_i w_i = 1$, is expressed as follows:*

$$\Phi_W (\langle u_1, p_1 \rangle, \ldots, \langle u_m, p_m \rangle) = \sum_{i=1}^{m} w_i \cdot p_{\sigma(i)},$$

being $\sigma : \{1, \ldots, m\} \to \{1, \ldots, m\}$ a permutation such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, $\forall i = 1, \ldots, m-1$.

The proposed Knowledge Degree based OWA operator can be computed as follows:

Definition 17 (KD-IOWA operator)**.** *Let $E_v = \{e_1, \ldots, e_m\}$ be the set of agents connected to a given agent e_h. These agents provide preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, using the reciprocal intuitionistic fuzzy preference relations, $\{B^1, \ldots, B^m\}$. A Knowledge degree IOWA (KD-IOWA) operator of dimension m, Φ_W^{KD}, is an IOWA operator whose set of order inducing values is the set of KD values, $\{KD^1, \ldots, KD^m\}$, associated with the set of agents.*

The KD is computed from each agent's C_F and C_T as in (12)

Therefore, the global reciprocal intuitionistic fuzzy preference relation $G = (g_{ij}) = (\langle \mu_{ij}^{KD}, \nu_{ij}^{KD} \rangle)$ is computed as follows:

$$\mu_{ij}^{KD} = \Phi_W^{KD} \left(\langle KD^1, \mu_{ij}^1 \rangle, \cdots, \langle KD^m, \mu_{ij}^m \rangle \right) = \sum_{h=1}^{m} w_h \cdot \mu_{ij}^{\sigma(h)} \tag{18}$$

$$v_{ij}^{KD} = \Phi_W^{KD}\left(\langle KD^1, v_{ij}^1\rangle, \cdots, \langle KD^m, v_{ij}^m\rangle\right) = \sum_{h=1}^{m} w_h \cdot v_{ij}^{\sigma(h)} \tag{19}$$

such that $KD^{\sigma(h-1)} \geq KD^{\sigma(h)}$, $w_{\sigma(h-1)} \geq w_{\sigma(h)} \geq 0$ ($\forall h \in \{2, \cdots, m\}$) with $\sum_{h=1}^{m} w_h = 1$, C_T^h the consistency level associated to $R^h = F(B^h)$, C_F^h the confidence level associated to B^h, and $\delta \in [0,1]$ a parameter to control the weight of both consistency and confidence criteria in the inducing variable.

To allocate different importance degrees, $\{u_1, \ldots, u_m\}$, to the different agents when doing the fusion of all the preference relations into a global one, it is required to transform the values to fuse under the importance degrees and with these transformed values carry out the aggregation by means of an aggregation operator. Concretely, in the area of quantifier guided aggregations, it has been proposed a procedure that asses the global satisfaction of m important criteria (agents) by an alternative x by calculating the weighting vector associated to an OWA operator, [62], in the following way:

$$w_h = Q\left(\frac{S(h)}{S(m)}\right) - Q\left(\frac{S(h-1)}{S(m)}\right)$$

being Q the membership function of the linguistic quantifier, $S(h) = \sum_{k=1}^{h} u_{\sigma(k)}$, and σ the permutation used to produce the ordering of the values to be aggregated. This approach for the inclusion of importance degrees associates a zero weight to those agents with zero importance degree. The linguistic quantifier is a Basic Unit-interval Monotone (BUM) function $Q : [0,1] \to [0,1]$ such that $Q(0) = 0$, $Q(1) = 1$ and if $x > y$ then $Q(x) \geq Q(y)$.

This procedure was extended by Yager, [61], to the case of IOWA operators. More concretely, each component in the aggregation consists of a triple with the first element being the value to aggregate, the second element representing the importance weight and third element the order inducing value . The same expression as above is used with σ being the permutation that order the induce values from largest to lowest. In our contribution we will take as well and IOWA approach and the consistency/confidence values associated with each agent will be used as both the importance weights and the order inducing values. Therefore the weights of the KD-IOWA operator are calculated as follows:

$$w_h = Q\left(\frac{\sum_{k=1}^{h} KD^{\sigma(k)}}{T}\right) - Q\left(\frac{\sum_{k=1}^{h-1} KD^{\sigma(k)}}{T}\right)$$

with $T = \sum_{k=1}^{m} KD^k$.

3.5. Consensus assessment

The consensus level has been regarded as the degree of similarity existing between the agents preferences evaluated at three different levels [3]. In order to asses the level of agreement between the agents with respect to the global aggregated preference and, as well, measure the similarities between the agents opinions, in this contribution the Global consensus is assessed as the average similarity between each agent's preference relation and the global aggregated one.

Definition 18. *Global Consensus*

The overall consensus level C_S in the decision making between the H agents taking part in the process is assessed in the following way:

$$C_S = \frac{\sum_{i=1}^{H} S_G^h}{H} \tag{20}$$

Where S_G^h is the similarity of the agent e_h with respect to the aggregated Global preference relation G expressed in (16)

This level is used to decide whether the feedback mechanism is applied or not to give advice to the agents, or when the consensus reaching process has to come to an end. When C_S satisfies a minimum threshold value $\kappa \in [0.5, 1)$, then the consensus reaching process ends, and the selection process is applied to achieve the solution of consensus.

4. Experimental results

In this section the proposed similarity-confindence-consistency network is leveraged in an iterative group decision making scenario to provide personalized recommendations to the users to increase the general agreement as depicted in Fig.3.

In the following we present the overall model representation with all its components together with the purpose of providing the reader with an overview of the proposed approach. The model is composed of four main phases: (1) Calculation of agents Knowledge Degree and agent profile identification(2) development of an influence network in which the inter agent influence is calculated based on the Knowledge Degree and similarity of the agents ; (2) Calculation of the level consensus among all the agents (3)Influence based information fusion and feedback spreading in the network (4) Resolution process.

- Phase 1, Calculation of agents knowledge degree and agent profile identification: In this phase the quality of the information provided by each agent, that is, the consistency is assess, as well as the agents self confidence, as referred in section 2.1.2 and 2.1.4 respectively. These measurements are applied to calculate the agents KD in (12) and to classify them as HCC, MCC and LCC as agents as referred in section **??**.

- Phase 2, Development of the influence network: By referring to Section 3.3, a novel influence network is built. This network is represented as a directed graph in which the influence of each agent is based in the similarity between the agents opinions, detailed in section 3.2, and their Knowledge Degree in (12). The agent's profile is taken into consideration to asses the agents influence in the network in terms of providing and receiving advise. The proposed similarity measure in (15) expresses the strength of agents connections sharing most similar preferences, known as structural equivalence relation.

- Phase 3, Influence network based feedback mechanism: When the group consensus is not high enough, i.e. it is lower than a threshold value representing consensus reaching state, a feedback mechanism phase that integrates both SNA and GDM methodologies is proposed as referred in section 3.4. This influence based feedback approach consist on a recommendation system that focuses on the agents who present less confidence and consistency in their opinions, MCC and LCC agents, and that are guided the HCC agents that presents the highest influence in the network. In order to fuse the information and to provide personalized recommendation to each agent based on the agents in his/her vicinity we propose to use the new KD-IOWA defined in (def. 17).

- Phase 4, Resolution process: Finally all agent preferences are fused into one collective preference relation by means of the KD-IOWA that induces the ordering of the preferences to aggregate based on the agents influence in the network. The exploitation procedure is then carried out by implementing the OWA quantifier guided dominance degree (QGDD) to derive the final ranking of alternatives from which the maximum dominance element is chosen as the solution of consensus for "most of" the agents in the network.

