Received: 16 March 2023 | Revised: 30 November 2023 | Accepted: 27 February 2024

DOI: 10.1002/hec.4826

RESEARCH ARTICLE

Health Economics WILEY

Optimal self-protection and health risk perceptions: Exploring connections between risk theory and the Health Belief Model

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Abstract

In this contribution to the longstanding risk theory debate on optimal self-protection, we aim to enrich the microeconomic modeling of self-protection, in the wake of Ehrlich and Becker (1972), by exploring the representation of risk perception at the core of the Health Belief Model (HBM), a conceptual framework extremely influential in Public Health studies (Janz and Becker, 1984). In our two-period model, we highlight the crucial role of risk perception in the individual decision to adopt a preventive behavior toward a generic health risk. We discuss the optimal prevention effort engaged by an agent displaying either imperfect knowledge of the susceptibility (probability of occurrence) or the severity (magnitude of the loss) of a health hazard, or facing uncertainty on these risk components. We assess the impact of risk aversion and prudence on the optimal level of self-protection, a critical issue in the risk and insurance economic literature, yet often overlooked in HBM studies. Our results pave the way for the design of efficient information instruments to improve health prevention when risk perceptions are biased.

KEYWORDS

Health Belief Model, prudence, risk aversion, risk perception, self-protection, uncertainty

JEL CLASSIFICATION

D81, I12, D9

1 | INTRODUCTION

Since the seminal paper of Ehrlich and Becker (1972), a vast literature in risk theory has addressed the topic of self-protection and self-insurance. It aims at a better understanding of the decision-making process guiding rational agents in allocating resources to mitigate financial or health risks. As shown in the extensive literature review of Courbage et al. (2013), this issue raises fascinating questions on the role risk aversion (Dionne & Eeckhoudt, 1985; Jullien et al., 1999), ambiguity aversion (Courbage & Peter, 2021) and prudence (Chiu, 2005; Eeckhoudt & Gollier, 2005) can play in optimal individual prevention. In addition, extending the time scale of the modeling framework to two periods can also yield contradictory results (Menegatti, 2009). Beyond a perfect information setting, some contributions to this strand of research have explored the impact of imperfect information on optimal self-protection. On the one hand, a biased estimation by the agent of the probability component of the risk has been tackled by including probability weighting (Bleichrodt & Eeckhoudt, 2006; Baillon et al., 2022) and pessimism (Etner and Jeleva, 2013). On the other hand, Brianti et al. (2018) have introduced uncertainty on the actual effect of the disease and on the effectiveness of the cure.

In parallel to this vibrant theoretical literature in Economics, the determinants of individual prevention under subjective beliefs has been widely studied in the Public Health Literature through the lens of the Health Belief Model (HBM). Since its early developments in the 1950s by the U.S. Public Health Service, it has gained massive momentum within the Public Health discipline and has become "one of the most widely used social cognition models in health psychology" (Conner & Norman, 2005), at the conceptual root of numerous empirical studies of prevention behavior for various health issues.[1] Its predictive power for many health afflictions has been assessed regularly and remains satisfying (Carpenter, 2010; Janz & Becker, 1984). The HBM distinguishes various channels explaining the adoption of health-related prevention behavior and underlines in particular the prominent role of subjective risk perception in each individual's decision to engage in preventive behavior. This risk perception at the core of the model is broken down into two essential constructs: *severity perception* and *susceptibility perception*, reflecting respectively the subjective utility loss sustained if the health risk materializes and the subjective probability of this health impairment. Other complementary constructs such as *perceived barriers*, *perceived benefits*, and more recently *self-efficacy* (Rosentstock et al., 1988) are also factored in the HBM as potential determinants of health prevention behavior, but we will not include them explicitly in our model.

Although it has been designed initially for operative Public Health studies, this HBM framework displays organic ties with expectancy theory since it addresses self-protection through the same expected-value prism as the optimal self-protection literature cited previously.[2] Given the large HBM-based empirical evidence gathered on the impact of risk perception on health prevention behavior, it seems only natural to investigate the connections between this Public Health model and the modeling of optimal self-protection in Economics. In the latter strand of literature, Crainich and Eeckhoudt (2017) have pondered over the case of heterogeneous baseline probability of disease, but to our knowledge no economic model has addressed subjective risk perception in line with the HBM by disentangling risk into two distinct channels: severity perception on the one side, susceptibility perception on the other. Hence our attempt to build a health-focused microeconomic self-protection model that isolates the role of these beliefs. The impact of these beliefs will be explored in two directions. First, we will assume that the agent has her own biased perceptions and second, we will confront the agent with uncertainty on severity and/or susceptibility, a situation overlooked by HBM studies so far. Throughout the paper we will differentiate the behavior of risk averse and risk neutral agents, a distinction that is largely ignored in the HBM approach but that can nonetheless have a significant impact as we will show.

The paper will present in Section 2 our original HBM-inspired optimal self-protection model suited for risk averse and risk neutral agents displaying subjective severity and susceptibility beliefs. Section 3 solves the model and delves into the comparative statics of optimal self-protection for the risk perception parameters and the discount factor. The impact of risk aversion on self-protection is analyzed in Section 4. In Section 5 we discuss how the agent's self-protection decision will shift when she is facing uncertainty on either or both severity or susceptibility. Section 6 concludes with the downstream research avenues opened up by our introduction of these two channels of risk perception in the self-protection decision process.

2 | A MODEL OF RISK PERCEPTION-BASED SELF-PROTECTION

2.1 | A two-period health risk self-protection model based on Health Belief Model constructs

In order to address the role of risk perceptions in optimal individual prevention, we assume that the agent has subjective beliefs regarding both the susceptibility of infection and the severity of the health risk. In line with the wide range of HBM-based empirical studies, our model can encompass a large spectrum of health risks such as infectious[3] or chronic diseases against which a preventive behavior reducing the health risk probability can be adopted. This self-protection is costly in the broadest sense: it consists either in actual spending in goods or services reducing this health susceptibility or in intangible costs reflecting the tediousness of the prevention efforts ε (exercising, eating healthier food…), with $0 \leq \varepsilon \leq \varepsilon_{\max}$, where $\varepsilon_{\max}$ is exogenous and strictly positive. The increasing convex cost of self-protection $C(\varepsilon)$ echoes the concept of *perceived barriers* in the HBM constructs (see Figure 1) as the cost of self-protection is not necessarily monetary but diminishes utility nonetheless. As our model focuses on health behaviors that can imply intangible adoption costs, neither consumption nor wealth are included explicitly in the utility function and saving from one period to another is not an option. However, the trade-off between the benefits of self-protection and its cost in terms of utility remains. Since our model aims at capturing the largest span possible of health issues, we need to operate a clear timing distinction between the prevention efforts and the occurrence of the health risk. Indeed, chronic diseases, such as diabetes or cardiovascular conditions, demand prevention efforts to avoid health impairments in the long haul. Hence a two-period model where the health loss potentially occurs in the second period, similar to Menegatti (2009).

The first period utility U_1^d is thus modeled as the difference between the baseline constant utility U_F of *business as usual* and the tangible and intangible costs of engaging in self-protection efforts $C(\varepsilon)$. The second period utility U_2^d, weighted in by a discount factor $0 < \beta < 1$, is determined by an increasing concave health utility function u applied either to a healthy status $u(H)$ or to a degraded health status $u(H - M)$ where M is the severity of the disease[4] and $H \geq M > 0$. In the HBM, this severity encompasses "both medical/clinical consequences and possible social consequences" (Janz & Becker, 1984).

The probability to suffer the health condition, that is, to catch an infectious disease or be affected by a chronic ailment, will be captured by the probability function $\alpha(\varepsilon; \pi)$ where π is the susceptibility, namely the perceived baseline probability "of contracting a [health] condition" (Janz & Becker, 1984) with $\pi \in [0, 1]$. The probability function α increases naturally with π. Self-protection efforts reduce α with diminishing returns such that $\frac{\partial \alpha(\varepsilon;\pi)}{\partial \varepsilon} < 0$ and $\frac{\partial^2 \alpha(\varepsilon;\pi)}{\partial^2 \varepsilon} > 0$. Considering that when the baseline susceptibility is higher, an additional protection effort decreases more strongly the probability to contract the condition, we have $\frac{\partial^2 \alpha(\varepsilon;\pi)}{\partial \varepsilon \partial \hat{\pi}} < 0$. For instance in the case of an infectious disease, a higher prevalence rate mechanically raises the marginal efficiency of self-protection measures since it protects against a larger presence of infectious agents.

