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A novel formulation of low voltage distribution network equivalents for reliability analysis

Mike Brian Ndawula [a,b,*], Sasa Z. Djokic [c], Mikka Kisuule [d], Chenghong Gu [a], Ignacio Hernando-Gil [e,f]

[a] Centre for Sustainable Power Distribution, University of Bath, United Kingdom
[b] National Grid Electricity Transmission, United Kingdom
[c] School of Engineering, The University of Edinburgh, Edinburgh, United Kingdom
[d] Department of Electrical and Computer Engineering, CEDAT, Makerere University, P.O. Box 7062, Kampala, Uganda
[e] ESTIA Institute of Technology, University of Bordeaux, Bidart F-64210, France
[f] Institute for Systems and Computer Engineering, Technology and Science (INESC TEC), Porto, Portugal

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ABSTRACT

Reliability analysis of large power networks requires accurate aggregate models of low voltage (LV) networks to allow for reasonable calculation complexity and to prevent long computational times. However, commonly used lumped load models neglect the differences in spatial distribution of demand, type of phase-connection of served customers and implemented protection system components (e.g., single-pole vs three-pole). This paper proposes a novel use of state enumeration (SE) and Monte Carlo simulation (MCS) techniques to formulate more accurate LV network reliability equivalents. The combined SE and MCS method is illustrated using a generic suburban LV test network, which is realistically represented by a reduced number of system states. This approach allows for a much faster and more accurate reliability assessments, where further reduction of system states results in a single-component equivalent reliability model with the same unavailability as the original LV network. Both mean values and probability distributions of standard reliability indices are calculated, where errors associated with the use of single-line models, as opposed to more detailed three-phase models, are quantified.

1. Introduction

Due to their volume and complexity, three-phase models (TPMs) of low voltage (LV) networks are often represented by equivalent single-line models (SLMs), which assume equally distributed customers and balanced operating conditions. Furthermore, during the reliability performance analysis of medium voltage (MV) and high voltage (HV) networks, SLMs of LV networks are typically not represented in much detail. This is due to a general lack of detailed information on LV network configurations (particularly customer service entry connections), protection systems, and fault rates and repair times of power components (PCs). An additional reason for these simplifications is that detailed modelling of LV networks significantly increases the complexity of calculations and therefore leads to long computational times. The SLMs are less accurate for LV than for MV networks, as LV networks feature more unbalanced loading due to diversity of customer demand patterns, uneven distribution and number of per-phase connected customers, as well as phase voltage variations (usually within ±10 % of the nominal voltage) [1]. Finally, the use of lumped SLMs to further simplify detailed LV TPMs is often inaccurate due to neglected spatial distribution of load points, different types of constituent PCs and variations in applied protection devices and schemes. All that is further worsened by the limited visibility of the real-time LV network utilisation [2].

Although the contribution of LV supply interruptions to the system quality of supply (QoS) metrics varies significantly in different countries/networks (it is usually estimated from notifications of interrupted customers), it is not negligible. Permanent faults in LV networks significantly contribute to both total customer minutes lost and customer interruptions [3]. Therefore, it is important to develop improved equivalent models of LV networks, to allow distribution network operators to perform a more accurate reliability analysis of larger MV/HV networks without increasing computational requirements and overall analysis complexity.

* Correspondence to: Centre for Sustainable Power Distribution, University of Bath, BA27AY, United Kingdom.
E-mail addresses: M.B.Ndawula@bath.ac.uk (M.B. Ndawula), Sasa.Djokic@ed.ac.uk (S.Z. Djokic), C.Gu@bath.ac.uk (C. Gu), I.Hernandogil@estia.fr, ignacio.gil@inesctec.pt (I. Hernando-Gil).

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Nomenclature

Variables

C_0^m	Normal operation system state, where all m components are operational.
C_k^m	All system states with k simultaneously faulted components.
ε	Accuracy for SMCS stopping criterion.
λ	Mean failure rate.
λ_{eqv}	Equivalent mean failure rate.
μ	Mean repair time.
μ_{eqv}	Equivalent mean repair rate.
m	Number of power components.
r	Reduced number of system states.
R_i	Uniformly distributed random number.
P_j	Probability of j^{th} component's state change.
σ	Standard deviation from SMCS.
S_k	System state.
T_i	State duration of the i^{th} component.
U	Total system unavailability.

Acronyms

AEM	Alternative existing method for equivalents.
CDF	Cumulative distribution function.
ED	Enumeration depth.
ENS	Energy not supplied.
Eqv-PC	Single equivalent power component.
HV	High voltage.
LP	Load point.
LV	Low voltage.
MCS	Monte Carlo simulation.
$MTTR_{eqv}$	Equivalent mean repair time.
MV	Medium voltage.
PC	Power component.
PDF	Probability distribution function.
QoS	Quality of Supply.
SAIFI	System average interruption frequency index.
SAIDI	System average interruption duration index.
SDS	State duration sampling.
SE	State enumeration.
SMCS	Sequential Monte Carlo simulation.
SLM	Single-line model.
STS	State transition sampling.
TPM	Three-phase model.

The aggregation of LV networks for reliability analysis must consider the per-phase distribution of customer supply points (e.g., residential customers in some countries are usually single-phase connected), the predominant use of single-pole protection components (i.e., single-phase fuses) and subsequent disaggregation of LPs. This will prevent large cumulative errors due to inaccurate representation of LV networks in the reliability analysis of MV/HV networks, allowing to correctly characterise the QoS levels for different customers and provide meaningful metrics to inform network planning and operation. Moreover, standard reliability analysis of larger MV and HV networks usually employs some equivalent form of LV SLMs [4,5]. The most common one is a simple lumped balanced load model [6], specifying only the number of customer load points (LPs) and their peak active and reactive power demands downstream of the MV/LV substation, where customers on different phases typically experience varying QoS levels.

Accordingly, previous research in [7–10] provided equivalent fault rates and mean repair times by calculating the sum of all PC fault rates and average of their mean repair times. The key drawback of this approach is that it does not differentiate between different protection types (e.g., 1-phase fuses and 3-phase circuit breakers), which, in the case of asymmetrical faults, can lead to large inaccuracies for networks with low reliability levels. Another common approach is to reduce the entire system to a single equivalent component by combining series and parallel branches of the network, such that the reliability of the obtained equivalent component equals the reliability of the original network. The main drawbacks of these methods are [11,12]:

- Limited applicability, typically only to networks with relatively simple topologies;
- Difficulty to accommodate various failure modes, maintenance schedules and weather effects;
- Inability to calculate customer-based reliability indices e.g., customer average interruption duration and frequency indices, because of inaccurate aggregation of demands at different network nodes;
- Difficulties in distinguishing the impact of areas with different PCs on the overall system reliability indices.

Existing techniques, such as the decomposition method, are based on conditioning a complex system on the states of key PCs, but the model becomes unmanageable as the number of key PCs increases [11]. Similarly, the minimal cut set method has "combinatorial explosion" when used for larger systems [13]. Although some techniques reduce a TPM into a single-line equivalent impedance model, e.g., for load-flow or voltage stability analysis [14,15], such equivalent models are generally not suitable for reliability analysis, as they are limited to radial networks. Other network reduction methods, such as variants of Ward and Dimo, cannot sufficiently reduce LV networks to single-component equivalents for reliability assessment of large and complex LV/MV networks [16,17].

The reliability assessment methods commonly used for assessing the probabilities of system states in distribution networks can be divided into two categories: simulation methods [18] and analytical methods [4]. Monte Carlo simulation (MCS) method is often used for assessment of network reliability performance [19–21], with sequential MCS (SMCS) providing flexibility for modelling different state duration distributions and allowing to consider the ageing effects of the PCs [22]. A significant advantage of MCS is reduced correlation between system dimensions and result accuracy. Compared to analytical methods, the MCS method is generally better suited for practical applications. However, the SMCS has substantial computational time and storage requirements, which increase with the size of the modelled network [4]. Also, for systems with high reliability, sampling of faulted states remains as a challenging aspect.