The steps that comply each of the previous phases are detailed in the following:

Step 1 Experts provide their IFPRs

Step 2 Confidence and consistency are calculated directly from the opinion of the agents expressed

Step 3 The opinion of the agents are aggregated and the consensus level is computed. If it is enough the procedure goes to Step 8 to calculate the final ranking of the alternatives, otherwise it continues with Step 4

Step 4 The profile of each agent is computed, HCC, MCC and LCC

Step 5 The adjacency matrix for the network is calculated taking into account both the proximity between agents, and their profiles

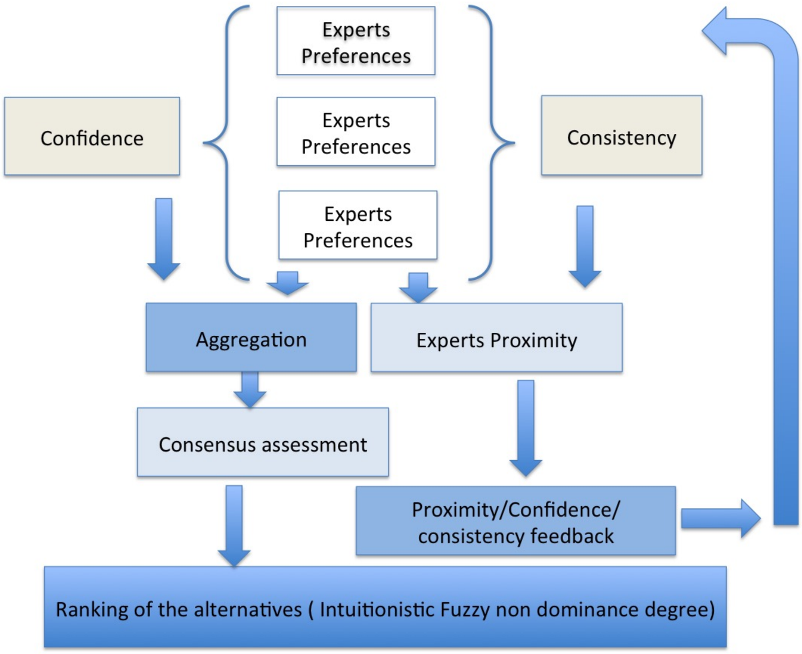


Figure 3: GDM problem resolution steps.

Step 6 The recommendations for the agents are computed by aggregating the opinions of the agents connected to the one receiving the recommendations.

Step 7 The agent will accept the recommendations provided by its vicinity with a $probability = probAccept$, the system will go to Step 2.

Step 8 The ranking of the alternatives is carried out, computing the Dominance and Non Dominance approaches.

To measure the success of the proposed approach the following measurements are going to be evaluated:

- Global consensus in each round measured as in (20).
- The evolution of the average similarity between the agents individuals opinions and the global solution. The similarity is measured according to (16)
- The evolution of the average agents consistency and the evolution of the individual agents consistency as measured as in (3), along the different rounds.
- The Steady State time ST defined as the minimum time it takes all agents opinions to reach a stable state. That is the minimum number of iterations the system requires to have an stable consensus level.

4.1. Simulation environment and experimental setup

In order to validate the proposed approach a computer simulation method in R inspired by the GDM-R Framework proposed by the authors in [53] is adopted to investigate the influence power of the different agents and the evolution of the collective consensus. In order to reproduce the experiments both the code and the dataset can be download from [1].

For all of the computer experiments, the Monte Carlo simulation is conducted 1000 times as in [13] and the dataset with the agents preferences is generated synthetically with the following constraints:1. The probability of an agent being completely consistent in this data set follows a binomial distribution set by the parameter called $probConsistent$. 2. The probability that an agent accept the proposed feedback from the network follows a binomial distribution in which the probability of succeed, that is, that the agent accept the recommendation is a parameter called p.

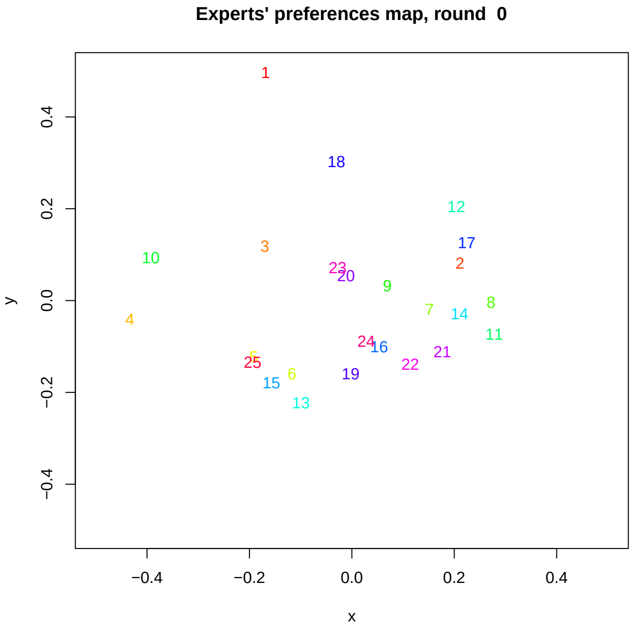
4.1.1. Evolution of the network topology

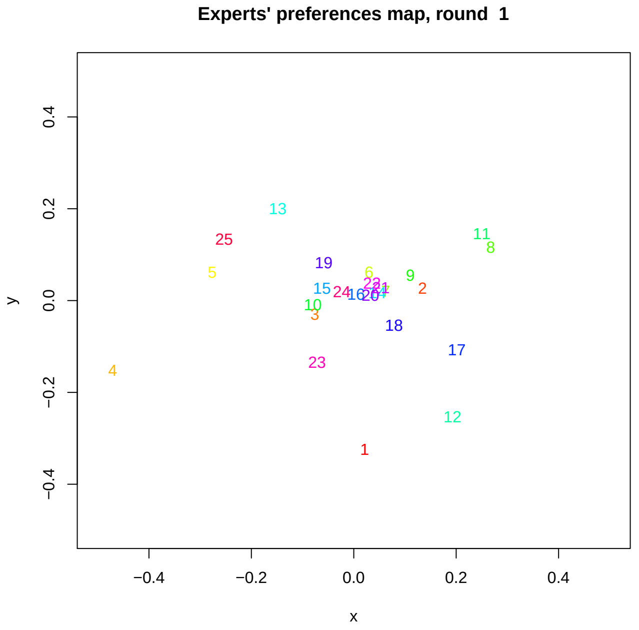
First of all the evolution of the network topology in subsequent iterations is studied in a normal scenario in which the agents accept the recommendations provided by the network with a medium accept probability. For the sake of simplicity this simulation is carried out with N= 25 agents. The assumptions and parameter setting for this simulation are indicated in table 1.

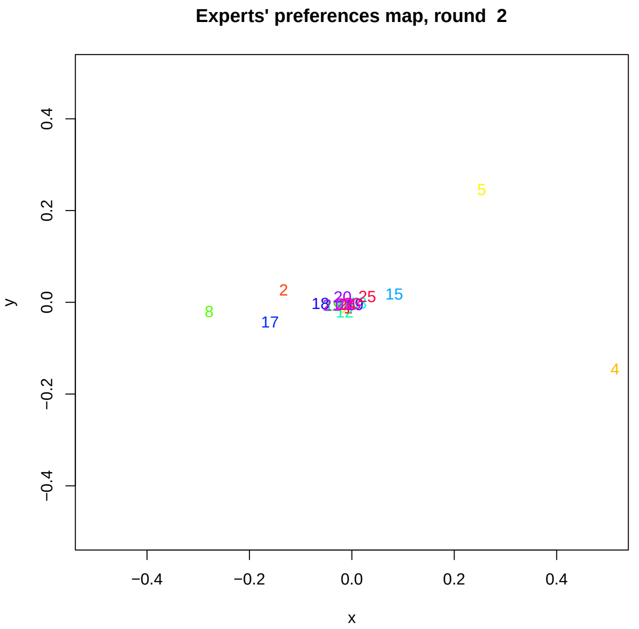
Table 1: Parameter setting for the experimental simulation