2.2 | Risk perception and decision utility

We assume that the individual agent is characterized by her subjective susceptibility perception $\hat{\pi}$ and her severity perception $\hat{M}$, that differ from the true risk parameters. Consequently, her self-protection effort decision is based on the

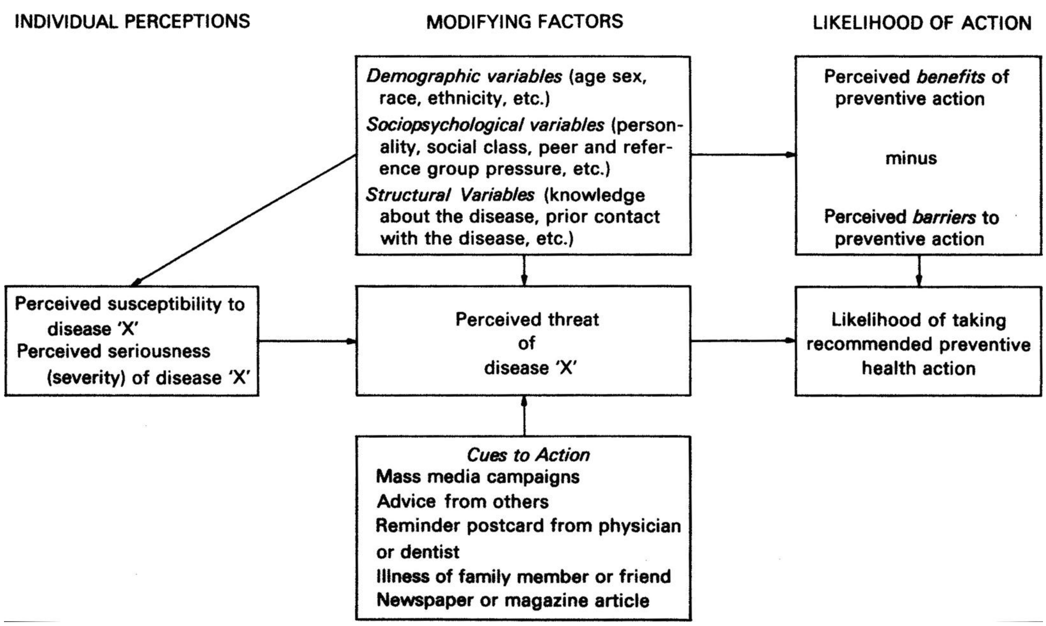

FIGURE 1 The Health Belief Model (HBM), Davidhizar (1983).

maximization of a perception-based utility function, while the actual utility she will experience results from the injection of this prevention effort into a utility function set on the true parameters. The larger the information bias between perceived and true parameters, the larger the gap between these two utilities. Hence the notation U^d we adopt to keep in mind that this utility function operates as the decision function of the agent but does not yield her actual welfare (Jimenez-Gomez, 2018).

Moreover, to disentangle health preferences and risk aversion, we resort to an expected utility function *à la* Kihlstrom-Mirman (1974). Such a utility function enables to consider risk aversion in a two-period expected utility framework (Bommier et al., 2012; Bommier & Le Grand, 2014) as it keeps ordinal preferences unchanged and also satisfies the monotonicity property.[5] The intertemporal utility function for an agent with biased risk perceptions is thus written as follows:

$$U^d\left(\varepsilon;\hat{M},\hat{\pi}\right) \ = \mathbb{E}\left[\phi\left(U_1^d(\varepsilon)+\beta U_2^d(\varepsilon)\right)\right],$$

with ϕ an increasing and concave function. The risk aversion of this function is characterized by the concavity of ϕ.[6]

Injecting the specific utility components presented in subsection 2.1 yields

$$U^d\left(\varepsilon;\hat{M},\hat{\pi}\right) = \alpha(\varepsilon;\hat{\pi})\left(\phi\left(U_F - C(\varepsilon) + \beta u\left(H-\hat{M}\right)\right) + (1-\alpha(\varepsilon;\hat{\pi}))\phi(U_F - C(\varepsilon) + \beta u(H)\right). \tag{1}$$

3 | OPTIMAL SELF-PROTECTION WITH RISK PERCEPTION

After solving our model in 3.1 we discuss the salient comparative static results in 3.2 that question some HBM postulates.

3.1 | Optimal self-protection with subjective risk perception

For the sake of clarity we use subsequently, for any given function f, the notation $f^{(i)}(\varepsilon)$ that stands for $\frac{\partial^{(i)} f(\varepsilon)}{\partial \varepsilon^i}$, for $i = 2, 3$. For $i = 1$, the notation $f'(\varepsilon)$ will be used.

We shall work under Assumption 1 defined as follows:

Assumption 1 $\alpha^{(2)}(\varepsilon;\hat{\pi})\alpha(\varepsilon;\hat{\pi}) > 2(\alpha'(\varepsilon;\hat{\pi}))^2$.

Assumption 1 is required to ensure that self-protection can be a desirable choice and to guarantee the concavity in ε of the utility function U^d (Jullien et al., 1999, Condition C1, p. 23). Huber (2022) also stresses the need for this assumption when a Kihlstrom-Mirman expected utility function is used.

To facilitate the reading, we denote $w_1 = U_F + \beta u\left(H-\hat{M}\right)$ and $w_2 = U_F + \beta u(H)$ the utility levels associated respectively with the infected state and the healthy state. We have $w_1 < w_2$ by definition.

We can thus rewrite (1) as

$$U^d\left(\varepsilon;\hat{M},\hat{\pi}\right) \ = \alpha(\varepsilon;\hat{\pi})\phi(w_1) + (1-\alpha(\varepsilon;\hat{\pi}))\phi(w_2).$$

Then, the following Lemma holds:

Lemma 1 *For an agent with subjective risk perception ($\hat{\pi}$, $\hat{M}$) and a risk aversion function ϕ, the optimal level of self-protection $\varepsilon_\phi^*\left(\hat{M},\hat{\pi}\right)$ exists and is unique. If ε_ϕ^* is an interior solution, ε_ϕ^* is the solution of the following equation:*

$$\begin{aligned} &C'(\varepsilon)[\alpha(\varepsilon;\hat{\pi})\phi'(w_1 - C(\varepsilon)) \\ &\quad +(1-\alpha(\varepsilon;\hat{\pi}))\phi'(w_2 - C(\varepsilon))] = -\alpha'(\varepsilon;\hat{\pi})(\phi(w_2 - C(\varepsilon)) - \phi(w_1 - C(\varepsilon))). \end{aligned} \tag{2}$$

Proof. The proof is given in Appendix A1.

Equation (2) captures the standard trade-off in optimal individual prevention between the marginal cost of the protection effort on the left-hand side and the marginal benefit of this protection on the right-hand side. The former reflects the reduction of expected utility in both states of the world induced by an increase in effort. The latter consists in the avoidance of the health loss risk through a decreased infection probability $\alpha'(\varepsilon;\hat{\pi})$. This interpretation is particularly straightforward in the risk neutral case where (2) simplifies to $C'(\varepsilon) = -\alpha'(\varepsilon;\hat{\pi})(w_2 - w_1)$.

3.2 | Comparative statics of the risk perception channels and of the discount factor

By construction, the HBM implicitly assumes an unambiguous positive correlation between severity or susceptibility perception and individual prevention. While it has been confirmed empirically by a vast body of studies (Carpenter, 2010), this correlation lacks theoretical foundation. Our model allows us to explore the validity of this postulate through comparative statics on $\hat{M}$ and $\hat{\pi}$ for the risk averse and the risk neutral case.

As

$$\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \hat{M}} = \frac{\beta u'(H-\hat{M})\left[C'\left(\varepsilon_\phi^*\right)\alpha\left(\varepsilon_\phi^*;\hat{\pi}\right)\phi^{(2)}\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) - \alpha'\left(\varepsilon_\phi^*;\hat{\pi}\right)\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)\right]}{-U^{d(2)}\left(\varepsilon_\phi^*\right)}, \tag{3}$$

it appears that the sign of $\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \hat{M}}$ is ambiguous. Expression 3 yields straightforwardly the following Lemma:

Lemma 2 *For risk averse agents, if* $C'\left(\varepsilon_\phi^*\right) \lesseqgtr s_\alpha^* A_\phi^*$ *then* $\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \hat{M}} \gtreqless 0$, *where* $s_\alpha^* = \frac{-\alpha'\left(\varepsilon_\phi^*,\hat{\pi}\right)}{\alpha\left(\varepsilon_\phi^*,\hat{\pi}\right)}$ *is the absolute value of semi elasticity of* α *with respect to* ε *and* $A_\phi^* = -\frac{\phi^{(2)}\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)}{\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)}$ *is the absolute risk aversion of* ϕ.

Regarding perceived susceptibility we have

$$\begin{aligned}\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \hat{\pi}} = &\frac{-C'\left(\varepsilon_\phi^*\right)\left[\frac{\partial \alpha\left(\varepsilon_\phi^*;\hat{\pi}\right)}{\partial \hat{\pi}}\left(\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) - \phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)\right)\right]}{-U^{d(2)}\left(\varepsilon_\phi^*\right)} \\ &- \frac{\frac{\partial^{(2)}\alpha\left(\varepsilon_\phi^*;\hat{\pi}\right)}{\partial \hat{\pi}\partial \varepsilon_\phi^*}\left(\phi\left(w_2 - C\left(\varepsilon_\phi^*\right)\right) - \phi\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)\right)}{-U^{d(2)}\left(\varepsilon_\phi^*\right)},\end{aligned} \tag{4}$$

which leads directly to the following Lemma:

Lemma 3 *For risk averse agents, if* $C'\left(\varepsilon_\phi^*\right) \lesseqgtr s^*_{\frac{\partial\alpha}{\partial\hat{\pi}}} \frac{\phi\left(w_2 - C\left(\varepsilon_\phi^*\right)\right) - \phi\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)}{\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) - \phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}$ *then* $\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \hat{\pi}} \gtreqless 0$, *where* $s^*_{\frac{\partial\alpha}{\partial\hat{\pi}}} = \frac{-\frac{\partial^{(2)}\alpha\left(\varepsilon_\phi^*;\hat{\pi}\right)}{\partial\hat{\pi}\partial\varepsilon_\phi^*}}{\frac{\partial\alpha\left(\varepsilon_\phi^*;\hat{\pi}\right)}{\partial\hat{\pi}}}$ *is the absolute value of semi elasticity of* $\frac{\partial\alpha}{\partial\pi}$ *with respect to* ε.