Conversely, analytical methods such as state enumeration (SE) risk assessment techniques [23–25] are often used to reduce network complexity through the selection of modified system states related to low-order contingencies [4]. This is because SE methods directly enumerate all system states while disregarding the transition relationships between states [23], making them unsuitable for sequential analysis. More recently, SE methods have focused on the assessment of high impact-low probability events [26] and have been reported for reliability assessment, e.g., in [27], but SE has not been used for the formulation of network reliability equivalents.

Modern power distribution systems generally present looped or multi-sectional, multi-tie configurations, where multiple tie switches are used to connect different load points. Therefore, when considering higher-order faults, the indices produced by traditional analytical methods, i.e., SE, may exhibit significant discrepancies from the actual values [28–30]. The SE method is thus more effective for networks with a smaller number of network components, but it cannot be used to model chronological time-dependent events, which can be more accurately captured by the SMCS methods [4].

To address all these constraints, the combination of SE and SMCS methods proposed in this paper first applies the state selection in SE, and then uses SMCS to ensure the accurate modelling of chronological time-dependent events, spatially distributed PCs and connected customers. This helps to overcome limitations of each method when used separately.

To the best of the authors' knowledge, this paper is the first attempt to combine SE and SMCS methods for an efficient and more accurate formulation of LV network reliability equivalents, presenting the following main contributions:

- A novel comparison of the (typically used) simplified single-line models (SLMs) and fully detailed three-phase models (TPMs) of LV networks, avoiding system performance overestimation by applying a comprehensive reliability assessment of a realistic LV distribution network;
- Development of a novel LV/MV network reliability assessment methodology that, for the first time, combines SE and SMCS to enable a more accurate formulation of network reliability equivalents with significantly reduced computational complexity;
- Development of a new simple single-component equivalent that offers the same unavailability (and therefore reliability performance) as the original LV/MV network.
- Evaluation of accuracy, computational efficiency, and scalability of the proposed LV equivalents, which are tested and validated in more complex and larger MV networks, illustrating replicability of the proposed methodology.

The paper also presents a detailed comparison of the results for both mean values and probability distributions of the commonly used reliability indices: system average interruption duration index (SAIDI), system average interruption frequency index (SAIFI), and energy not supplied (ENS) [31]. All reliability indices are obtained from probabilistic-based SMCS calculations and presented values are expected (ex-ante) values, as opposed to ex-post values. The rest of the paper is organised as follows: the test LV network used for analysis is presented in Section II, Section III details the development of the combined SE-SMCS methodology, while Section IV evaluates the performance of the proposed approach. Finally, the main conclusions are presented in Section V.

2. Low voltage network design

Residential LV networks differ in geographical locations, numbers of supplied customers and their peak demand, PC types and ratings, as well as used protection systems [32]. The main design criteria of the secondary (LV) distribution systems are voltage regulation requirements and current-carrying capacities, which determine transformer ratings and feeder types for the required lengths, while the network strength (i.e., fault levels) and earthing arrangements determine types and settings of protection [32].

The UK residential LV networks can be divided into three general types: urban, rural and suburban [3]. This paper uses the suburban LV network as a test-case, because their characteristics (e.g., mix of different PCs and different topologies) allow to demonstrate the capability of the proposed equivalenting methodology for a wider range of LV networks. In general, suburban networks represent individual house dwellings located in city suburban areas and in small towns. Due to the lower costs and greater space availability, they are characterised by a medium power density and use of radial overhead lines, as opposed to underground cable systems in urban areas. Service entry connections are single-phase, with either overhead lines or underground cables. Fig. 1 represents the TPM of a generic suburban UK LV network [5] protected with fuses and auto-reclosing circuit breakers (AR-CB), and 27 LPs supplying 76 customers, each with after-diversity maximum demand (ADMD) of 2.27 kW.

Fig. 2 represents the SLM of the same network with per-phase clustering of customers to only 9 LPs [5]. The PC data used for both models including buses, transformers, overhead lines, cables, circuit breakers and fuses are provided in [5].

3. Methodology

3.1. Modelling component failures and fault types

The aggregate fault statistics used for the SLM network are obtained from [7]. For application on the TPM, the historical fault rates of all *single-phase* PCs are divided by three, to disaggregate reported values proportionally. It should be noted that depending on the network design, characteristics, and operational conditions, one of the phases is usually affected more than the other two, but to the best of the authors' knowledge, the reliability data required for a more detailed per-phase analysis is not available in the existing literature. The mean repair time values for PCs in both TPM and SLM networks are assumed to be the same. Although detailed statistics on fault types in LV networks are scarcely available, the fault types are disaggregated as follows: 50 % of the faults are single-line-to-ground faults, 28 % are double-line and double-line-to-ground faults, while 22 % are three-phase faults, based mostly on statistics in [33] and [34].

3.2. State enumeration (SE) method

For the SE-based reliability analysis of complex networks, an iterative process is usually followed according to four steps: i) system state selection, ii) analysing the impact of the state, iii) calculating the reliability indices, and iv) updating the cumulative indices. The state selection process in SE depends on the enumeration of all possible system states. For a system of m PCs, the total number of all system states is 2^m, so it is not computationally feasible to enumerate all system states for large systems. Usually, the analysis stops at a given enumeration depth (ED), which corresponds to a failure level. For example, the first failure level (ED_1) denotes all faulted system states that contain exactly one faulted PC:

$$ED_1 = C_0^m + C_1^m \tag{1}$$

where C_0^m corresponds to the 'normal operation' system state with all m components operational, and C_1^m to all system states where exactly one PC is faulted. The second failure level (ED_2) corresponds to all ED_1 states plus all system states containing two simultaneous PC failures, as given by (2), and so on, as generalised by (3).

$$ED_2 = C_0^m + C_1^m + C_2^m \tag{2}$$

$$ED_k = C_0^m + C_1^m + \ldots + C_k^m \tag{3}$$

where k is the depth of the failure level and all system states are enumerated if $k = m$. All EDs contain the normal state (C_0^m), where all PCs are in normal operation as given by (4).

$$ED_1 < ED_2 << 2^m \tag{4}$$

Table 1 provides the number of states for the first two ED levels for the TPM and SLM LV networks in Figs. 1 and 2.

After the state selection, the next step is to determine the frequency of occurrence and mean duration of each system state. Then, the impact of each state must be identified in terms of interrupted loads/customers. The SE assumes mutual exclusiveness between all system states, describing each state as a continuous-time Markov chain (CTMC) with