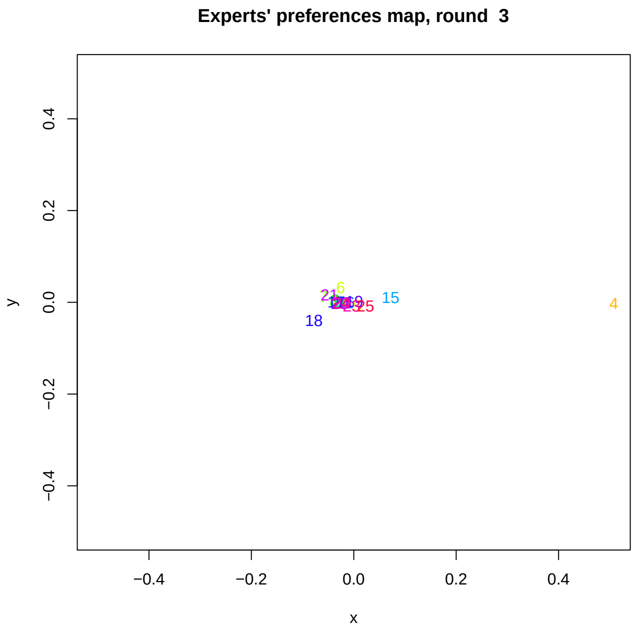
Parameter	**Value**
Number of agents	$N = 25$
Number of Alternatives	$A = 3$
Probability of fully consistent agent	$probConsistent = 0.3$
Adoption probability	$p = 0.7$
KD minimum Threshold	$KD_{THmin} = 0.3$
KD superior Threshold	$KD_{THsup} = 0.8$
Similarity Threshold	$\alpha_{sim} = 0.6$

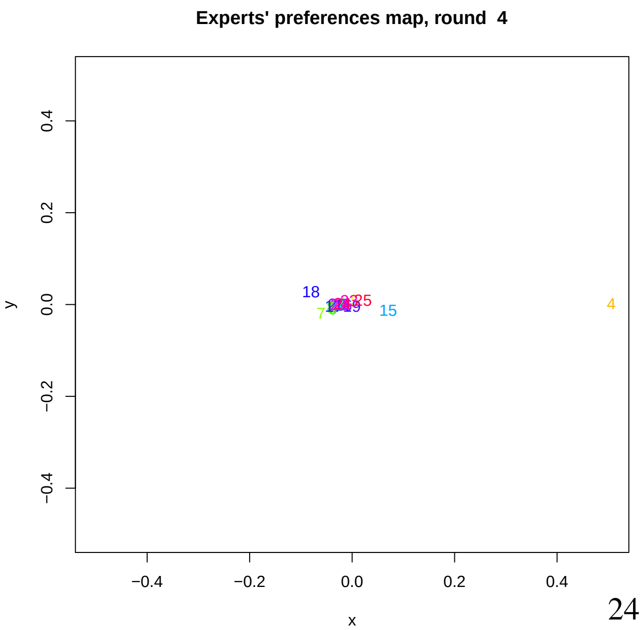
Table 2 illustrates a two dimension reduction of the agents preferences in each of the iterations until a stable state is achieved. This map-like representation allows to identify how the agents opinions are located with respect to each other and how the majority of them converges towards a common solution within the different iterations. However, there exist some exceptions, as is the case of agent number 4, whose opinions remind far from the other agents. This is due to its initial lack of influence in the network and to have opinions far from from the accepted ones.


Experts' preferences map, round 0
x
y


Experts' preferences map, round 1
x
y


Experts' preferences map, round 2
x
y


Experts' preferences map, round 3
x
y


Experts' preferences map, round 4
x
y

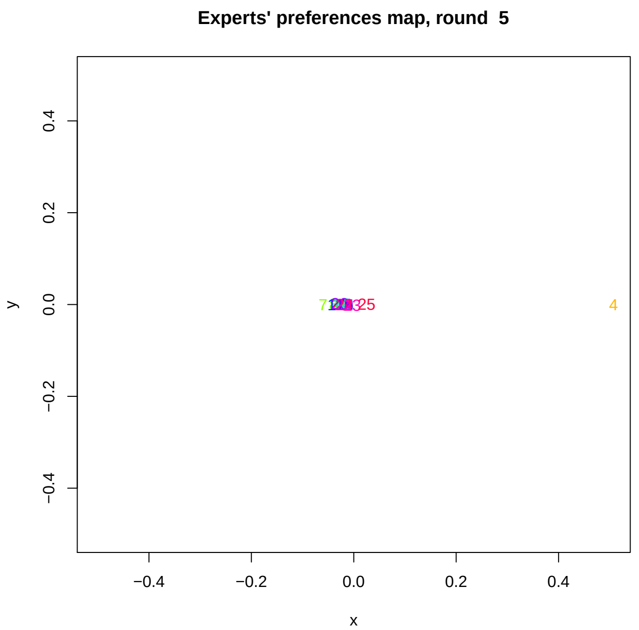

Experts' preferences map, round 5
x
y

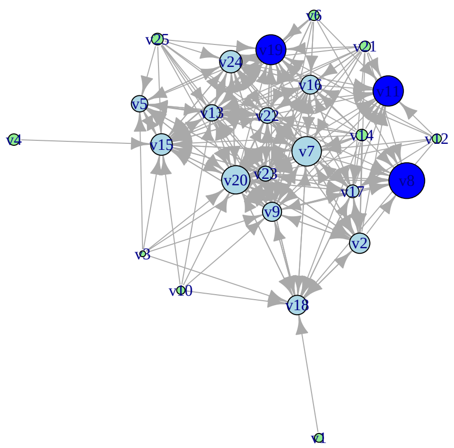
Table 3 depicts the evolution of the network topology across the different rounds until a stable state is reached. The size of the nodes is proportional to the agents knowledge degree, KD, in each iteration. As aforementioned the KD is used as a measure of the influence that each agent exert in his neighbourhood, therefore in this representation, the bigger the node, the higher the influence. The colour of the nodes represents the type of agent(deep blue, light blue and light green for HCC, MCC and LCC agents respectively). Note that the networks generated in each iteration exhibit the typical properties of a real world social network namely [47, 13]: 1. The small-world effect, the majority of the pairs of nodes are connected by a short path through the network. 2. The degree distribution follows a power law. More concretely, we can observe the HCC agents present a high-in degree, many nodes are connected to them and so they are the most influential agents in the network, whereas the LCC agents presents a high out-degree but a null in-degree, and therefore they are deeply exposed to the opinions from the other agents without exerting any influence. However, the influence of the LCC agents that get connected to the network may increase as they adopt the advise coming from the network and so they produce higher consistent opinions, for example, agent 15 evolves from LCC to HCC. On the other hand the ones that do not get connected to the network, as it is the case of agent 4, get isolated and so they barely evolve within the iterations, as it is the desired behaviour to deal with malicious users. Obviously the evolution of the agent's influence in the network highly depends on the agent' willingness to accept the recommendations from his neighbours the higher the probability of acceptance p the better the evolution of the influence of the agents.

Table 4 depicts the evolution of the global consensus and average consistency in the different rounds. Note that the biggest variation is reported during the three first iterations, then the consensus level reminds pretty stable, around 0.95. Therefore we can conclude that the iterations where there is more exchange of opinions between the agents take place during the first three rounds, and them the system converges. The evolution of each agent consistency and the global average consistency experiment a similar behaviour verifying that the proposed system helps to increase the individual consistency level almost to 1.