Lemmas 2 and 3 reveal that, for some sets of beliefs on severity and susceptibility, an increase in perceived severity or susceptibility for risk averse agents may have the counter-intuitive impact of decreasing optimal self-protection $\left(\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \hat{M}} < 0 \text{ or } \frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \hat{\pi}} < 0\right)$. However, we can easily show that for risk neutral agents a higher perceived severity/susceptibility systematically yields a higher self-protection effort. This discrepancy prompts serious caution on the causal mechanics attributed to risk perceptions by the HBM theory. According to this framework, policy-makers can rely on empirical HBM studies not only to increase risk perception among the population through prevention campaigns in order to raise the average level of self-protection, but also to identify the most efficient channel(s) to do so: severity, susceptibility or another construct. Our findings alert that these campaigns may trigger differentiated responses depending on the risk preference profile, which is not envisioned by the HBM that expects an homogeneous reaction.

Another insight conveyed by the previous Lemmas concerns the key role of the cost of self-protection. The conditions expressed in Lemmas 2 and 3 reflect that if the marginal cost of self-protection is low (resp. high) enough compared to the elasticities of the probability α, an increase in perceived severity/susceptibility will lead to more (resp. less) self-protection. Given the convexity of the protection cost function, risk averse agents that have already adopted a high level of self-protection measures will be facing a high marginal cost and will thus be less likely to respond positively to a stimulus increasing their perceived severity/susceptibility.[7] This behavior results from their reluctance to engage in more efforts and thus risk suffering a higher net loss if they get infected despite the protection efforts already engaged. In terms of health policies, our Lemmas thus indicate that reducing the financial or psychological costs of self-protection can be a necessary complementary step to catalyze the impact of a prevention campaign enhancing risk perceptions.

From the perspective of the HBM, this role of the protection cost can be analyzed through the concept of *perceived barriers*. This construct (see Figure 1) often emerges as a strong predictor of preventive behavior against various health conditions (Carpenter, 2010). Our theoretical observations of the impact of one construct (perceived barriers in the form of marginal cost), on the comparative statics with respect to another construct (severity or susceptibility) highlights the need for empirical study to address the cross effects of the former on the latter. Most applied HBM studies capture the main effect of various variables but do not consider cross effects.[8] Investigating further these indirect effects in future empirical studies, through moderated mediation models for instance (Jones et al., 2015), could contribute, in the light of our Lemmas, to the identification of the cost/barriers threshold above which the counter-intuitive comparative statics occur.[9]

We conclude this comparative statics subsection with a Lemma on the role of the discount factor β.

Lemma 4 *For risk averse and risk neutral agents,* $\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \beta} > 0$.

Proof. The proof is given in Appendix A2.

In this setting, the discount factor can be interpreted as a subjective estimation of the delay between the exposure to the health risk and the potential health status degradation (the higher β, the shorter the delay). Therefore, in terms of health policies this result adds qualitative insights on an effective component of a prevention campaign. Indeed, if agents are under-protected due to their overestimation of the delay, an information campaign that unveils the actual time frame of the health condition would lead these agents to increase their protection.

4 | PREFERENCES TOWARDS RISK AND SELF-PROTECTION UNDER HEALTH RISK PERCEPTION

As mentioned in the Introduction, the theoretical economics literature has long pondered over the puzzling impact of risk-aversion on optimal self-protection. After Dionne and Eeckhoudt (1985), showed that self-protection, contrary to self-insurance, can decrease when risk aversion increases, a loss probability threshold over which this effect takes place has been identified in a one-period setting by Jullien et al. (1999) and others (see the exhaustive review in Courbage et al., 2013). A similar result was confirmed by Menegatti (2009) and Huber (2022) for a two-period model. This probability threshold that determines if self-protection increases or decreases with risk aversion is considered endogenous in the sense that it depends jointly on the preferences of both the benchmark agent and the "more risk averse" agent involved in the comparison. Through an elegant method based on risk neutral probabilities, Peter (2021) managed to remove this endogeneity in the definition of the threshold, now exclusively linked to the benchmark agent's preferences. We apply this approach to our Kihlstrom-Mirman function that respects ordinal preferences and define the following risk neutral probability $\overline{\alpha}$ for a risk aversion function ϕ, with ε_ϕ^* the corresponding optimal effort:

$$\overline{\alpha}\left(\varepsilon_\phi^*;\hat{\pi}\right) = \frac{\alpha\left(\varepsilon_\phi^*,\hat{\pi}\right)\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)}{\alpha\left(\varepsilon_\phi^*,\hat{\pi}\right)\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha\left(\varepsilon_\phi^*,\hat{\pi}\right)\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}. \tag{5}$$

$\overline{\alpha}$ corresponds to the loss probability that needs to be applied to a risk neutral agent for the latter to derive the same utility from the lottery as our benchmark agent does with the *real* probability $\alpha\left(\varepsilon_{\phi}^{*};\hat{\pi}\right)$ and the same payoffs (Heaton, 2018). In the case of a risk averse benchmark agent, $\overline{\alpha}$ is thus greater than $\alpha\left(\varepsilon_{\phi}^{*};\hat{\pi}\right)$, and they are equal for a risk neutral benchmark agent. Let us consider another agent whose risk preferences are characterized by φ, defined by $\varphi = k(\phi)$, where $k' > 0$ and $k^{(2)} < 0$ so that she is more risk averse than our benchmark agent. Her optimal effort is denoted $\varepsilon_{k\phi}^{*}$. In the wake of Peter (2021), we obtain the following Lemma:

Lemma 5

1. *If* $\overline{\alpha}\left(\varepsilon_{\phi}^{*},\hat{\pi}\right) \geq \frac{1}{2}$ *and if* $k^{(3)} > 0$, *then* $\varepsilon_{k\phi}^{*} < \varepsilon_{\phi}^{*}$.
2. *If* $\overline{\alpha}\left(\varepsilon_{\phi}^{*},\hat{\pi}\right) \leq \frac{1}{2}$ *and if* $k^{(3)} < 0$, *then* $\varepsilon_{k\phi}^{*} > \varepsilon_{\phi}^{*}$.

Proof. The proof is given in Appendix A3.

Lemma 5 confirms, under an additional condition on $k^{(3)}$, the ambiguous effect of risk aversion on self-protection in a two-period Kihlstrom-Mirman setting respecting ordinal preferences.[10] This result is in line with Proposition 4 in Huber (2022) that was established for a pure monetary cost of effort while we use a general effort cost function. Given that spending in self-protection reduces utility in both the loss and the no-loss state compared to abstaining from it, self-protection is not necessarily higher among agents that are more risk averse (Briys & Schlesinger, 1990).

As established by Keenan and Snow (2009), in a Kihlstrom-Mirman framework $\phi^3 > 0$ indicates a prudent agent and the condition $k^3 > 0$ reflects an increase in the degree of prudence. The latter is needed to wave off the ambiguity on the impact of risk aversion in the previous Lemma. Since decreasing the loss probability through self-protection increases variance when $\overline{\alpha} \in \left[\frac{1}{2},1\right]$, a greater risk aversion combined with a greater degree of prudence $\left(k^{(3)} > 0\right)$ will lower self-protection. This behavior can be explained by the will to avoid above all the worst outcome among all states of the world, that is, being sick and having borne additional self-protection costs. The opposite effect is at play for less prudent agents whose self-protection increases with risk aversion when $\overline{\alpha} \in \left[0,\frac{1}{2}\right]$.[11]

It must be noted that there exist two cases for which our results and the previous ones in the literature are indeterminate due to conflicting effects: if $\overline{\alpha}\left(\varepsilon_{\phi}^{*},\hat{\pi}\right) > \frac{1}{2}$ and $k^{(3)} < 0$, and if $\overline{\alpha}\left(\varepsilon_{\phi}^{*},\hat{\pi}\right) < \frac{1}{2}$ and $k^{(3)} > 0$.[12] The latter case is problematic because it is a relevant situation for health policy purposes since *"the great majority of risky situations that require self-protection [...] and public decisions on safety are characterized for events with a probability lower than* $\frac{1}{2}$*"* (Dachraoui et al., 2004). Our own findings nevertheless push back on this serious limitation through the introduction of perceived susceptibility. Indeed, the first component of our Lemma 5 actually encompasses a much wider range of situations since we can show that, under the action of $\hat{\pi}$, the distorted probability $\overline{\alpha}$ increases in a concave manner with $\alpha\left(\varepsilon_{\phi}^{*},\hat{\pi}\right)$, the subjective probability. As illustrated in Figure 2 for a numerical simulation of (5), low levels of $\alpha\left(\varepsilon_{\phi}^{*},\hat{\pi}\right)$ can translate very quickly into $\overline{\alpha} \geq \frac{1}{2}$. Risk averse agents with levels of subjective probability that are low enough to be realistic for actual health risks will also reduce self-protection under increased prudence. This broader result is consistent with the emerging body of experimental studies (see a brief survey in Bleichrodt (2022)). Although this lasting discussion on the role of prudence does not translate straightforwardly into policy recommendations, our conclusions highlight indirectly the significant impact of susceptibility on this behavior and stresses the importance for health policies to pay greater attention to subjective risk perceptions.