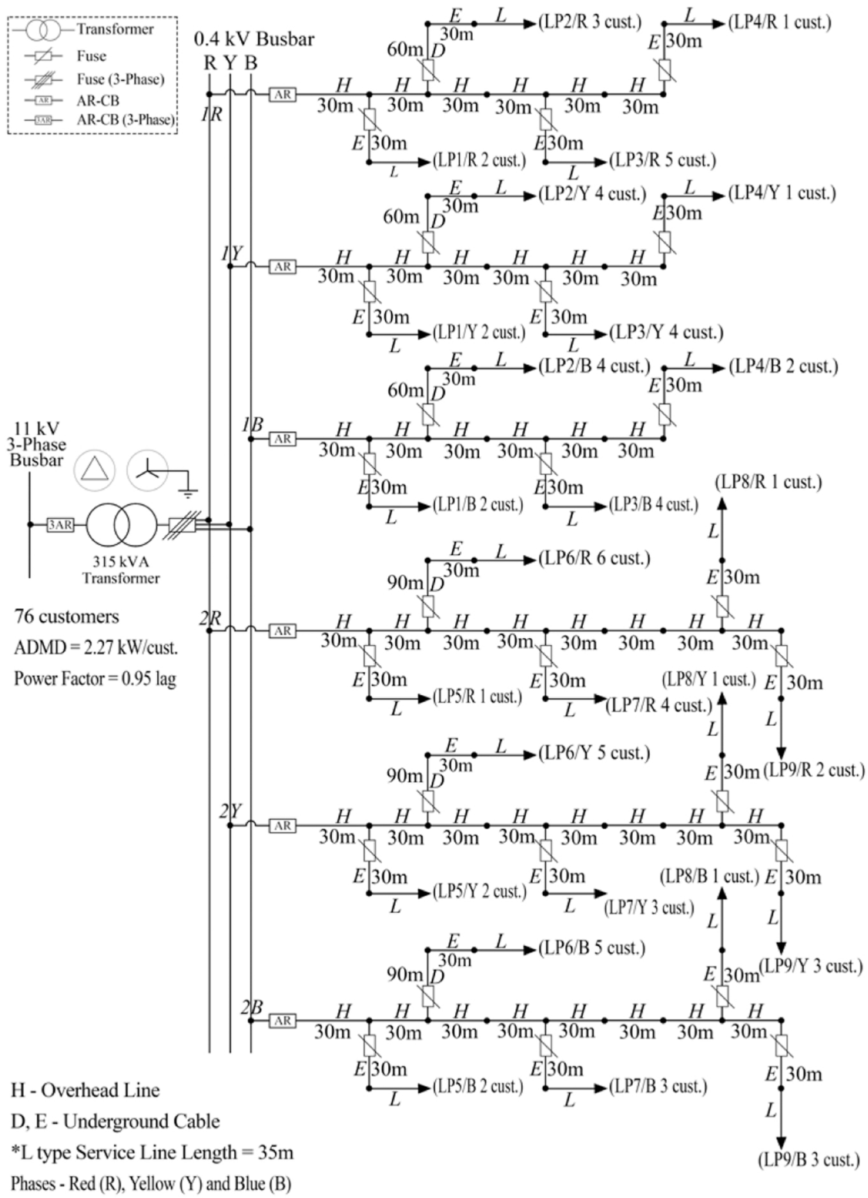

Fig. 1. Three-phase model (TPM) of a generic suburban LV network.

Poisson distributions modelled for the fault/repair times of all PCs. In that way, system states are determined by transitions of individual component states, derived from their failure and repair rates. The analysis may not include all possible system states in the CTMC, e.g., when the state space is reduced by adopting the SE state selection procedure based on the specific ED. Thus, an artificial cycle of system 'operating' and 'failure states' is created with a sequential MCS, allowing for the calculation of relevant reliability indices. Considering low-order contingencies will result in reduced computational efforts without impacting accuracy, as successive transitions between two system failure states are very rare, while transitions between normal and failure system states are dominant in real power systems. System states with several simultaneously failed PCs are also very rare [4]. Building reliability network equivalents is inevitably a trade-off between accuracy and computational time, where the use of low-order EDs (Table 1) should typically satisfy the accuracy requirements. However, the use of higher-order EDs (e.g., higher than in Table 1) should be carefully considered for other analysis e.g., system resilience and protection design.

3.3. *Sequential Monte Carlo simulation (SMCS) method*

Sequential MCSs are usually performed using state duration sampling (SDS), where each PC is assigned a mean failure rate (λ) and mean repair time (μ), and where state durations for time-to-fail and time-to-repair of each PC are generated, to allow for the creation of a state transition process of the up and down cycles of all system PCs. The SDS is suited to the assessment of large networks, allowing to apply different probability distributions for PC failure and repair processes [22].

The proposed combination of SE and SMCS requires to modify SDS, because the new system description considers a reduced number of states (r) that do not distinctly equate to an equivalent number of PCs. For example, $r = C_0^{97} + C_1^{97} = 98$ states for the SLM ED_1 in Table 1, which corresponds to a fictitious number of PCs between 6 ($2^6 = 64$) and 7 ($2^7 = 128$). Thus, to perform sequential analysis, it is necessary to adopt MCS based on state transition sampling (STS), which assumes that PC failure and repair processes are both modelled with exponential distributions [35]. To prevent from any possible departure from exponential, in [36] the authors assessed the suitability of several probability

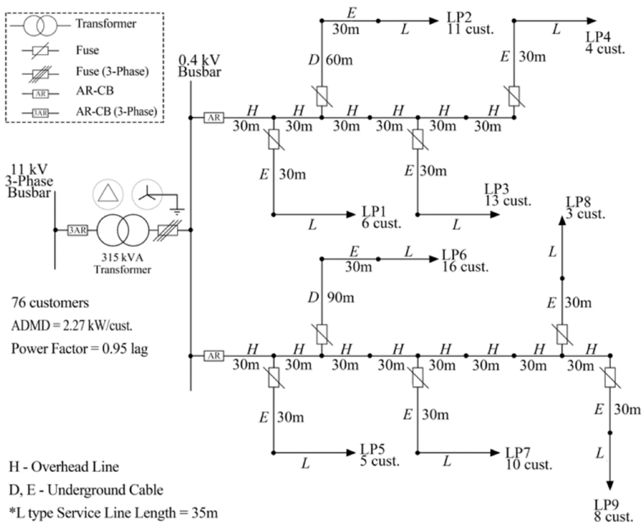

Fig. 2. Single-line model (SLM) of a generic suburban LV network.

Table 1
Number of system states for two enumeration depths (EDs).

Type	LPs	PCs	ED_1 States	ED_2 States
TPM	27	281	282	39,622
SLM	9	97	98	4,754

distributions (exponential, Weibull, Rayleigh and Gamma) to model the failure and repair process of network components. Accordingly, the STS focuses on state transitions of the entire system, rather than on individual PC states or durations. The system will transit from one system state S_k into the next state S_{k+1} depending on the random state duration of the PC that first departs from its present state (up or down) S_k:

$$T = \min_i(T_i) \tag{5}$$

where: T is the duration of system state S_k and T_i is the state duration of the i^{th} PC, for $i = 1, 2, ..., m$, where m is the number of PCs. Using the conditional probability theory for state transition sampling [35], it is possible to determine if the transition from state S_k to S_{k+1} is caused by the departure of j^{th} PC from its present state S_k. The probability of the j^{th} PC departing from present state at time t_0 is given by:

$$P_j = \frac{\lambda_j}{\sum_{i=1}^{m} \lambda_i} \tag{6}$$

where P_j is the probability of departure of the j^{th} PC from its present state. This means that the state transition of any PC (according to P_j) leads to a possible state transition, and for a system of m PCs, there can be m possible reached states in each transition. Using (6), the probability of each of the m PCs transiting from their present state can be calculated. The probability of the m states that could be reached can then be successively placed in the interval [0,1], as shown in Fig. 3 (because $\sum_{j=1}^{m} P_j = 1$) [35].

The next step is to generate a uniformly distributed random number

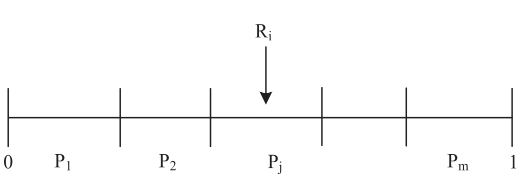

Fig. 3. Transition to different system states using random number generation in the state transition sampling technique [35].

R_i between 0 and 1. If R_i falls within the segment corresponding to P_i, then the transition of the i^{th} PC leads to the next system state. The consequences of each system state, i.e., the impact on frequency and duration of customer interruptions, are analysed by considering ADMD values of demands of affected customers. Afterwards, all cumulative indices are updated before generating a new random number R_i and simulations are repeated in 1-hour time steps until the stopping criterion, i.e., the required accuracy ε, is satisfied:

$$\varepsilon = \sqrt{\text{var}(x)} \Big/ \left(\bar{x} \cdot \sqrt{N}\right) \tag{7}$$

where: $\sqrt{\text{var}(x)}$ is the standard deviation, $\bar{x}$ is the mean, and N is the number of samples [4].