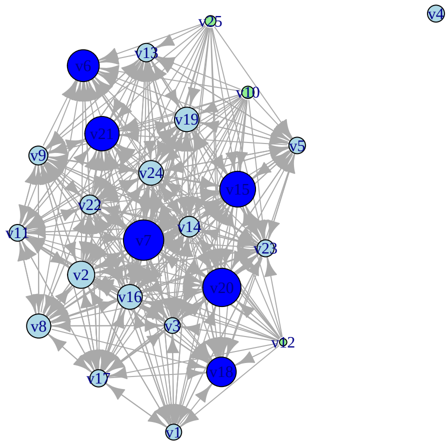
The numbers of cluster among the iterations varies between 1 to 2. This is motivated by the existence of agent number 4 that it gets isolated from the rest of the network. The number of clusters depends on the minimum similarity threshold to warranty a communication between agents, the higher the threshold the bigger the resulting number of clusters.

Table 5 depicts the histogram of the consistency level at the beginning of the process, where we can observe that the majority of the agents report levels lower than 0.8 and at the end of the process, where the agents reports consensus levels over 0.95, demonstrating the efficacy of the proposed approach to increase the quality of the opinions provided in the process.

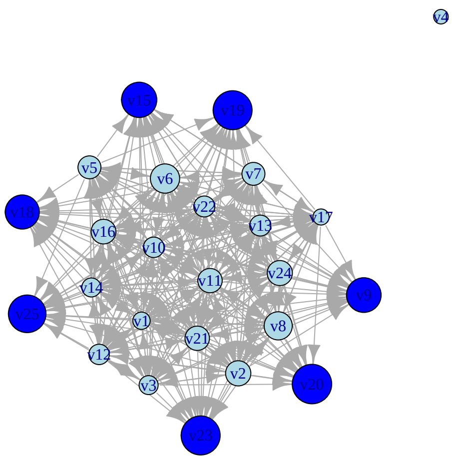
Network evolution round 0



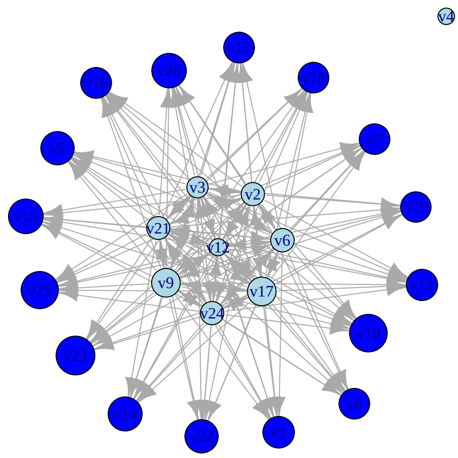
Network evolution round 1



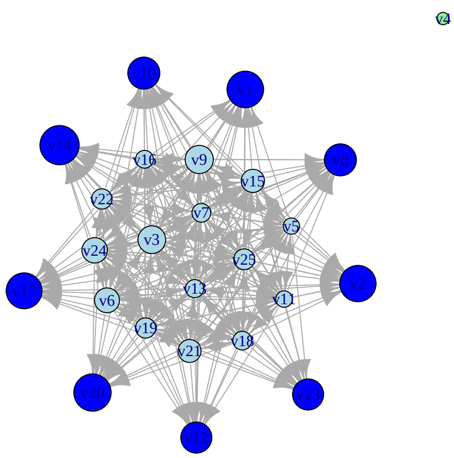
Network evolution round 2



Network evolution round 3



Network evolution round 4



Network evolution round 5

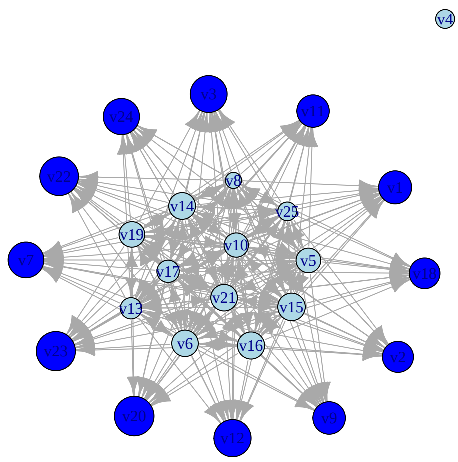


Table 6 depicts the histogram of the agents profiles, (HCC, MCC,LCC) at the beginning and in the last iteration of the process. Showing how the influence of the agents increases with the iterations, since there is no remaining LCC, and the number the HCC evolves from 3 to 12.

4.1.2. Impact of the similarity threshold

In the following we investigate the impact of the network Similarity Threshold, α_{sim} and the Agents Adoption Probability p in the proposed approach in terms of the following criteria:

- Steady state time T, defined as the minimum time it takes all agents' opinions to reach a stable state [13].
- Number of cluster in the stable state, N_{CL} defined as the number of different opinion clusters among the agents in the stable state. Larger N_{CL} values indicate more different opinions among the agents in the stable state, whereas $N_{CL} = 1$ indicates that all agents reach a consensus [13].
- Global consensus level in the stable state

Figs. 4, 5 and 6 reveal the impact of α_{sim} and p on the steady-state T, Global Consensus C_S and Number of Clusters at the stable state, N_{CL}, respectively.

In Fig. 4 we can observe that for a given value of p for medium to low values of α_{sim} the network is almost fully connected and so it stabilizes in few iterations because almost all the agents communicate between each others leading to a network topology dominated by a big main cluster (Fig. 6). For medium values of α_{sim} the social network progressively changes its topology to a collection of big clusters, however there is still a high level of communication that leads to longer T but still with high values of consensus as shown in Fig. 5. Finally, for high values of α_{sim}, T decreases since there is less interaction between the agents and so they stabilize faster. However, this low interaction produces lower values of C_S, (see Fig. 5) and the network topology is composed of a high number of little clusters as depicted in Fig. 6.

From the point of p we can observe that as p decreases T increases as well, reaching values of T higher than 10 when p is lower than 0.4. However for very low values of p T is very low because, as there is very low adoption probability the agents' opinions barely changes and consequently the system stabilizes very fast, but with very low consensus levels.

Fig. 5 reveals the impact of α_{sim} and p on the final global consensus C_S reached when the system gets a stable point. For a given value of p as α_{sim} increases C_S decreases since the higher the value of α_{sim} the lower the interaction between agents, and therefore lower information exchange to reach a consensus solution. In these scenarios, the ones with high α_{sim} , the network topology is composed by a few number of stables clusters, that interact between each others but do not communicate with