5 | UNCERTAINTY ON SEVERITY AND SUSCEPTIBILITY

Let us now address the issue of uncertainty which is central in the kind of health risk prevention we are looking at, given our focus on risk perception. The HBM postulates that individuals facing a health risk adjust their preventive behavior according to their subjective perception of the risk at stake. However, it overlooks the likely situation,

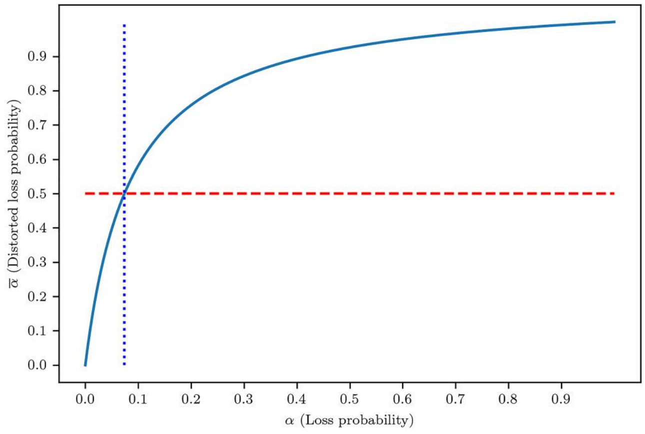

FIGURE 2 Risk neutral probability $\overline{\alpha}$ as a function of subjective probability α with $\frac{\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}{\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)} = 0.08$.

especially for emerging health risks, where individuals are so misinformed that they do not have a subjective estimation but are in fact facing a form of uncertainty. This state of beliefs has been identified empirically in the recent Public Health literature, notably for COVID-19 (Chen et al., 2022). It is thus relevant for both risk theory and the HBM approach to examine the introduction of uncertainty on severity and on susceptibility, as an advanced stage of biased perceptions.

In their one-period self-protection model, Brianti et al. (2018) have discussed the impact of applying uncertainty on either cure effectiveness or disease effect, while keeping the other parameter known. We expand their approach to study how the optimal self-protection effort changes when we introduce uncertainty on severity or on susceptibility, or on both. Our risk components now take the form of random variables $\tilde{M}$ (with support in $[0, H]$) and $\tilde{\pi}$ (with support in $[0,1]$). For this analysis that implies simultaneous uncertainty, we need to make an assumption in the spirit of Assumption 1:

Assumption 2 $\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]\mathbb{E}\left[\alpha^{(2)}(\varepsilon;\tilde{\pi})\right] > 2\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]^2$.

We also consider the following assumption:

Assumption 3 *$\tilde{M}$ and $\tilde{\pi}$ are independent.*

This assumption is rather natural considering that each parameter constitutes a distinct component of the overall risk. It is backed up by empirical evidence on various forms of health impairments, in particular infectious diseases. Cummings et al. (1978) show that these two perceptions are substantially independent for a wide range of diseases.[13]

We now compare simultaneously the optimal level of self-protection in the deterministic case (ε^*) with the case where only severity $\left(\varepsilon^*_{\tilde{M}}\right)$ or only susceptibility $\left(\varepsilon^*_{\tilde{\pi}}\right)$ is a random variable, and with the case where uncertainty affects both risk components $\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)$. We have thus

$$\varepsilon^* = \underset{\varepsilon}{\operatorname{argmax}}\, E[U(\varepsilon,\mu_M,\mu_\pi)],$$

$$\varepsilon^*_{\tilde{M}} = \underset{\varepsilon}{\operatorname{argmax}}\, E\left[U\left(\varepsilon,\tilde{M},\mu_\pi\right)\right],$$

$$\varepsilon^*_{\tilde{\pi}} = \underset{\varepsilon}{\operatorname{argmax}}\, E[U(\varepsilon,\mu_M,\tilde{\pi})],$$

$$\varepsilon^*_{\tilde{M},\tilde{\pi}} = \underset{\varepsilon}{\operatorname{argmax}}\, E\left[U\left(\varepsilon,\tilde{M},\tilde{\pi}\right)\right],$$

where $\mu_M = E\left[\tilde{M}\right]$ and $\mu_\pi = E[\tilde{\pi}]$.

The following Lemma is then set under Assumption 2 and 3.

Lemma 6 *If the agent is prudent (i.e., $\phi^{(3)} > 0$), the following assertions hold (henceforth denoted* Lemma 6.1, 6.2 and 6.3).

1. *Introducing uncertainty on severity increases self-protection:* $\varepsilon^*_{\tilde{M},\tilde{\pi}} > \varepsilon^*_{\tilde{\pi}}$.
2. *If* $\alpha^{(3)}(\varepsilon) < 0$*, introducing uncertainty on susceptibility increases self-protection:* $\varepsilon^*_{\tilde{M},\tilde{\pi}} > \varepsilon^*_{\tilde{M}}$.
3. *If* $\alpha^{(3)}(\varepsilon) < 0$*, then the following inequalities hold:* $\begin{cases} \varepsilon^*_{\tilde{M},\tilde{\pi}} > \varepsilon^*_{\tilde{\pi}} > \varepsilon^*, \\ \varepsilon^*_{\tilde{M},\tilde{\pi}} > \varepsilon^*_{\tilde{M}} > \varepsilon^*. \end{cases}$

Proof. The proof of this Lemma is given in Appendix A4.

These results offer a complementary perspective on the behavior of risk averse agents who are also prudent ($\phi^{(3)} > 0$, see Keenan & Snow, 2009), under Assumptions 2 and 3. By definition, in the setting of a self-protection model these agents face an endogenous health risk they can act upon through ε. Lemma 6 shows that if they are also exposed to an exogenous risk in the form of uncertainty on the severity or susceptibility of the disease, they will increase their self-protection effort, and increase it even further if the uncertainty weighs on both risk parameters. For severity, a similar mechanism has been identified by Felder and Mayrhofer (2017) from the physician perspective in a model with co-morbidity.[14] In terms of prevention policy, our findings can be relevant to address a health risk that is moderate but misconstrued by the population. Dissipating uncertainty through information and education can avoid the over-protection of prudent agents triggered by one or two sources of uncertainty and thus a misallocation of resources toward self-protection.

Furthermore, the comparison between Lemma 6.1 and Lemma 6.2 shows that if the effect of uncertainty on the severity channel is unambiguous, when it comes to the susceptibility channel an additional specific third order property is required on the probability function α, which can be interpreted as the prevention technology (Peter, 2021). This asymmetry between the two channels of risk perception is also manifest in the case of risk neutral agents for which the following Corollaries hold:

Corollary 7 *For risk neutral individuals, introducing uncertainty on severity increases optimal self-protection.*

Corollary 8 *For risk neutral individuals, if* $\alpha^{(3)} < 0$*, introducing uncertainty on susceptibility increases optimal self-protection. On the contrary, if* $\alpha^{(3)} > 0$*, uncertainty on susceptibility strictly decreases optimal self-protection.*

Proof. The proof of these Corollaries is given in Appendix A5 and A6. These results hold whether the other risk parameter is also a random variable or if it is deterministic.

Whereas introducing uncertainty on severity drives unambiguously a prudent risk neutral agent to increase her self-protection,[15] uncertainty on susceptibility can have opposite effects depending on the properties of the prevention technology. When $\alpha^{(3)} > 0$ the agent will be reluctant to engage in self-protection costs that yield a lower expected marginal efficiency. For risk averse as well as for risk neutral agents, reactions to uncertainty on susceptibility are thus less straightforward than on severity, and can even lead to a decrease of self-protection for the risk neutral category. The policy implication of this finding can be found in the situation where health authorities are concerned by under-protection against a serious but misunderstood risk. In that case, prevention policies should dedicate resources in priority to information campaigns on susceptibility rather than on severity since dissipating uncertainty on the former is more likely to increase self-protection.

6 | CONCLUSION

Through our original take on the HBM we have introduced two-way risk perception into optimal self-protection modeling. In doing so we broaden the theoretical perspective on self-protection and bring out complementary results for health prevention. We were able to disentangle the severity perception and the susceptibility perception channel within the decision mechanism and we showed how this asymmetry generates differentiated reactions to risk aversion and to prudence, making the lasting debate on the role of these risk preferences even more complex.

Combining the traditional study of risk aversion and prudence with the introduction of uncertainty has allowed us to characterize more thoroughly, under certain conditions, the behavior of prudent agents. Our findings emulate the results of Brianti et al. (2018) and extend their scope to the case of simultaneous uncertainty. We confirm that a more prudent agent can exert less self-protection effort when faced with a deterministic low infection risk. But we also find that if there is uncertainty on either component of the risk perception, a prudent agent will increase her self-protection.