3.4. Formulation of equivalent reliability network model

The STS-based SMCS procedure allows to formulate a new network reliability equivalent using state selection procedure derived from the SE method. The new system description, based on a selected number of system states (e.g., ED_1 or ED_2), can be further reduced to produce an equivalent single-component reliability model, where failure and repair rates of that single-equivalent PC are obtained by setting its unavailability to be the same as the unavailability of the SE-reduced network (with considered ED_1 or ED_2 states). The aim of a reliability assessment is to identify and deploy network solutions that will ensure customers are provided with continuous supply and, when supply interruptions occur, to quantify their impact. For that purpose, frequency-based indices (e.g., SAIFI) will provide information on the frequency of supply interruptions, which could be used to assess customers' costs/losses per interruption event, while duration-based indices (e.g., SAIDI) will provide information on the duration of supply interruptions, which could be used to assess customers' costs/losses per minute or hour of an interruption event. The use of ENS, however, combines both frequency- and duration-based indices, and it is therefore a composite reliability performance indicator that quantifies the combined effects of the numbers and durations of supply interruptions with the amount of interrupted demand. For example, ENS allows for a direct evaluation of "savings" due to unsupplied energy and for assessing customers' willingness-to-pay for supply restoration [37].

In this paper, energy-not-supplied index (ENS) is used for an alternative formulation of unavailability, U, which is suited for representing a lumped symmetrical load model, as both ENS and unavailability can be simply added to the information on the peak active and reactive power demands. In that way, this methodology allows to significantly improve accuracy of reliability analysis of larger MV and HV networks (e.g., for a total of ~3000 customers and ~5000 PCs) employing traditional lumped models of LV/MV networks (e.g., with ~100 PCs), where the corresponding new equivalent models can be obtained without substantially increasing computational complexity. The reliability dependency between MV and LV networks would thus be accurately modelled. Even at MV, the proposed SE-SMCS method would account for all PC failure rates (both MV and LV) considering their impact on network reliability and their spatial distribution/location in the network.

Accordingly, the analysis outlined by (8)-(11) summarises the general procedure for obtaining accurate reliability network equivalents after the initial system reduction using SE and selecting the ED. The equivalent failure rate (λ_{eqv}) of a single-component equivalent is derived from SAIFI as:

$$\lambda_{eqv} = SAIFI \times LPs \tag{8}$$

while the total system unavailability, U, is formulated as:

$$U = \frac{Annual\ ENS}{Connected\ demand \times hours\ inayear} \tag{9}$$

The equivalent repair rate (μ_{eqv}) and equivalent repair time ($MTTR_{eqv}$) of a single-component equivalent reliability model can then be derived using (10) and (11).

Fig. 4 illustrates the general algorithm for using SE-SMCS to obtain single-component equivalent network reliability models, while Fig. 5 illustrates a high-level schematic of the proposed equivalenting methodology.

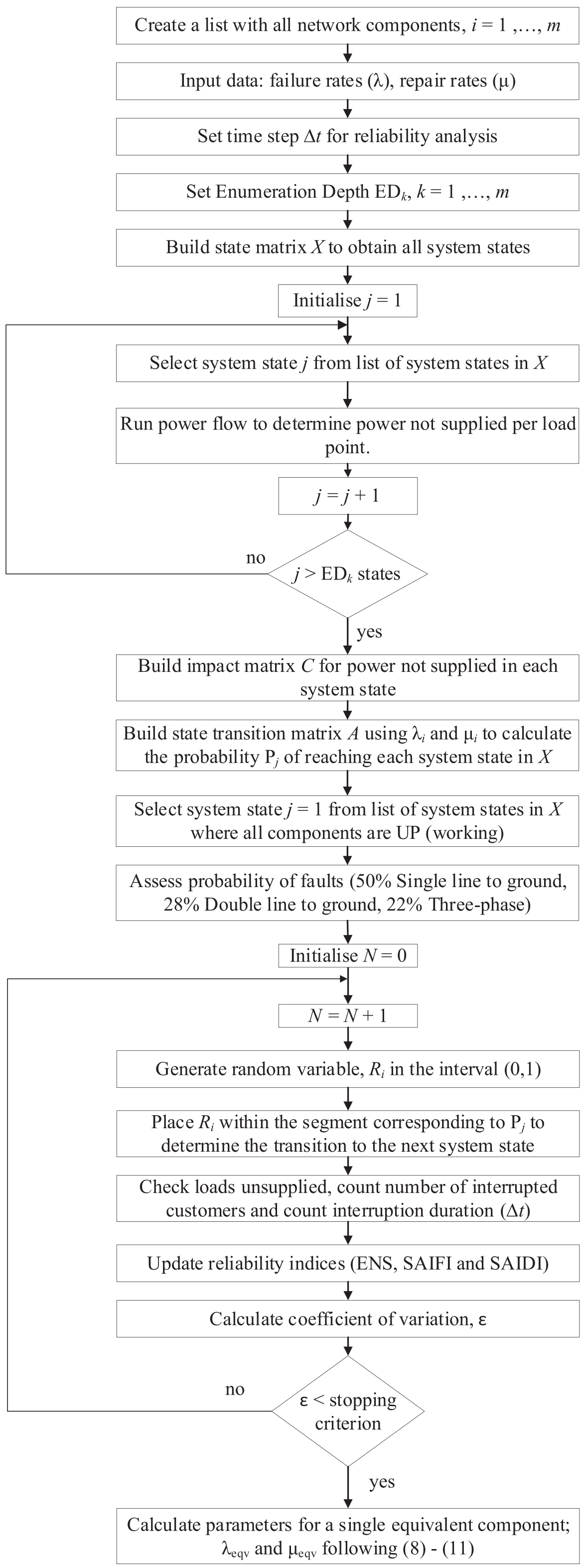

Fig. 4. General algorithm of the proposed equivalenting methodology.

$$U = \frac{\lambda_{eqv}}{\lambda_{eqv} + \mu_{eqv}} \tag{10}$$

$$\mu_{eqv} = \frac{(1-U)\cdot\lambda_{eqv}}{U}, MTTR_{eqv} = \frac{1}{\mu_{eqv}} \tag{11}$$

4. Results

4.1. Single-line model (SLM) vs three-phase model (TPM)

Fig. 6 presents the three main reliability indices calculated for the comparison of the SLM and TPM networks. The accuracy is evaluated using the standard error from SMCS:

$$Std.error = \sigma \Big/ \sqrt{N} \tag{12}$$

where: σ is standard deviation; N is the number of samples. As expected, the use of the TPM of the LV network results in a more accurate reliability analysis, mainly due to its more detailed representation of the actual network. The SLM results in an underestimated ("worse-than-actual") reliability performance, which is not as obvious for SAIFI and SAIDI results as it is for the ENS results. The reason is that in both SLM and TPM, SAIDI and SAIFI are calculated in the same way for any fault i.e., for single-line-to-ground, double-line (to-ground) and three-phase faults, which are also evaluated with the same durations. However, the interrupted loads/demands are significantly different in the two models, which is indicated by a three-fold difference in the ENS results, confirming that the TPM correctly acknowledges different types of faults and correctly represents both single-phase connection of customers and operation of single-pole protection components (fuses). Since the TPM offers a more realistic evaluation than the SLM, Sections IV.B and C focus only on the TPM representation of the generic LV network.

4.2. Combining SE and SMCS for LV reliability equivalents

As described in Section III.B, the next step is to use SE to reduce the complexity of the TPM to a lower number of system states, representing the first and second EDs (Fig. 7). This section compares the indices obtained using their mean values to demonstrate that the use of low-order EDs is sufficient to assess network reliability performance with good accuracy, while requiring significantly shorter computational times. The term 'original' in Fig. 7 refers to the TPM of the LV test network, while the terms 'ED1' and 'ED2' refer to the two corresponding EDs in the SE approach.