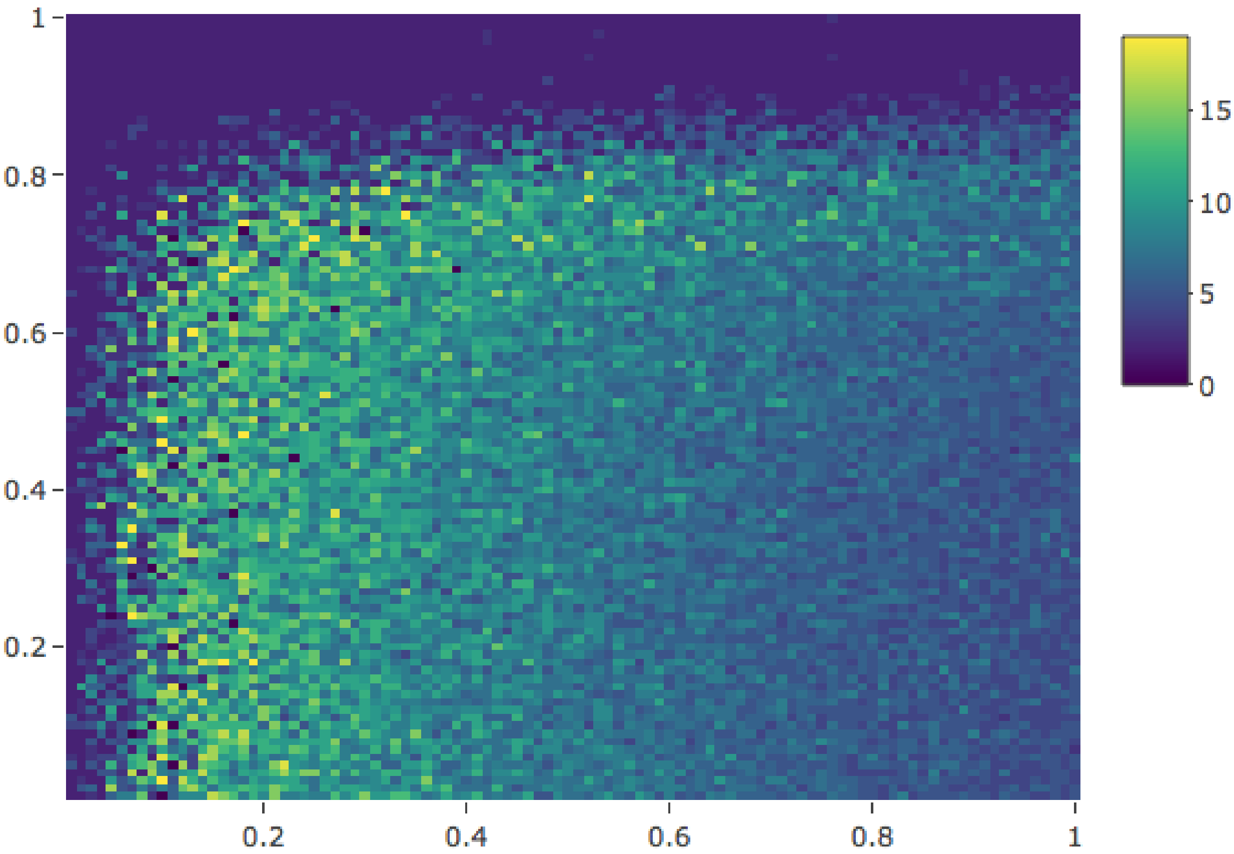


Figure 4: The average Steady State time T values (represented by the colours) under different p (x axis) and α_{sim} (y axis)

external members. Note that even for values of p close to 1, that means that the agents accept all the recommendations, the level of consensus is very low. On the contrary, for medium and low values of α_{sim} the consensus levels are almost 1 even for low values of p, proving that the system is able to converge to a consensus solution even when the agents are more reluctant to change their opinions.

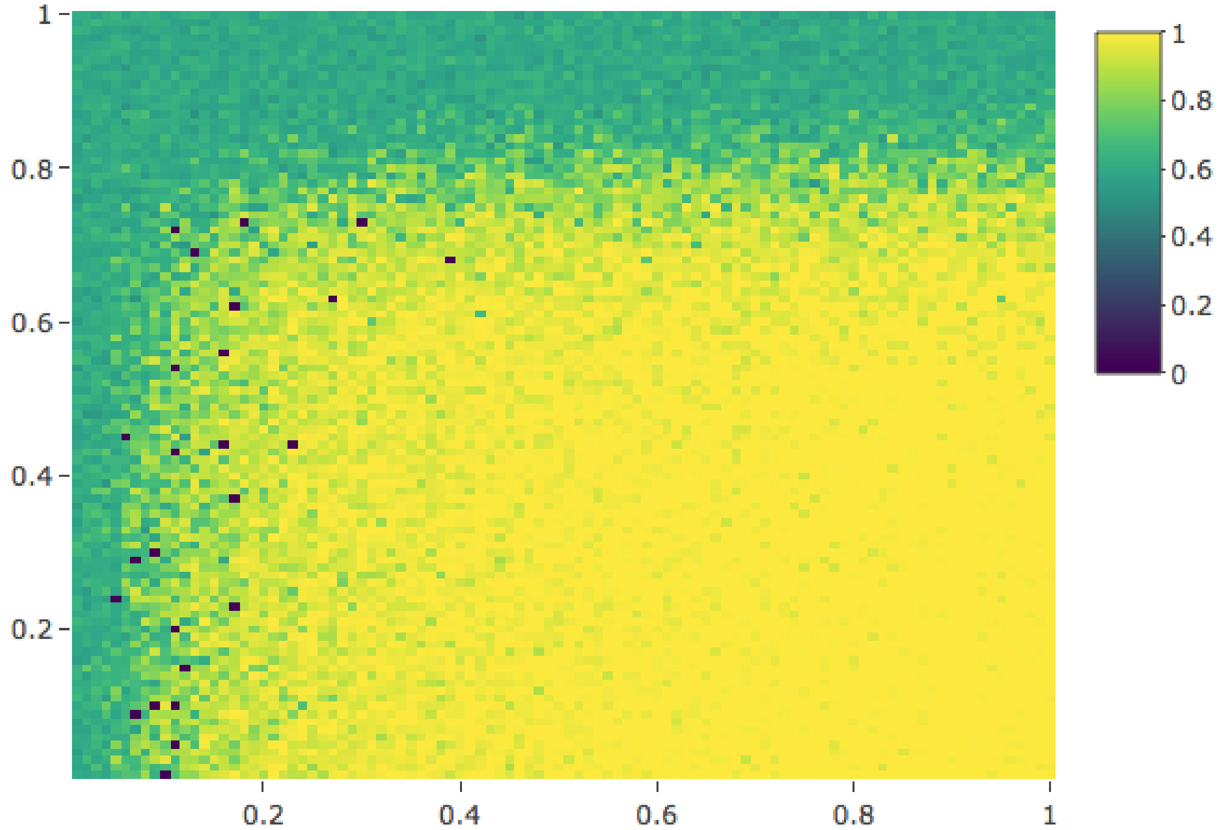


Figure 5: The Global Consensus level at the stable state C_S (represented by the colours) under different p (x axis) and α_{sim} (y axis)

5. Conclusions

With the wide incursion of networked communication mechanisms in every day life, such as social networks, it is necessary to develop mechanisms to reach decisions taking into consideration the opinions from multiple heterogeneous users in web communities. In addition, many scenarios such as the ones involving e-politics, marketing or e-health require that the majority of the people agrees with the decision. To do so, in this contribution we propose a new similarity based influence social network that leverage the knowledge of the crowds to model the public opinions dynamic and to reach consensus among the different agents involved in the decision making process. We have conducted extensive simulations to investigate the evolutions of the agents' opinions concluding that the proposed network permits the agents to increase the consistency of their opinions as well as bring