Our comparative statics on risk perception channels questions the theoretical foundations of the HBM, that assumes a systematic unidirectional causality between risk perception and individual prevention, and thus ignores heterogeneity in risk preferences. Given the conceptual proximity of the HBM to expected value theory, our results call for a deeper dive in the postulates of this public health framework, in particular regarding risk aversion. The latter could be introduced more rigorously in empirical HBM-bases studies to better observe its effects on self-protection. Beyond their direct role on preventive behavior previously identified in the empirical literature (Carpenter, 2010), we have shown that *perceived barriers* can exhibit cross effects on the impact of severity and susceptibility. This particular mechanism makes the reduction of these barriers even more relevant for prevention policies. It also reflects that all kinds of cross effects between HBM variables should be the subject of advanced empirical investigation.

Given the insights gained from our attempt at modeling the core HBM constructs in a risk theory framework, an extension of this work could be the introduction of additional HBM variables such as *self-efficacy*. From a policy perspective, the next natural step of this research would be to scale up our model to study the differentiated impact of an information campaign applied to a population that is heterogeneous in risk preferences and/or in risk perceptions.

AUTHOR CONTRIBUTION

Both authors of this paper have actively contributed to all aspects of the research presented in this manuscript.

ACKNOWLEDGMENTS

The authors thank the MSH-Paris Saclay for the *Maturation* funding of their project (PR)^{2}ÉMATIQUE. Marc Leandri is grateful to the CNRS and the EconomiX center for hosting him as a Visiting Researcher in 2022–23. The authors thank the editor and referees for the valuable comments and suggestions that have significantly improved this contribution.

CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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ENDNOTES

[1] According to Google Scholar and Scopus approximately 5000 publications in the last decade resort to the HBM to ground their empirical design in an established theoretical framework.

[2] It must be reminded that alternative criteria to expected utility, such as Rank Dependent Utility, are also investigated in various economic contributions (Konrad & Skaperdas, 1993) but we focus here on the standard expected utility criteria.

[3] This approach is indeed well-fitted to address self-protection behaviors against COVID-19 for instance (Dedonno et al., 2022).

[4] We assume that the individual receives adequate treatment: the efficiency of healthcare and cure will not be addressed here.

[5] According to this latter property, an agent will not choose a decision if another available decision leads to better utility in all circumstances. Non-monotonicity yields counter-intuitive results about the role of risk aversion (Bommier & Le Grand, 2019).

[6] The case of a risk neutral individual can be addressed by taking ϕ as the identity function such that $U^d\left(\varepsilon;\hat{M},\hat{\pi}\right) = \mathbb{E}\left[U_1^d(\varepsilon) + \beta U_2^d(\varepsilon)\right] = U_1^d(\varepsilon) + \beta\mathbb{E}\left[U_2^d(\varepsilon)\right]$.

[7] For risk neutral agents, the response to an increase in risk perception is independent from the initial level of self-protection, which reflects the general definition of risk neutrality as preferences unaffected by the initial level wealth.

[8] The few studies that observe interactions between constructs through structural equation modeling, such as Lim et al. (2021), detect an indirect effect of perceived barriers on self-efficacy but not on severity or susceptibility.

[9] We are grateful to one of the reviewers for this relevant suggestion.

[10] Extensive calculations left the impact of either $\hat{M}$ or $\hat{\pi}$ on the risk neutral probability $\overline{\alpha}$ undetermined.

[11] Our generic result holds for any type of benchmark agent and can thus be used to reformulate the seminal result of Eeckhoudt and Gollier (2005) for a risk neutral benchmark by substituting ϕ by the identity function and $\overline{\alpha}$ by α.

[12] For the interested reader, Peter (2021) addresses these instances of indeterminacy through additional conditions on the relative curvature of the marginal transformation function (Proposition 3, p. 10).

[13] More recently, Dedonno et al. (2022) establish their independence in the case of COVID-19.

[14] Formally, our modeling of perceived severity amounts to a form of comorbidity in their setting or to a form of background risk (Lee, 2012).

[15] This result is in line with Brianti et al. (2018, Proposition 2) for uncertainty on the disease effect.

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How to cite this article: Augeraud-Véron, E., & Leandri, M. (2024). Optimal self-protection and health risk perceptions: Exploring connections between risk theory and the Health Belief Model. *Health Economics*, 1–19. https://doi.org/10.1002/hec.4826

APPENDIX A

A1 | Proof of Lemma 1

The aim of this proof is to show the concavity of $U^d(\varepsilon)$.

$$\begin{aligned} U^{d(2)}(\varepsilon) = & -C^{(2)}(\varepsilon)[\alpha(\varepsilon,\hat{\pi})\phi'(w_1 - C(\varepsilon)) + (1-\alpha(\varepsilon,\hat{\pi}))\phi'(w_2 - C(\varepsilon))] \\ & -\alpha^{(2)}(\varepsilon,\hat{\pi})(\phi(w_2 - C(\varepsilon)) - \phi(w_1 - C(\varepsilon))) \\ & +2\alpha'(\varepsilon,\hat{\pi})C'(\varepsilon)(\phi'(w_2 - C(\varepsilon)) - \phi'(w_1 - C(\varepsilon))) \\ & +(C'(\varepsilon))^2\left[\alpha(\varepsilon,\hat{\pi})\phi^{(2)}(w_1 - C(\varepsilon)) + (1-\alpha(\varepsilon,\hat{\pi}))\phi^{(2)}(w_2 - C(\varepsilon))\right]. \end{aligned}$$

The third term being positive, the sign of $U^{d(2)}(\varepsilon)$ is a priori undefined. However, using the interior optimality condition, it can be noticed that at the optimum the following equation holds:

$$\phi(w_2 - C(\varepsilon)) - \phi(w_1 - C(\varepsilon)) = \frac{-C'(\varepsilon)}{\alpha'(\varepsilon,\hat{\pi})}[\alpha(\varepsilon,\hat{\pi})\phi'(w_1 - C(\varepsilon)) + (1-\alpha(\varepsilon,\hat{\pi}))\phi'(w_2 - C(\varepsilon))].$$

Thus

$$\begin{aligned}\frac{\alpha'(\varepsilon,\hat{\pi})U^{d(2)}(\varepsilon)}{C'(\varepsilon)} > & \left(\alpha^{(2)}(\varepsilon,\hat{\pi})\alpha(\varepsilon,\hat{\pi}) - 2(\alpha'(\varepsilon,\hat{\pi}))^2\right)[\phi'(w_1 - C(\varepsilon)) - \phi'(w_2 - C(\varepsilon))] \\ & + \alpha^{(2)}(\varepsilon,\hat{\pi})\phi'(w_2 - C(\varepsilon)) \\ & + \alpha'(\varepsilon,\hat{\pi})C'(\varepsilon)\left[\alpha(\varepsilon,\hat{\pi})\phi^{(2)}(w_1 - C(\varepsilon)) + (1-\alpha(\varepsilon,\hat{\pi}))\phi^{(2)}(w_2 - C(\varepsilon))\right].\end{aligned}$$

Using Assumption 1, the previous expression enables us to conclude the proof: $U^d(\varepsilon)$ is concave at the optimum, and thus the optimum is a maximum.

A2 | Proof of Lemma 4

From (2), we know that the sign of $\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \beta}$ is the same as the sign of

$$\begin{aligned}&-C'(\varepsilon)\left[u\left(H_S - \hat{M}\right)\alpha(\varepsilon,\hat{\pi})\phi^{(2)}(w_1 - C(\varepsilon)) + u(H_S)(1-\alpha(\varepsilon,\hat{\pi}))\phi^{(2)}(w_2 - C(\varepsilon))\right] \\ &-\alpha'(\varepsilon,\hat{\pi})\left(u(H_S)\phi'(w_2 - C(\varepsilon)) - u\left(H_S - \hat{M}\right)\phi'(w_1 - C(\varepsilon))\right).\end{aligned}$$

According to the properties of α, u and ϕ, we have

$$-\alpha'(\varepsilon,\hat{\pi})\left(u(H_S) - u\left(H_S - \hat{M}\right)\right)\phi'(w_2 - C(\varepsilon)) > 0,$$

and thus

$$-\alpha'(\varepsilon,\hat{\pi})\left(u(H_S)\phi'(w_2 - C(\varepsilon)) - u\left(H_S - \hat{M}\right)\phi'(w_1 - C(\varepsilon))\right) > 0.$$

It is also easy to see that

$$-C'(\varepsilon)\left[u\left(H_S - \hat{M}\right)\alpha(\varepsilon,\hat{\pi})\phi^{(2)}(w_1 - C(\varepsilon)) + u(H_S)(1-\alpha(\varepsilon,\hat{\pi}))\phi^{(2)}(w_2 - C(\varepsilon))\right] > 0,$$

which enables us to conclude that $\frac{\partial \varepsilon_\phi^*(\hat{M},\hat{\pi})}{\partial \beta} > 0$.