Since the ENS has the lowest rate of convergence [4], the SMCS stopping criterion ε, given by (7), is set to 12 % as in [37]. The standard error is highest for the original network, owing to the lower overall number of MCS samples. This is due to the computational memory "bottleneck" encountered while checking the consequences of all system states, which can be avoided using the smaller ED networks (allowing for more MCS samples). In addition, Table 2 presents the index values for the TPM, reporting a low error (less than 2.5 %) for all indices, showing that ED_1 and ED_2 produce nearly identical results, which is due to the previously discussed low probability of double faults.

The higher number of states in ED_2 results in much longer simulation times than for ED_1, even though both networks present a significant reduction in the computational time: when compared to the MCS analysis of the original network, computational time is reduced by 99.9 % for ED_1 and by 91.1 % for ED_2 models.

The time required to perform equivalenting is also higher in ED_2 (33.8 hours) than ED_1 (2.5 hours) due to the considerably higher number of system states. Table 2 demonstrates that ED_1 could be sufficiently representative of the original network in terms of reliability performance, while allowing for the generation of more MCS samples due to

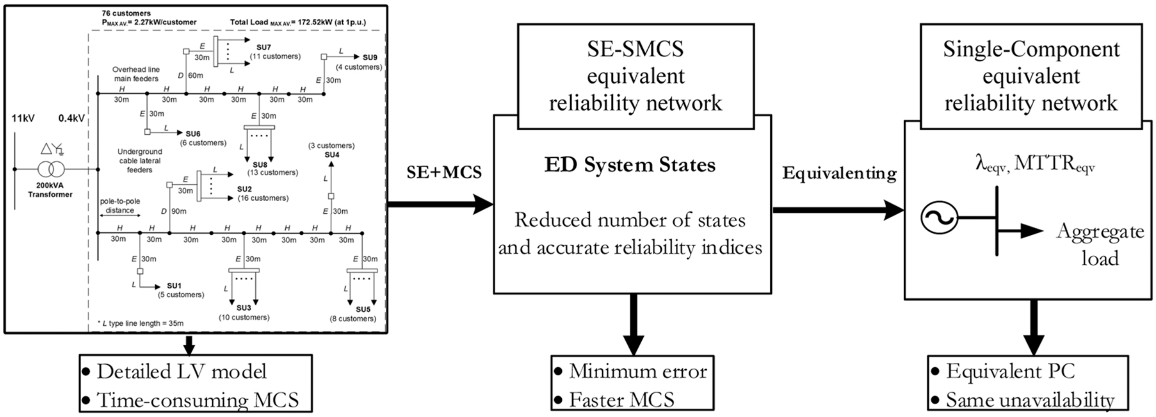

Fig. 5. Schematic representation of the proposed equivalenting methodology.

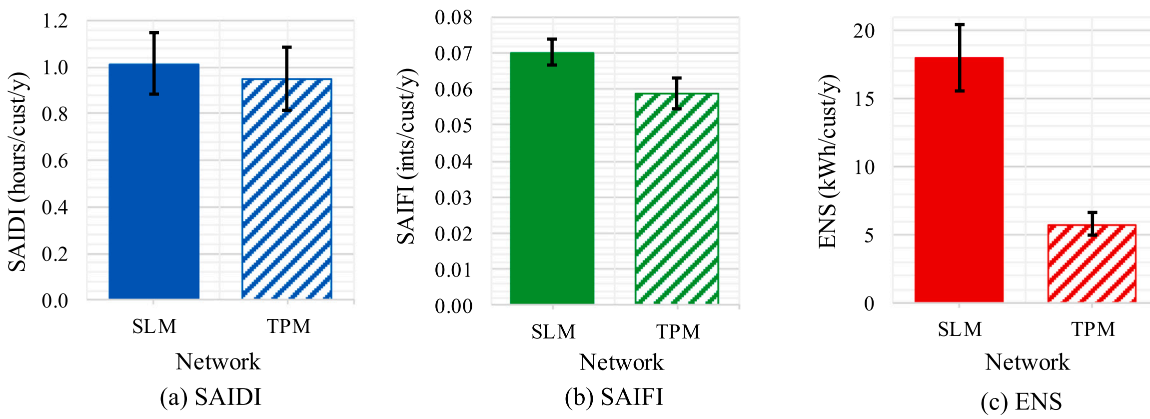

Fig. 6. Reliability indices for the SLM and TPM of a suburban LV network.

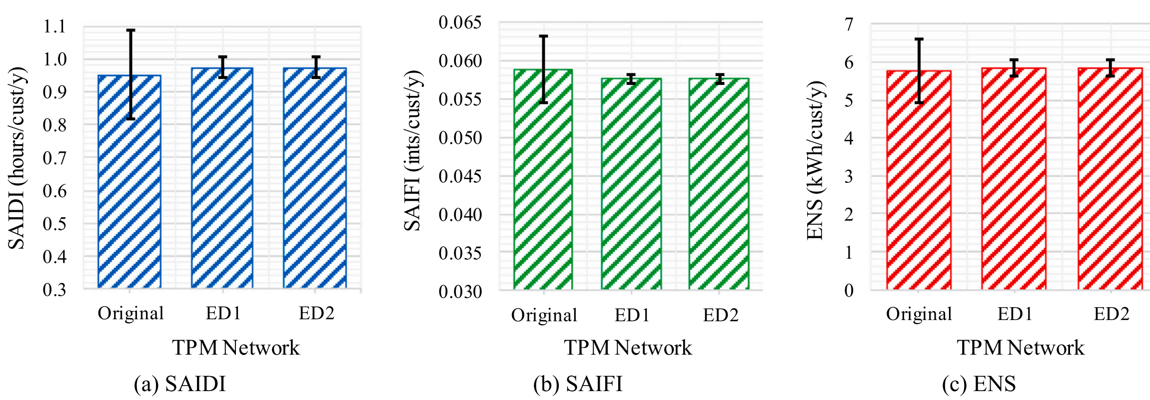

Fig. 7. Reliability indices comparing the performance of the original and reduced order networks for the TPM of the suburban LV network.

the shorter computational times.

Table 2
Reliability indices for the TPM original and SE networks.

Network	SAIDI		SAIFI		ENS	
	hrs/c/y	Error	ints/c/y	Error	kWh/c/y	Error
Orig.	0.9508	-	0.0588	-	5.7648	-
ED_1	0.9739	2.4 %	0.0576	2.0 %	5.8437	1.4 %
ED_2	0.9741	2.4 %	0.0576	2.0 %	5.8446	1.4 %

4.3. Combined SE-SMCS for single-component equivalents

As previously introduced in Section III.D, SE is used to reduce the detailed LV test network to its most probable system states. This allows for the execution of a fast and reasonably accurate sequential MCS analysis that reveals the network reliability performance in terms of standard indices (SAIDI, SAIFI and ENS). Afterwards, the single equivalent component (Eqv-PC) that represents the modelled LV network with the same average unavailability is calculated according to (8)-(11). This Eqv-PC has an equivalent failure rate and repair time that correctly represent not only the SAIFI and SAIDI performance, but also the ENS. Therefore, this Eqv-PC can be connected to a larger (MV) network as a "lumped reliability equivalent", ensuring accurate analysis.

The results presented in this subsection for the Eqv-PCs are limited to only ED_1, since results in Section IV.B demonstrate that there is no significant difference between ED_1 and ED_2 results. Table 3 presents failure rate (λ_{eqv}) and repair time ($MTTR_{eqv}$) obtained for the Eqv-PC, which has the same unavailability as the TPM ED_1 network. Also, the Eqv-PC requires significantly shorter computational time than the MCS analysis of the actual TPM network (-99.98 %).

Table 4 presents the MCS results when this Eqv-PC is connected to the lumped/aggregate load (representing the total network demand), as would be typically the case during the analysis of larger MV networks. The results show a very good matching with the values obtained using the reduced ED_1 network and demonstrate a relatively low error when compared to the original TPM network.