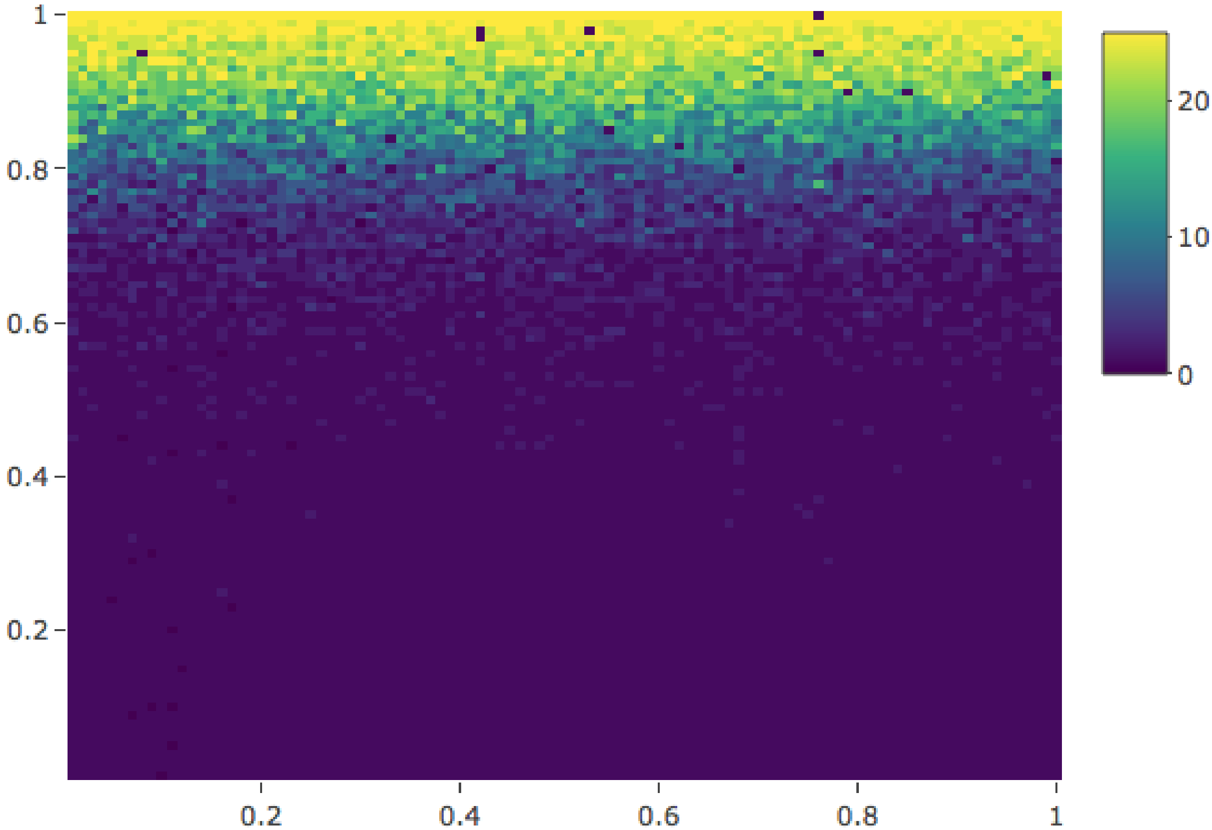


Figure 6: The number of opinion clusters at stable state, N_{CL} (represented by the colours) under different p (x axis) and α_{sim} (y axis)

their opinions closer in few iterations overcoming the initial reluctance of the agents to change their minds and isolating those agents that might present a malicious behaviour. Moreover It has been proved that when the generated advices are implemented, consensus increases. In fact given that the consensus is bounded this result guarantees the convergence of the feedback process to consensus reaching state.

The main contributions to the literature of the proposed approach are the following:

- The proposed approach implements an influence network in which the inter agents influence is calculated by means of a dynamic combination between the similarity in the opinions of the agents as well as each agent confidence and consistency, levels.

- The system is implemented as a directed graph, and so it allows to point out in each iteration the most influential agents and use their opinions as recommendations to other like-minded agents. Moreover the opinions of those agents presenting lower confidence and consistency levels can evolve towards higher influential positions thanks to the advice received.

- In order to avoid malicious users behaviours the system is able to recognize and isolate those agents with opinions very different from the rest and those whose coherence levels, that is the consistency, are very low. Moreover the use of an IOWA operator that dynamically calculates the ordering of the opinions in the aggregation to provide the personalized feedback avoids malicious user to "learn how the system works" and to take advantage of the system to impose their opinions.

- This influence network has been applied in a Group decision making process with the purpose of filling the gap between social networks and the classical consensus reach process which is a dynamic and iterative process guided by a moderator and composed by several rounds in which the individuals express, discuss and modify their opinions until reaching an agreed decision. In this model we leverage well known opinion dynamics models to carry out opinion diffusion with the final objective of reaching consensus.

As future work the challenge is to leverage the proposed model including trust propagation mechanisms in e-health and e- marketing scenarios, and to test the proposing approach using other type of propagation models in social networks such us the cascading propagation model. In order to do that we plan to take advantage of the labelled Graph rewriting methodologies, as the one presented in the framework PORGY to test and visually compare the network topology and the convergence speed using different social networks scenarios [55, 19, 20].

Another challenge consists on measuring the similarity between users depending on the exact context, for example between patients in a medical social network.

Apart from the previous ones, another important issue to take into consideration as future research is concerned with the bias in social networks scenarios, and how to address it in when fusing the agents information in order to provide recommendations.

6. Acknowledgements

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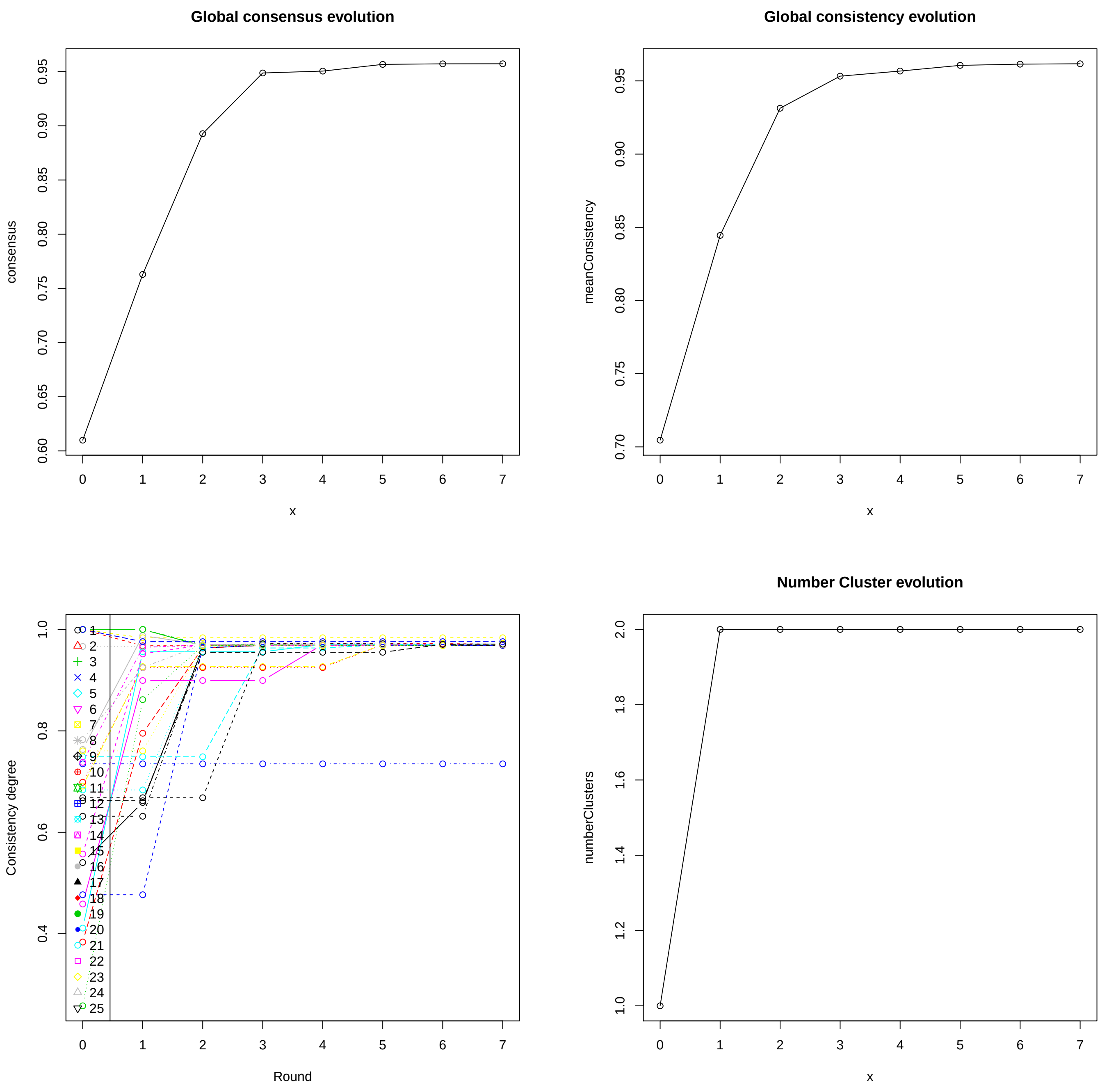
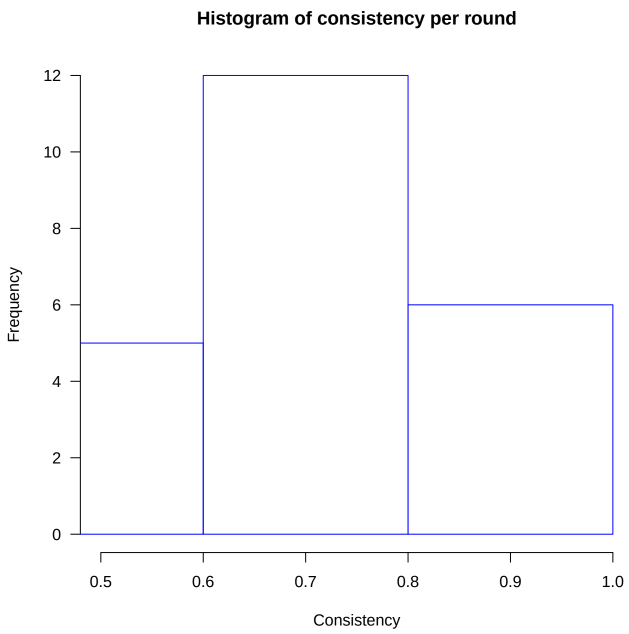