A3 | Proof of Lemma 5

The proof is an adaptation of the proof of Proposition 2 in Peter (2021) for our framework of a two-period model with an expected utility function *à la* Kihlstrom-Mirman. The optimal self-protection effort of a risk averse agent with risk function ϕ is given as a solution of the following equation:

$$\begin{aligned}0 = & -C'(\varepsilon)[\alpha(\varepsilon,\hat{\pi})\phi'(w_1 - C(\varepsilon)) + (1-\alpha(\varepsilon,\hat{\pi}))\phi'(w_2 - C(\varepsilon))] \\ & - \alpha'(\varepsilon,\hat{\pi})(\phi(w_2 - C(\varepsilon)) - \phi(w_1 - C(\varepsilon))).\end{aligned}$$

We denote $V^d(\varepsilon)$ the expected utility of a more risk averse agent with utility function $k(\phi)$.

$$\begin{aligned}V^d(\varepsilon) &= \mathbb{E}\left[k\phi\left(U_1^d(\varepsilon) + \beta U_2^d(\varepsilon)\right)\right], \\ &= \alpha(\varepsilon,\hat{\pi})k\phi\left(U_F - C(\varepsilon) + \beta u\left(H - \hat{M}\right)\right) \\ &\quad + (1-\alpha(\varepsilon,\hat{\pi}))k\phi(U_F - C(\varepsilon) + \beta u(H)).\end{aligned}$$

The results of Lemma 1 hold and V^d is a concave function at its extremum, which is thus a maximum. It implies that for ε smaller than the optimum V^d is increasing, and for ε greater that the optimum V^d is decreasing. Notice that V^d is not necessarily globally concave, as inflection points may exist.

$$V^{d'}(\varepsilon) = \alpha'(\varepsilon,\hat{\pi})\left[k\phi\left(U_F - C(\varepsilon) + \beta u\left(H - \hat{M}\right)\right) - k\phi(U_F - C(\varepsilon) + \beta u(H))\right]$$
$$- C'(\varepsilon)\begin{bmatrix} \alpha(\varepsilon,\hat{\pi})k'\phi\left(U_F - C(\varepsilon) + \beta u\left(H - \hat{M}\right)\right)\phi'\left(U_F - C(\varepsilon) + \beta u\left(H - \hat{M}\right)\right) \\ +(1-\alpha(\varepsilon,\hat{\pi}))k'\phi(U_F - C(\varepsilon) + \beta u(H))\phi'(U_F - C(\varepsilon) + \beta u(H)) \end{bmatrix}.$$

According to our previous remark on V^d, $\varepsilon^*_{k\phi}$ is greater that ε^*_ϕ if and only if $V^{d'}\left(\varepsilon^*_\phi\right) \geq 0$.

To ease the notation, we denote $\alpha^*_\phi = \alpha\left(\varepsilon^*_\phi,\hat{\pi}\right)$.

Since

$$\alpha'\left(\varepsilon^*_\phi,\hat{\pi}\right) = -C'\left(\varepsilon^*_\phi\right)\frac{\left[\alpha^*_\phi\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)\right]}{\left(\phi\left(w_2 - C\left(\varepsilon^*_\phi\right)\right) - \phi\left(w_1 - C\left(\varepsilon^*_\phi\right)\right)\right)},$$

$V^{d'}\left(\varepsilon^*_\phi\right)$ has the opposite sign of

$$f\left(\varepsilon^*_\phi\right) = \frac{k\phi\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) - k\phi\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)}{\phi\left(w_2 - C\left(\varepsilon^*_\phi\right)\right) - \phi\left(w_1 - C\left(\varepsilon^*_\phi\right)\right)}$$
$$+ \frac{\alpha^*_\phi k'\phi\left(w_1 - C\left(\varepsilon^*_\phi\right)\right)\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)k'\phi\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)}{\alpha^*_\phi\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)}.$$

Letting $\phi\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) = \varphi_1$ and $\phi\left(w_2 - C\left(\varepsilon^*_\phi\right)\right) = \varphi_2$, the previous expression can be rewritten as follows:

$$f\left(\varepsilon^*_\phi\right) = \frac{k\varphi_1 - k\varphi_2}{\varphi_2 - \varphi_1} + \frac{\alpha^*_\phi k'(\varphi_1)\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)k'(\varphi_2)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)}{\alpha^*_\phi\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)},$$
$$= \frac{1}{\varphi_2 - \varphi_1}\int_{\varphi_2}^{\varphi_1} k'(z)dz + \frac{\alpha^*_\phi k'(\varphi_1)\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)k'(\varphi_2)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)}{\alpha^*_\phi\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)}.$$

Moreover,

$$\overline{\alpha}\left(\varepsilon^*_\phi,\hat{\pi}\right) = \frac{\alpha^*_\phi\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right)}{\alpha^*_\phi\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)},$$
$$1 - \overline{\alpha}\left(\varepsilon^*_\phi,\hat{\pi}\right) = \frac{\left(1-\alpha^*_\phi\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)}{\alpha^*_\phi\phi'\left(w_1 - C\left(\varepsilon^*_\phi\right)\right) + \left(1-\alpha^*_\phi\right)\phi'\left(w_2 - C\left(\varepsilon^*_\phi\right)\right)},$$

and thus

$$f\left(\varepsilon^*_\phi\right) = \frac{1}{\varphi_2 - \varphi_1}\int_{\varphi_2}^{\varphi_1} k'(z)dz + \overline{\alpha}\left(\varepsilon^*_\phi,\hat{\pi}\right)k'(\varphi_1) + \left(1 - \overline{\alpha}\left(\varepsilon^*_\phi,\hat{\pi}\right)\right)k'(\varphi_2).$$

Let us rewrite

$$\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)k'(\varphi_1)+\left(1-\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)\right)k'(\varphi_2)=\frac{1}{2}k'(\varphi_1)+\frac{1}{2}k'(\varphi_2)-\left(\frac{1}{2}-\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)\right)(k'(\varphi_1)-k'(\varphi_2)).$$

The sign of $V^{d'}\left(\varepsilon_\phi^*\right)$ is thus the same as the sign of $-f\left(\varepsilon_\phi^*\right)$, which is given by

$$\frac{1}{\varphi_2-\varphi_1}\int_{\varphi_1}^{\varphi_2}k'(z)dz-\frac{1}{2}k'(\varphi_1)-\frac{1}{2}k'(\varphi_2)+\left(\frac{1}{2}-\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)\right)(k'(\varphi_1)-k'(\varphi_2)).$$

As

$$\begin{aligned}\frac{1}{b-a}\int_a^b\frac{b-s}{b-a}ds &=\frac{1}{2},\\ \frac{1}{b-a}\int_a^b\frac{s-a}{b-a}ds &=\frac{1}{2},\end{aligned}$$

$$\begin{aligned}-f\left(\varepsilon_\phi^*\right)=&\frac{1}{\varphi_2-\varphi_1}\int_{\varphi_1}^{\varphi_2}\left[k'(s)-\frac{\varphi_2-s}{\varphi_2-\varphi_1}k'(\varphi_1)-\left(1-\frac{\varphi_2-s}{\varphi_2-\varphi_1}\right)k'(\varphi_2)\right]ds\\ &+\left(\frac{1}{2}-\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)\right)(k'(\varphi_1)-k'(\varphi_2)).\end{aligned}$$

If $\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)=\frac{1}{2}$, then the last term is null. Moreover, if k' is concave (i.e., $k^{(3)}<0$), then the term within brackets is positive, implying that $V^{d'}\left(\varepsilon_\phi^*\right)>0$. As a consequence, $\varepsilon_{k\phi}^*>\varepsilon_\phi^*$. Similarly, if $k^{(3)}>0$, $\varepsilon_{k\phi}^*<\varepsilon_\phi^*$.

If $\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)>\frac{1}{2}$, as $k^{(2)}<0$, then the second term is negative. If $k^{(3)}>0$, then $V^{d'}\left(\varepsilon_\phi^*\right)<0$, thus $\varepsilon_{k\phi}^*<\varepsilon_\phi^*$.

If $\overline{\alpha}\left(\varepsilon_\phi^*,\hat{\pi}\right)<\frac{1}{2}$, as $k^{(2)}<0$, then the second term is positive. If $k^{(3)}<0$, then $V^{d'}\left(\varepsilon_\phi^*\right)>0$, thus $\varepsilon_{k\phi}^*>\varepsilon_\phi^*$.

A4 | Proof of Lemma 6

$$\mathbb{E}\left[U\left(\varepsilon,\tilde{M},\tilde{\pi}\right)\right]=\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})\phi(\tilde{w}_1-C(\varepsilon))+(1-\alpha(\varepsilon;\tilde{\pi}))\phi(w_2-C(\varepsilon))]$$

with $\tilde{w}_1=U_F+\beta u\left(H-\tilde{M}\right)$.