Again, the MCS stopping criterion (7) is set to ε=12 % [38] to obtain the satisfactory accuracy and allow the simulation to converge to values that will permit a high-confidence comparison for both networks. Fig. 8 presents the cumulative distribution functions (CDFs) of the indices presented in Table 4, where the tails of the distributions are truncated for a clearer presentation. The CDFs are especially important in quantifying the risk of paying compensation to customers, e.g., when durations of supply interruptions are longer than those specified in relevant regulatory frameworks, such as the security and quality of supply legislation in [39].

The SAIDI CDF (Fig. 8a) demonstrates that the combined SE-SMCS model (with ED_1 states) can adequately represent the original network, both in terms of mean reliability indices and distributions/variations of these indices. Similar deductions can be made from the SAIFI and ENS CDFs in Fig. 8b and c, respectively. The single-component Eqv-PC model with equivalent failure rate (λ_{eqv}) and repair time ($MTTR_{eqv}$) aggregates composite reliability information in terms of the whole modelled LV network and incorporates it into a single component model, which is still able to adequately approximate the CDFs of more detailed original and ED_1 network models (which both contain information from a higher number of PCs within the LV network).

The differences in curve shapes are a consequence of substituting the original network with a single equivalent PC, which uses different input characteristics (i.e., SE and SMCS) to match the probabilistic reliability performance of the detailed original network. The limitation from equivalent models in terms of probabilistic/CDF results is compensated by the significant reduction of simulation complexity. This is especially important when dealing with large MV networks, typically with thousands of PCs/LPs, which may require prohibitively long simulation times if detailed LV networks are included.

For example, the reduction in the MCS time for the TPM Eqv-PC is 80.2 % when compared to the TPM ED_1 reduced system, and 99.98 % when compared to the original detailed TPM network (Table 3). The presented method allows for adequate quantification of the risk of longer customer interruption times and provides reasonably accurate mean values. These results are useful in evaluating the cost-benefit analysis of proposed investments to improve reliability at both planning and operational stages.

4.4. Comparison of results for reliability analysis at MV level with LV network equivalents: "Plug and Play" functionality

This subsection analyses the effectiveness of the presented SE-SMCS single-PC equivalent network model when connected ("plugged") into larger network models at higher MV levels. To validate the "plug and play" functionality of the proposed LV network equivalents, the analysis in this section considers a more complex MV network, which is a sample version of a realistic MV suburban network from [40], supplying a diverse range of 44 LV networks, each with 76 customers, for a total of 3,344 customers. In terms of modelling complexity, the original MV network would require modelling 520 PCs only in the MV part of the network, and 97 PCs within each supplied LV network, which would

Table 3
TPM single equivalent component parameters.

Eqv-PC	λ_{eqv} (failures/year)	$MTTR_{eqv}$ (hours)	Computational Time Saving (%)
TPM	1.555	0.588	99.98 %

Table 4
Reliability indices for the TPM Equiv. Component.

Network	SAIDI		SAIFI		ENS	
	hrs/c/y	Error	ints/c/y	Error	kWh/c/y	Error
Orig.	0.9508	-	0.0588	-	5.7648	-
ED_1	0.9739	2.4 %	0.0576	2.0 %	5.8437	1.4 %
Eqv-PC	0.9154	3.7 %	0.0577	1.9 %	5.8485	1.5 %

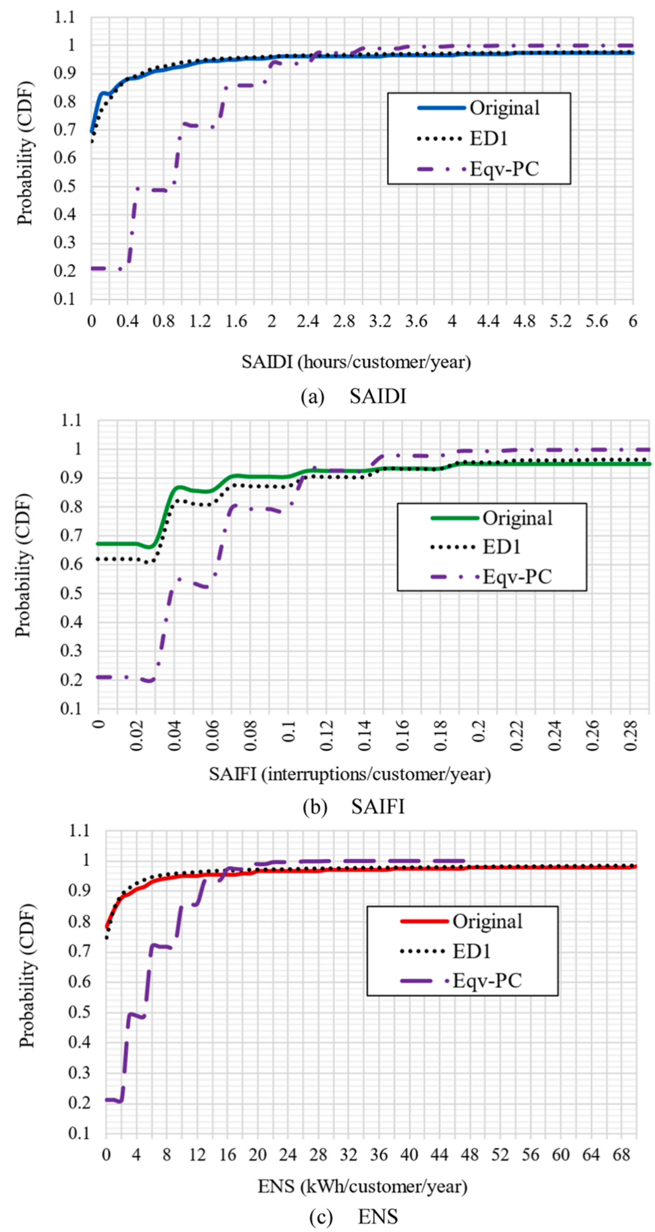

Fig. 8. The CDFs of the reliability indices obtained for the TPM networks.

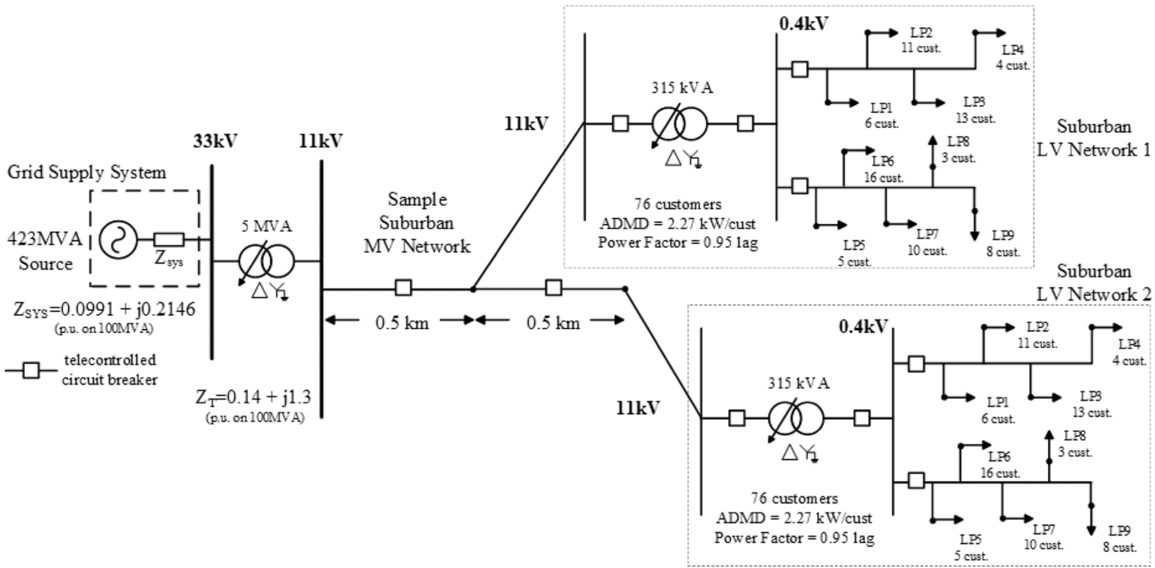

Fig. 9. Original (SLM) suburban MV network model with 213 PCs.

make a total of 4,788 PCs. Thus, for validation, a simplified MV network model supplying a reduced number of two LV networks (as illustrated in Fig. 9) is used for further demonstration of the scalability potential of LV network equivalents into larger and more complex MV systems. The sample MV/LV network used for analysis in this section contains a total of 213 PCs, supplying 152 customers through 18 load points. This test network is reasonably complex for a validation of this "plug and play" functionality, with respect to the SMCS model and the available computational resources required to model the full network in PSS/E software [40]. Each suburban LV network in Fig. 9 can be equivalented and represented by a single equivalent component (as previously demonstrated), resulting in a reduced suburban MV network with only 21 PCs and 2 LPs, supplying a total number of 152 customers.