Table 4: Evolution of the consensus and consistency during the different consensus rounds


Histogram of consistency per round
Frequency
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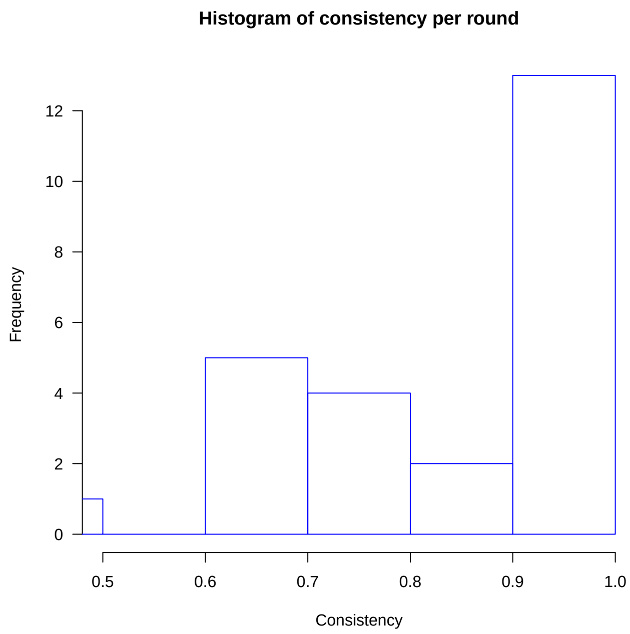
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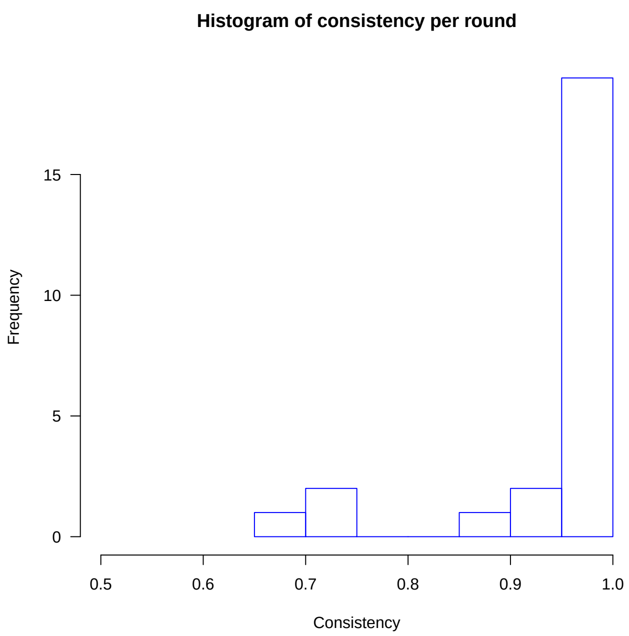
Histogram of consistency per round
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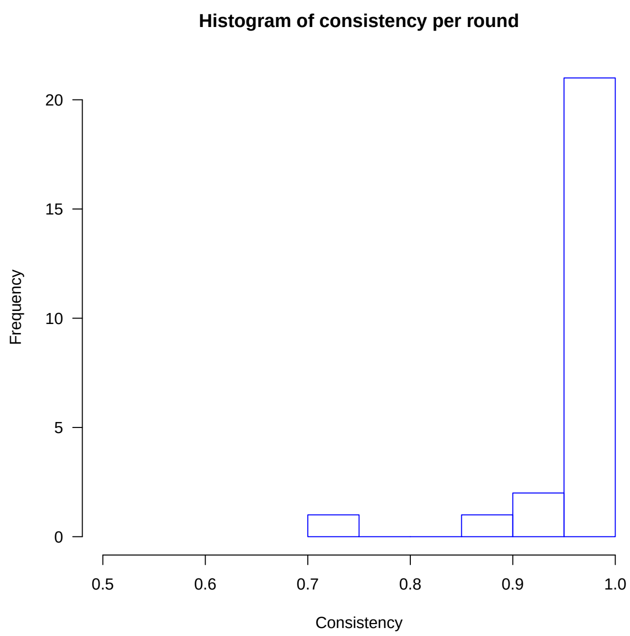
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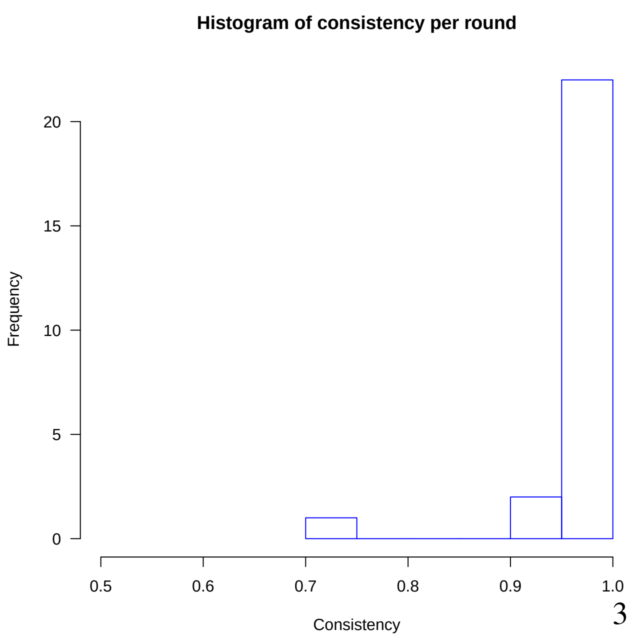
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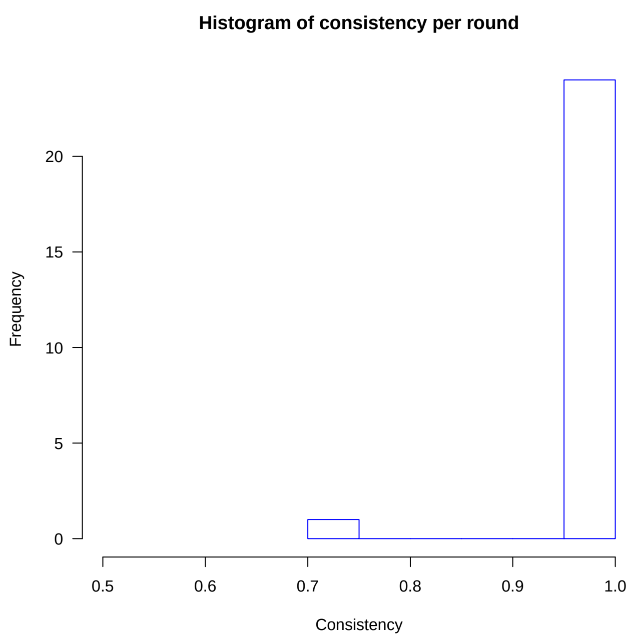