Since $\tilde{M}$ and $\tilde{\pi}$ are independent (Assumption 3),

$$\mathbb{E}\left[U\left(\varepsilon,\tilde{M},\tilde{\pi}\right)\right]=\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]E[\phi(\tilde{w}_1-C(\varepsilon))]+(1-\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})])\phi(w_2-C(\varepsilon)).$$

Let

$$\begin{aligned}\varepsilon^*&=\operatorname*{argmax}_\varepsilon\mathbb{E}[U(\varepsilon,\mu_M,\mu_\pi)],\varepsilon_{\tilde{M}}^*=\operatorname*{argmax}_\varepsilon\mathbb{E}\left[U\left(\varepsilon,\tilde{M},\mu_\pi\right)\right],\\ \varepsilon_{\tilde{\pi}}^*&=\operatorname*{argmax}_\varepsilon\mathbb{E}[U(\varepsilon,\mu_M,\tilde{\pi})],\varepsilon_{\tilde{M},\tilde{\pi}}^*=\operatorname*{argmax}_\varepsilon\mathbb{E}\left[U\left(\varepsilon,\tilde{M},\tilde{\pi}\right)\right].\end{aligned}$$

Let $\varepsilon_{\tilde{M},\tilde{\pi}}^*$ be an interior solution, it solves

$$g_{\tilde{M},\tilde{\pi}}\left(\varepsilon_{\tilde{M},\tilde{\pi}}^*\right)=-C'\left(\varepsilon_{\tilde{M},\tilde{\pi}}^*\right),\tag{6}$$

where

$$g_{\tilde{M},\tilde{\pi}}(\varepsilon) = \frac{\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})](\phi(w_2 - C(\varepsilon)) - \mathbb{E}[\phi(\tilde{w}_1 - C(\varepsilon))])}{(\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]\mathbb{E}[\phi'(\tilde{w}_1 - C(\varepsilon))] + (1 - \mathbb{E}[\alpha(\varepsilon;\tilde{\pi})])\phi'(w_2 - C(\varepsilon)))}.$$

Let $\overline{w}_1 = U_F + \beta u(H - \mu_M)$. As ϕ and u are concave functions, applying twice Jensen inequality (together with the fact that ϕ is an increasing function) yields the following result:

$$\mathbb{E}[\phi(\tilde{w}_1 - C(\varepsilon))] < \phi\left(U_F + \beta\mathbb{E}\left[u\left(H - \tilde{M}\right)\right] - C(\varepsilon)\right) < \phi(\overline{w}_1 - C(\varepsilon)).$$

Let us assume that $\phi^{(3)} > 0$.

We have then $\mathbb{E}[\phi'(\tilde{w}_1 - C(\varepsilon))] > \phi'\left(U_F + \beta\mathbb{E}\left[u\left(H - \tilde{M}\right)\right] - C(\varepsilon)\right)$. Due to the concavity of function u, and using that ϕ' is a decreasing function, we get the following inequality:

$$\mathbb{E}[\phi'(\tilde{w}_1 - C(\varepsilon))] > \phi'(\overline{w}_1 - C(\varepsilon)).$$

As a consequence,

$$g_{\tilde{M},\tilde{\pi}}(\varepsilon) < g_{\tilde{\pi}}(\varepsilon), \tag{7}$$

where

$$g_{\tilde{\pi}}(\varepsilon) = \frac{\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})](\phi(w_2 - C(\varepsilon)) - \mathbb{E}[\phi(\overline{w}_1 - C(\varepsilon))])}{(\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]\mathbb{E}[\phi'(\overline{w}_1 - C(\varepsilon))] + (1 - \mathbb{E}[\alpha(\varepsilon;\tilde{\pi})])\phi'(w_2 - C(\varepsilon)))}.$$

To prove Lemma 6, we will use an intermediate Lemma 9, defined and proved below.

Lemma 9 $g'_{\tilde{M},\tilde{\pi}}\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right) > 0$ *and* $g'_{\tilde{\pi}}\left(\varepsilon^*_{\tilde{\pi}}\right) > 0$.

Proof. $g'_{\tilde{M},\tilde{\pi}}(\varepsilon)$ has the same sign as $h_{\tilde{M},\tilde{\pi}}(\varepsilon)$, which is defined as follows:

$$\begin{aligned} h_{\tilde{M},\tilde{\pi}}(\varepsilon) = {} & \left(\mathbb{E}\left[\alpha^{(2)}(\varepsilon;\tilde{\pi})\right]\varphi - C'(\varepsilon)\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]\varphi_1\right)(\phi'(w_2 - C(\varepsilon)) - \mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]\varphi_1) \\ & + \left(\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]\varphi_1 + C'(\varepsilon)\left(\phi^{(2)}(w_2 - C(\varepsilon)) - \mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]\varphi_2\right)\right)\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]\varphi, \end{aligned}$$

where

$$\begin{aligned} \varphi &= \phi(w_2 - C(\varepsilon)) - \mathbb{E}[\phi(\tilde{w}_1 - C(\varepsilon))] > 0, \\ \varphi_1 &= \phi'(w_2 - C(\varepsilon)) - \mathbb{E}[\phi'(\tilde{w}_1 - C(\varepsilon))] < 0, \\ \varphi_2 &= \phi^{(2)}(w_2 - C(\varepsilon)) - \mathbb{E}\left[\phi^{(2)}(\tilde{w}_1 - C(\varepsilon))\right]. \end{aligned}$$

Since $\phi^{(3)} > 0$, then $\varphi_2 > 0$. Some computations enable us to rewrite $h_{\tilde{M},\tilde{\pi}}(\varepsilon)$ as

$$\begin{aligned} h_{\tilde{M},\tilde{\pi}}(\varepsilon) = {} & \left(\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]^2 - \mathbb{E}\left[\alpha^{(2)}(\varepsilon;\tilde{\pi})\right]\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]\right)\varphi\varphi_1 + \varphi\phi'(w_2 - C(\varepsilon))\mathbb{E}\left[\alpha^{(2)}(\varepsilon;\tilde{\pi})\right] \\ & + C'(\varepsilon)\varphi\left(\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]\phi^{(2)}(w_2 - C(\varepsilon)) - \mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]\varphi_2\right) \\ & - C'(\varepsilon)\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]\varphi_1\phi'(w_2 - C(\varepsilon)) + C'(\varepsilon)\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})]\varphi_1^2\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})]. \end{aligned}$$

At the optimum $\varepsilon^*_{\tilde{M},\tilde{\pi}}$, the following equality holds:

$$C'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)\mathbb{E}\left[\alpha\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]\varphi_1 = \mathbb{E}\left[\alpha'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]\varphi + C'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)\phi'\left(w_2 - C\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)\right).$$

Replacing the last term in the expression of $h_{\tilde{M},\tilde{\pi}}(\varepsilon)$ by this equality leads to the following expression of $h_{\tilde{M},\tilde{\pi}}(\varepsilon)$ at the optimum $\varepsilon^*_{\tilde{M},\tilde{\pi}}$:

$$\begin{aligned}
h_{\tilde{M},\tilde{\pi}}\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right) = {} & \\
& \left(2\mathbb{E}\left[\alpha'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]^2 - \mathbb{E}\left[\alpha^{(2)}\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]E\left[\alpha\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]\right)\varphi\varphi_1 \\
& + \varphi\phi'\left(w_2 - C\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)\right)\mathbb{E}\left[\alpha^{(2)}\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right] \\
& + C'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)\varphi\left(\mathbb{E}\left[\alpha'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]\phi^{(2)}\left(w_2 - C\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)\right)\right) \\
& - \mathbb{E}\left[\alpha'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]\mathbb{E}\left[\alpha\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]\varphi_2.
\end{aligned}$$

According to Assumption 2, $2\mathbb{E}\left[\alpha'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]^2 - \mathbb{E}\left[\alpha^{(2)}\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right]\mathbb{E}\left[\alpha\left(\varepsilon^*_{\tilde{M},\tilde{\pi}};\tilde{\pi}\right)\right] < 0$. Thus $g'_{\tilde{M},\tilde{\pi}}(\varepsilon)$ is positive at the optimum $\varepsilon^*_{\tilde{M},\tilde{\pi}}$, that is, $g_{\tilde{M},\tilde{\pi}}(\varepsilon)$ is increasing when crossing function $-C'(\varepsilon)$ at $\varepsilon = \varepsilon^*_{\tilde{M},\tilde{\pi}}$.

The proof of $g'_{\tilde{\pi}}\left(\varepsilon^*_{\tilde{\pi}}\right) > 0$ is similar.

Combined with (6), the positivity of $g_{\tilde{M},\tilde{\pi}}\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)$ demonstrates the uniqueness of the optimum. Indeed, as $-C'(\varepsilon)$ is a decreasing function, non uniqueness would mean that the other crossings would be with a negative slope, which is impossible.

With $g_{\tilde{M},\tilde{\pi}}\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right) = -C'\left(\varepsilon^*_{\tilde{M},\tilde{\pi}}\right)$ and $g_{\tilde{\pi}}\left(\varepsilon^*_{\tilde{\pi}}\right) = -C'\left(\varepsilon^*_{\tilde{\pi}}\right)$, Lemma 9, Equation (7) and $-C$ being a decreasing function of ε yield $\varepsilon^*_{\tilde{\pi}} < \varepsilon^*_{\tilde{M},\tilde{\pi}}$, which proves Lemma 6.1.

For the rest of the proof of the other components of Lemma 6, we introduce the following notations:

$$g(\varepsilon) = \frac{\alpha'(\varepsilon,\mu_\pi)(\phi(w_2 - C(\varepsilon)) - \phi(\overline{w}_1 - C(\varepsilon)))}{(\alpha(\varepsilon,\mu_\pi)\phi'(\overline{w}_1 - C(\varepsilon)) + (1 - \alpha(\varepsilon,\mu_\pi))\phi'(w_2 - C(\varepsilon)))},$$

$$g_{\tilde{M}}(\varepsilon) = \frac{\alpha'(\varepsilon,\mu_\pi)(\phi(w_2 - C(\varepsilon)) - \mathbb{E}[\phi(\tilde{w}_1 - C(\varepsilon))])}{(\alpha(\varepsilon,\mu_\pi)\mathbb{E}[\phi'(\tilde{w}_1 - C(\varepsilon))] + (1 - \alpha(\varepsilon,\mu_\pi))\phi'(w_2 - C(\varepsilon)))}.$$

Utilizing a similar proof as in Lemma 9, we establish the following Corollary:

Corollary 10 $g'_{\tilde{M}}\left(\varepsilon^*_{\tilde{M}}\right) > 0$, $g'(\varepsilon^*) > 0$.