The presented analysis uses the SLM of the LV network (not TPM), as this allows for reducing complexity and performing the analysis with a lower number of PCs and LPs, as compared to the use of the TPM. Another reason is to allow for a direct (and fair) comparison with an alternative existing method (AEM) used for LV network aggregation for unreliability cost assessment and in combination with other reliability equivalenting methodologies [7–10], which is also based on SLM. In the AEM, the equivalent fault rate is calculated as a sum of all PC fault rates, while the equivalent repair time is obtained as the average of the mean repair times of all PCs within the aggregated LV network. However, based on the results in Fig. 6, where the TPM of the LV network provides a better estimation of the reliability performance, it is expected that the use of the TPM would provide more accurate results for the evaluated reliability of the overall MV/LV system. The main aim of the presented analysis is to evaluate the impact of two different aggregate/equivalent representations of the LV network on the overall system reliability performance, i.e., MV plus LV networks (MV+LV), where a detailed whole-system model is used as the reference case.

After analysis, Table 5 compares the equivalent failure rate (λ_{eqv}) and repair time ($MTTR_{eqv}$) values obtained for the presented Eqv-PC LV network model (obtained using the combined SE-SMCS equivalenting approach), with the values obtained using the AEM equivalenting technique. However, while this might be a straightforward calculation for LV networks only, since the reduced MV/LV network after equivalenting the 2 LV networks (i.e., supplying only 2 LPs) represents the original 18 LPs (within the 2 LV networks), both SE-SMCS and AEM approaches require to ascertain the specific impact (i.e., "reliability weight") from those faults occurring on the MV part of the system on each downstream LV network. This impact analysis is based on the number of LPs served per each aggregate LV SLM network, which is 9 LPs.

Accordingly, Fig. 10 compares the average values of the three reliability indices calculated for the original detailed MV network (213 PCs) and two other reduced network models (each having 21 PCs): one using the presented SE-SMCS method for obtaining single-component representation, and another using the AEM method. Taking the 'original' network values as a reference (without the 2 LV networks aggregation), the results in Fig. 10 demonstrate that the SE-SMCS approach provides a more accurate evaluation of the original detailed MV+LV network reliability than the AEM, which provides a highly over/under-estimation of the different reliability indices under analysis. The proposed SE-SMCS method provides a more accurate approximation, where the lowest error is reported in the SAIFI index (5.5 %) due to the direct correlation of the LV SAIFI and the equivalent failure rate λ_{eqv}, as given by (8).

Similarly, the ENS value has a lower error (6.8 %) than SAIDI (13.8 %), due to the use of the LV ENS to calculate the equivalent unavailability according to (9). However, the standard error (i.e., black error bars in Fig. 10) shows that the proposed SE-SMCS approach is reasonably accurate, and it allows for a correct representation of the aggregated LV networks. Effectively, this proves accuracy of the SE-SMCS method and justifies the "plug and play" functionality of the proposed LV network equivalents, which will enable the reliability analysis of larger MV networks without increasing the complexity and, most importantly, without compromising the accuracy of the key indices obtained with the presented methodology for reliability network equivalents.

Fig. 11 compares probability and cumulative distribution functions (PDFs/CDFs) of ENS for the three scenarios. In these results, the tail of the PDF for ENS in the original network is indistinguishable for that of the reduced networks.

However, there is more variation between the distributions when looking at the ENS around the expected mean for each network i.e., original and the reduced networks (SE-SMCS and AEM). This is

Table 5
Comparison of two single-component models.

Eqv-PC	λ_{eqv} (failures/year)	$MTTR_{eqv}$ (hours)
SE-SMCS	1.052	1.217
AEM	0.472	6.235

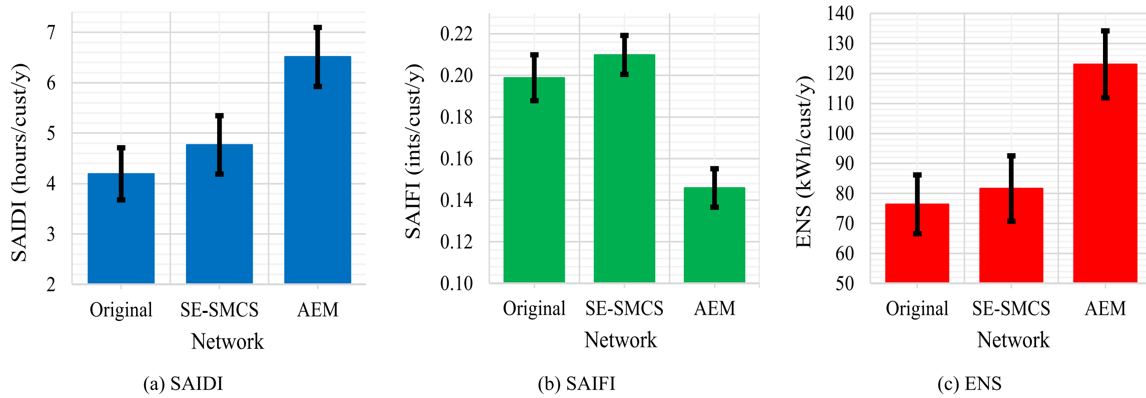

Fig. 10. Comparison of reliability indices for the original and two reduced network models for the generic suburban MV network in Fig. 9.

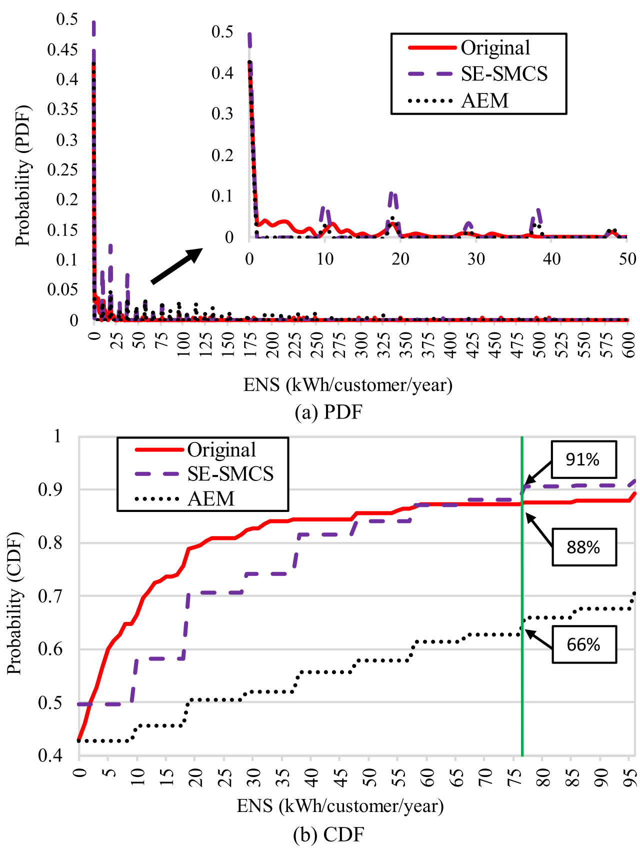

Fig. 11. Comparison of distribution functions of ENS for the MV network.