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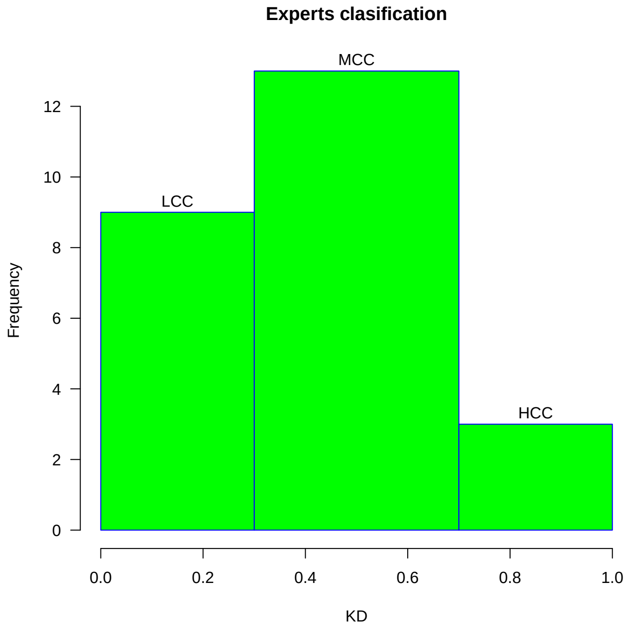
Experts clasification
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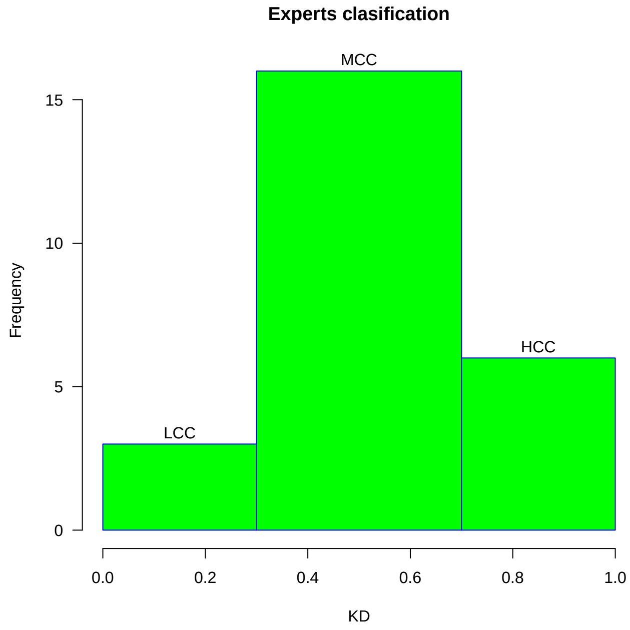
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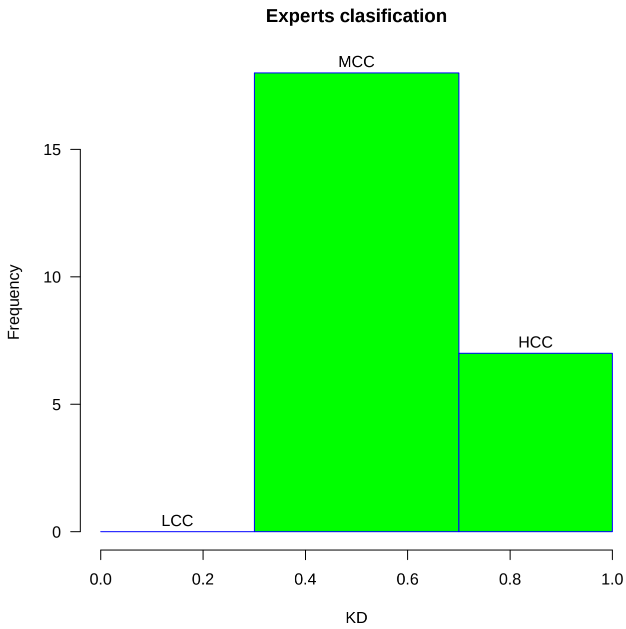
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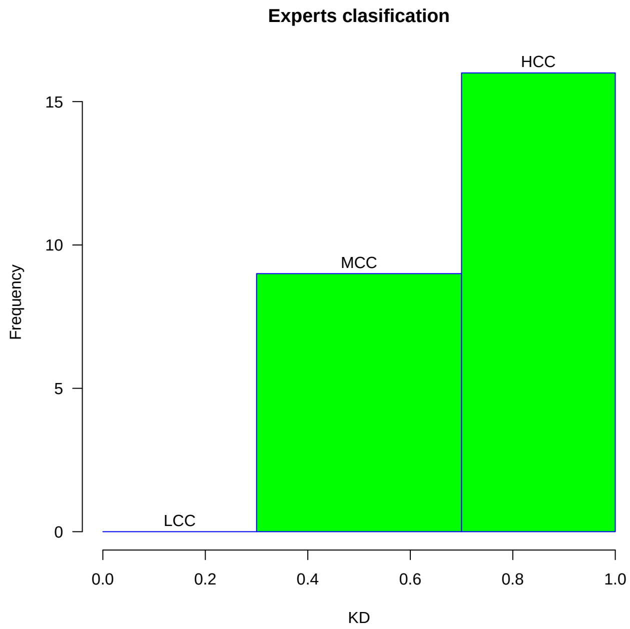
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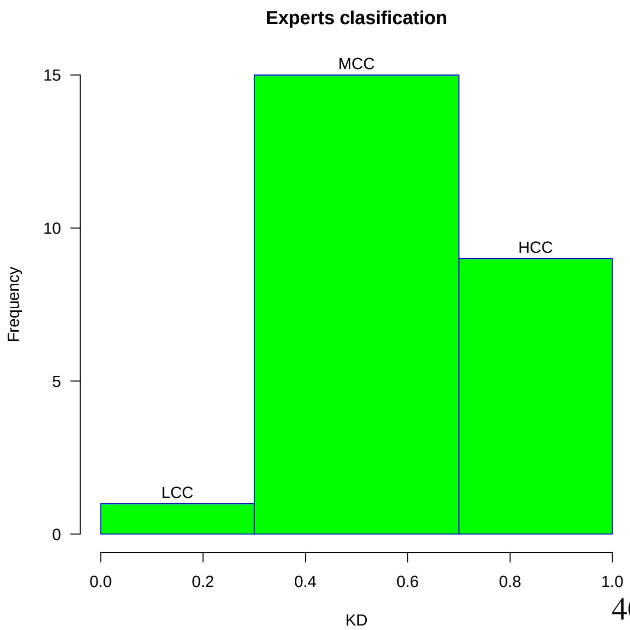
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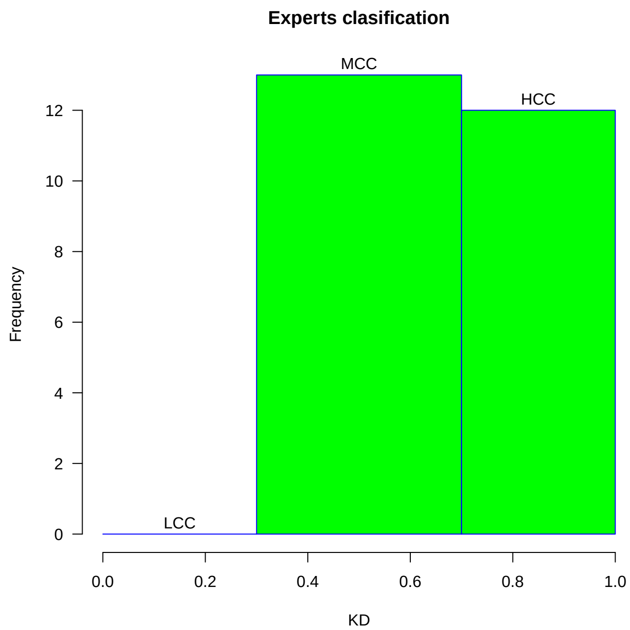
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