Let us assume that $\alpha^{(3)} < 0$.

We have then $\mathbb{E}[\alpha'(\varepsilon;\tilde{\pi})] < \alpha'(\varepsilon;\mu_\pi)$. As $\alpha^{(2)} > 0$ and $\phi'(\overline{w}_1 - C(\varepsilon)) - \phi'(w_2 - C(\varepsilon)) > 0$, the following inequalities hold:

$$\begin{aligned}
&\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})](\mathbb{E}[\phi'(\tilde{w}_1 - C(\varepsilon))] - \phi'(w_2 - C(\varepsilon))) + \phi'(w_2 - C(\varepsilon)) \\
&> \mathbb{E}[\alpha(\varepsilon;\tilde{\pi})](\phi'(\overline{w}_1 - C(\varepsilon)) - \phi'(w_2 - C(\varepsilon))) + \phi'(w_2 - C(\varepsilon)),
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}[\alpha(\varepsilon;\tilde{\pi})](\phi'(\overline{w}_1 - C(\varepsilon)) - \phi'(w_2 - C(\varepsilon))) + \phi'(w_2 - C(\varepsilon)) \\
&> \alpha(\varepsilon,\mu_\pi)(\phi'(\overline{w}_1 - C(\varepsilon)) - \phi'(w_2 - C(\varepsilon))) + \phi'(w_2 - C(\varepsilon)).
\end{aligned}$$

Thus, the following inequality (upon the assumption $\alpha^{(3)} < 0$ for the right part) is satisfied:

$$g_{\tilde{M},\tilde{\pi}}(\varepsilon) < g_{\tilde{\pi}}(\varepsilon) < g(\varepsilon).$$

As a consequence, in a similar fashion to the proof of Lemma 6.1, we can show that $\varepsilon^*_{\tilde{M},\tilde{\pi}} > \varepsilon^*_{\tilde{\pi}} > \varepsilon^*$. This proves the first inequality of Lemma 6.3.

It is straightforward that through a similar proof we can obtain, upon the same assumptions ($\phi^{(3)} > 0$ and $\alpha^{(3)} < 0$) the following inequality:

$$g_{\tilde{M},\tilde{\pi}}(\varepsilon) < g_{\tilde{M}}(\varepsilon) < g(\varepsilon),$$

and thus $\varepsilon^*_{\tilde{M},\tilde{\pi}} > \varepsilon^*_{\tilde{M}} > \varepsilon^*$. This proves Lemma 6.2 and the second inequality of Lemma 6.3.

A5 | Proof of Corollary 7

Let us define, for risk neutral agents:

$$\varepsilon n^*_{\tilde{M}} = \underset{\varepsilon}{\operatorname{argmax}}\, E\left[U\left(\varepsilon,\tilde{M},\mu_\pi\right)\right],$$

$$\varepsilon n^*_{\tilde{\pi}} = \underset{\varepsilon}{\operatorname{argmax}}\, E[U(\varepsilon,\mu_M,\tilde{\pi})],$$

$$\varepsilon n^*_{\tilde{M},\tilde{\pi}} = \underset{\varepsilon}{\operatorname{argmax}}\, E\left[U\left(\varepsilon,\tilde{M},\tilde{\pi}\right)\right].$$

Due to the assumption of risk neutrality, expected utilities can be written as follows:

$$\mathbb{E}\left[U^d\left(\varepsilon,\tilde{M},\tilde{\pi}\right)\right] = \mathbb{E}\left[U_F - C(\varepsilon) + \beta\left(\alpha(\varepsilon,\tilde{\pi})\left(u\left(H-\tilde{M}\right) - u(H)\right) + u(H)\right)\right].$$

Since $\tilde{M}$ and $\tilde{\pi}$ are independent we have

$$\mathbb{E}\left[U^d\left(\varepsilon,\tilde{M},\tilde{\pi}\right)\right] = U_F - C(\varepsilon) + \beta\left(\mathbb{E}[\alpha(\varepsilon,\tilde{\pi})]\left(\mathbb{E}\left[u\left(H-\tilde{M}\right)\right] - u(H)\right) + u(H)\right).$$

Thus if $\varepsilon n^*_{\tilde{M},\tilde{\pi}}$ is an interior solution, it is a solution of

$$C'\left(\varepsilon n^*_{\tilde{M},\tilde{\pi}}\right) = \beta\mathbb{E}\left[\alpha'\left(\varepsilon n^*_{\tilde{M},\tilde{\pi}},\tilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H-\tilde{M}\right)\right] - u(H)\right).$$

Furthermore, if $\varepsilon n^*_{\tilde{M}}$ is an interior solution, it is a solution of the following equation:

$$C'\left(\varepsilon n^*_{\tilde{M}}\right) = \beta\mathbb{E}\left[\alpha'\left(\varepsilon n^*_{\tilde{M}},\tilde{\pi}\right)\right](u(H-\mu_M) - u(H)).$$

As u is concave, we have $\mathbb{E}\left[u\left(H-\tilde{M}\right)\right] \le u(H-\mu_M)$. And since $\mathbb{E}\left[\alpha'\left(\varepsilon n^*_{\tilde{M}},\tilde{\pi}\right)\right] < 0$, then

$$\mathbb{E}\left[\alpha'\left(\varepsilon n^*_{\tilde{M}},\tilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H-\tilde{M}\right)\right] - u(H)\right) \ge \mathbb{E}[\alpha'(\varepsilon^*,\tilde{\pi})](u(H-\mu_M) - u(H)).$$

The functions $\varphi(\varepsilon,\tilde{M},\tilde{\pi}) = \mathbb{E}\left[\alpha'\left(\varepsilon n^*_{\tilde{M}},\tilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H-\tilde{M}\right)\right] - u(H)\right)$ and $\varphi(\varepsilon,\mu_M,\tilde{\pi}) = \mathbb{E}\left[\alpha'\left(\varepsilon n^*_{\tilde{M}},\tilde{\pi}\right)\right](\mathbb{E}[u(H-\mu_M)] - u(H))$ are decreasing in ε since $\mathbb{E}\left[\alpha^{(2)}\left(\varepsilon n^*_{\tilde{M}},\tilde{\pi}\right)\right] > 0$.

Moreover, function C being convex ($C^{(2)} > 0$), we finally get $\varepsilon n^*_{\tilde{M}} < \varepsilon n^*_{\tilde{M},\tilde{\pi}}$, which proves Corollary 7.

It is straightforward that this result holds if susceptibility is a known parameter $\hat{\pi}$ instead of a random variable $\tilde{\pi}$. In that case the assumption on $\alpha^{(3)} > 0$ is not needed, only the assumption on $\phi^{(3)} > 0$ is necessary.

A6 | Proof of Corollary 8

In a similar fashion to the previous proof, for uncertainty on susceptibility we know that if $\varepsilon n^*_{\tilde{M},\tilde{\pi}}$ is an interior solution, it is a solution of the following equation:

$$C'\left(\varepsilon n^*_{\tilde{M},\tilde{\pi}}\right) = \beta\mathbb{E}\left[\alpha'\left(\varepsilon n^*_{\tilde{M},\tilde{\pi}},\tilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H-\tilde{M}\right)\right]-u(H)\right).$$

Furthermore, if $\varepsilon n^*_{\tilde{\pi}}$ is an interior solution, it is a solution of

$$C'\left(\varepsilon n^*_{\tilde{\pi}}\right) = \beta\alpha'\left(\varepsilon n^*_{\tilde{\pi}},\mu_\pi\right)\left(\mathbb{E}\left[u\left(H-\tilde{M}\right)\right]-u(H)\right).$$

Assuming $\alpha^{(3)} < 0$

$$\mathbb{E}[\alpha'(\varepsilon,\tilde{\pi})] \leq \alpha'(\varepsilon,\mu_\pi),$$

which yields

$$\mathbb{E}[\alpha'(\varepsilon,\tilde{\pi})]\left(E\left[u\left(H-\tilde{M}\right)\right]-u(H)\right) \geq \alpha'(\varepsilon,\mu_\pi)\left(\mathbb{E}\left[u\left(H-\tilde{M}\right)\right]-u(H)\right).$$

As C' is increasing in ε we finally get $\varepsilon n^*_{\tilde{M},\tilde{\pi}} > \varepsilon n^*_{\tilde{\pi}}$.

Conversely, if $\alpha^{(3)} > 0$ then $\varepsilon n^*_{\tilde{M},\tilde{\pi}} < \varepsilon n^*_{\tilde{\pi}}$. This proves Corollary 8.

As in the previous proof, it is straightforward that this result holds if severity is a known parameter $\hat{M}$ instead of a random variable $\tilde{M}$. In that case, the assumption on $\alpha^{(3)} > 0$ is not needed, only the assumption on $\phi^{(3)} > 0$ is necessary.