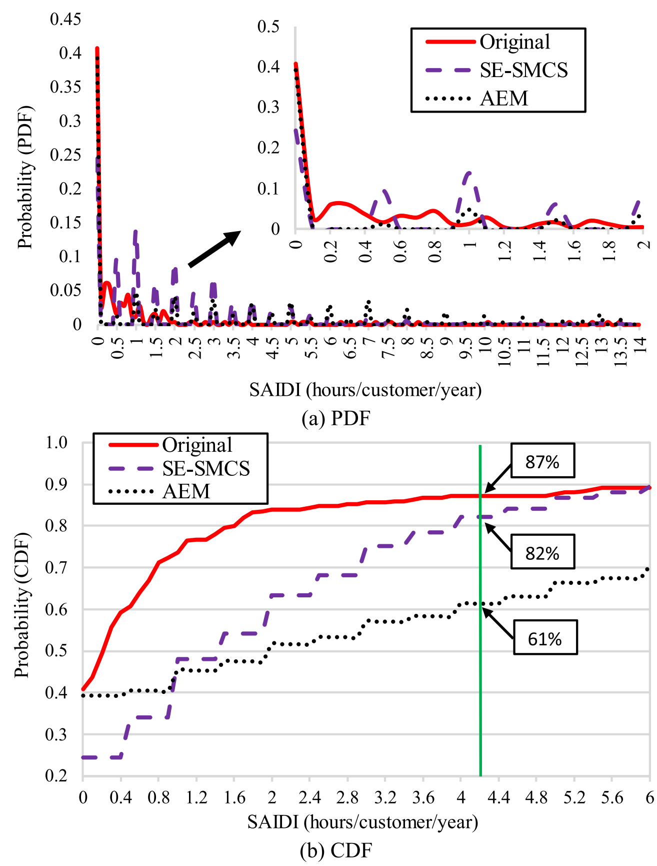

Fig. 12. Comparison of distribution functions of SAIDI for the MV network.

illustrated in Fig. 11b which examines the CDFs. The average ENS of the original network is 77 kWh/customer/year (green line) and Fig. 11b shows that there is an 88 % probability of sampling the mean ENS or less. Using the SE-SMCS reliability equivalents method, this probability is 91 %, which is significantly more accurate than the 66 % probability which is obtained using the conventional AEM. This further affirms that the proposed SE-SMCS method provides good matches for the PDF/CDF of ENS of the 'original' non-aggregated MV network.

Furthermore, Fig. 12a shows the PDFs of SAIDI for each network model, while Fig. 12b shows the CDFs. It can be seen that the probability of sampling mean SAIDI values of 4.2 hours/customer/year (green line) is 87 %, while SE-SMCS

provides around 82 % probability, which is again more accurate than the 61 % probability obtained using the AEM. Finally, Fig. 13a shows the PDFs and Fig. 13b shows the CDFs for SAIFI. The probability of sampling the mean SAIFI of 0.2 interruptions/customer/year (green line) is 74 % for the original network, while the SE-SMCS method provides a probability of 79 %. Again, this is much more accurate than the 90 % probability obtained with the AEM.

Based on the presented results, the reliability dependency between MV and LV networks is accurately modelled by the proposed approach. This is important when performing reliability analysis of MV networks with high failure rates of some LV components, for example a couple of long and highly exposed overhead lines supplying a low number of more remote customers. As previously illustrated, the AEM-based equivalent failure rate will simply provide an additive and therefore possibly misleading value of poor reliability, as it accounts neither for the location of the network components, nor for the actual impact of their failures. On the contrary, the proposed SE-SMCS method overcomes these

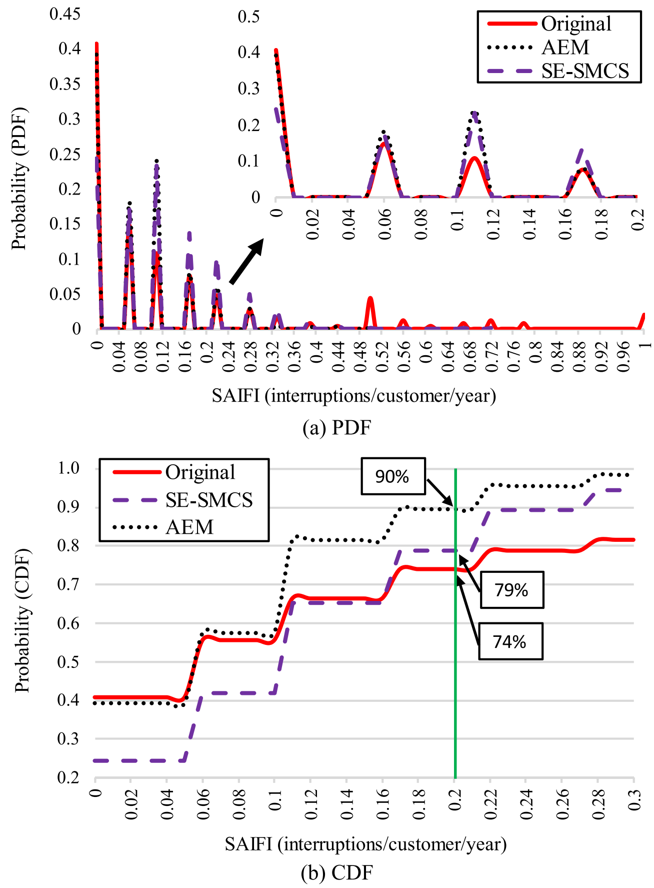

Fig. 13. Comparison of distribution functions of SAIFI for the MV network.

issues and ensures that the single-component equivalent network model is a more accurate reliability representation of the modelled LV network. Before performing the equivalenting, it accounts for all PC failure rates considering their spatial distribution/location in the network, as well as the impact of their failures on network reliability in terms of customer interruptions and energy not supplied.

It should be noted that the proposed methodology can be very useful in planning studies related to, e.g., the expansion of existing MV networks, where reliability equivalents of LV networks can be obtained independently and then simply "plugged" into the MV network model for a fast, efficient, and accurate evaluation. Finally, due to reduced computation times, the SE-SMCS method allows for a comprehensive evaluation of available options to optimise network planning.

5. Conclusions

This paper presents a novel methodology for formulating improved reliability equivalents of LV networks, which considers differences in spatial distribution of demands, types of network components and applied protection systems. The paper first compares the impact of using single-line and three-phase models for assessing reliability performance of LV networks. Then, a combined state enumeration (SE) and sequential Monte Carlo simulation (SMCS) method is introduced, where SE reduces the network to its most probable system states, while state transition sampling (STS)-based SMCS provides much faster and reasonably accurate reliability assessment. Furthermore, the presented SE-SMCS method allows for further network reduction into the simplest possible form: a single-component network equivalent, which has the same unavailability as the original network. The equivalent models allow to make further trade-offs between reducing calculation times and minimising the loss of accuracy, in terms of both mean values and probability distributions of the considered reliability indices. The benefits of using single-component reliability equivalents for the analysis of larger (MV) networks are illustrated and compared with a commonly used method.

Further work will integrate time-varying demand profiles, which are required to formulate accurate reliability equivalents of networks with distributed renewable energy resources, where coincidental generation and load patterns are required input data to correctly evaluate PC failure rates. It will also derive intermediate simplified reliability equivalents, representing distinctive smaller parts of the equivalented network (e.g., substation and main feeders), to improve accuracy of the single-component reliability equivalent without significant increase of the complexity. In addition, a more accurate per-phase disaggregation of historical fault rates for use in full three-phase network models (TPMs) will account for differently affected phases and allow for improved assessment of reliability performance of the "worst supplied customers", which is particularly relevant in terms of single-phase connected residential customers.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Further reading

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