

Agroecology and biodiversity: A benchmark dynamic model

Emmanuelle Augeraud-Véron ^{*} Raouf Boucekkine [†] Rodolphe Desbordes [‡]

Abstract

Conventional agriculture not only neglects but also harms the ecosystem services provided by biodiversity, inducing a negative feedback loop. In a theoretical inspired by agroforestry ('agriculture with trees'), a common agroecological practice in developing countries, we highlight how the choice between expanding agricultural land or retaining forest land is shaped by the bi-directional relationship between agriculture and biodiversity as well as the utility derived from biodiversity consumption. The static case shows that a high stock of biodiversity may be deliberately maintained as long as the agroecological productivity effect is important enough. This result holds in the dynamic case. However, in the latter case, a large intertemporal discount rate can lead to total biodiversity loss along with the full collapse of the economy. Another key implication of our model, among other results, is that the effect of a shift of consumer preferences towards agricultural goods (instead of biodiversity goods) on the biodiversity stock is much more ambiguous in the dynamic case than in the static case, depending on the strength on the agroecological productivity effect. These results have profound implications for biodiversity conservation.

Keywords: agriculture, agroecology, agroforestry, biodiversity, dynamic, feedback loop, static, sustainability.

^{*}Bordeaux School of Economics

[†]*Corresponding author.* Rennes School of Business and Aix-Marseille School of Economics

[‡]Skema School of Economics

1 INTRODUCTION

Over the period 1960-2019, land use change has affected almost a third of global land area and has involved substantial expansion of agricultural land (Winkler et al., 2021; Potapov et al., 2022a,b). These land transformations have considerably harmed biodiversity. Newbold et al. (2015) estimate that, in 2005, species richness and abundance have globally declined by 11-14%, with much stronger local impacts (40-50%) for intensively exploited agricultural land characterised by monocultures, heavy use of synthetic fertilisers and pesticides, and mechanisation. This biodiversity loss implies both the neglect and degradation of key benefits provided by ecosystems to agriculture (Zhang et al., 2007), notably in terms of supporting services (e.g. soil structure and fertility) and regulating services (e.g. pollination and pest control). Negative feedback loops may thus emerge between conventional agricultural practices and ecosystem services, progressively harming agricultural output, especially if any negative effects cannot be compensated by external inputs (Foley et al., 2005; Ortiz et al., 2021).

In response to this trade-off between short-run gains and long-run losses as well as the large environmental and human costs (e.g. water pollution, freshwater scarcity, emerging infectious diseases, climate change) associated with food production, more sustainable farming approaches have been proposed (Tscharntke et al., 2012; Bommarco et al., 2013; Nair, 2014; Tittone et al., 2016; Garibaldi et al., 2017; Perfecto et al., 2019). This ‘biodiversity’-based agriculture seeks to harness, rather than substitute, the ecosystem services provided by biodiversity. It aims at optimally integrating the biological and ecological processes within the agroecosystem while minimising the use of external non-renewable inputs that cause environmental or human harm (Pretty, 2008; Kremen et al., 2012; Wezel et al., 2014; Duru et al., 2015). Among the various ESR (efficiency/substitution/redesign) agroecological practices, related to crop or landscape management, that may be adopted (Wezel et al., 2014), agroforestry (‘agriculture with trees’) is relatively widespread in tropical and developing countries. Defined as “the purposeful growing or deliberate retention of trees with crops and/or animals in interacting combinations for multiple products or benefits from the same management

unit” (Nair et al., 2021), agroforestry is estimated to involve 43% of all agricultural land (at least 10% tree cover) and may provide direct subsistence to about a billion people (Zomer et al., 2014, 2016). Several meta-analyses have highlighted that agroforestry, in comparison to a less diverse agrosystem and even to other crop diversification strategies, tend to increase associated biodiversity, raise soil quality and fertility, improve pest and diseases control as well as pollinator abundance, and generate yield improvements (Pumariño et al., 2015; Torralba et al., 2016; Dainese et al., 2019; Kuyah et al., 2019; Staton et al., 2019; Udawatta et al., 2019; Beillouin et al., 2021; Baier et al., 2023; Centeno-Alvarado et al., 2023).¹ These benefits have induced various authors and international organisations, such as the IPCC (The Intergovernmental Panel on Climate Change), to advocate greater use of agroforestry to meet a large number of key sustainable development goals, (Waldron et al., 2017; Noordwijk et al., 2018; Shukla et al., 2019).

Our paper contributes to the field of agroforestry, and more broadly agroecology, by theoretically highlighting the relatively ignored negative ‘feedback loop’ existing between agricultural expansion and ecosystem services. Indeed, in their review, Ortiz et al. (2021) consider that the bi-directional relationship between agricultural production and biodiversity is not sufficiently considered and understood.² We build up a benchmark non-spatial dynamic model incorporating this bi-directional relationship allowing to assess in depth the potential long-term implications of a well-defined feedback effect of biodiversity on the productivity of the agricultural sector in accordance with the empirical literature surveyed above. We call this effect, the *agroecological productivity effect*. Other potentially important ingredients are also incorporated.

The model runs as follows. An agricultural economy produces an agricultural good with a simple Cobb-Douglas technology with land as a principal input (labor is normalised to 1). No storage of this good is possible, so it’s consumed entirely at every period of time. The

¹The biodiversity improvements do not necessarily mean that the environment is not affected by agroforestry. For example, Chaudhary et al. (2016) find that agroforestry, while the least detrimental non-timber producing management regimes, still reduces on average species richness found in nearby natural forests by 32%.

²Perfecto et al. (2005) illustrate a potential trade-off between biodiversity and agricultural production in a very simplified static model, without any feedback.

key addition to this otherwise standard agricultural economy model is a feedback effect of the biodiversity stock on the productivity of the agricultural sector: the larger this stock, the more productive the agricultural land is (agroecological productivity effect or boost). Biodiversity dynamics are driven by a simple accumulation law of motion depending on the initial stock of biodiversity, the remaining forest (non-agricultural) land and harvesting of this biodiversity by the population. We identify harvesting with wild food consumption. The representative farmer maximises an intertemporal discounted flow of utility derived from consuming both the agricultural and wild good subject to biodiversity dynamics. It turns out by construction that the farmer has two independent controls, the land use variable (precisely, the size of the agricultural vs forest areas) and the consumption of wild food.

In order to have a first idea on the qualitative implications of the agroecological productivity effect, we study a preliminary static optimisation counterpart. We highlight the main effects at work in the general case (that is under general functional specifications for the feedback effect and the instantaneous utility). However, to derive analytical results, we specialise in Cobb-Douglas utility functions and linear feedback effects. We solve the corresponding optimisation problem and highlight the main qualitative properties of the optimal controls, notably through comparative statics with respect to three key parameters: the regeneration rate of biodiversity, the elasticity of instantaneous utility with respect to the agricultural good (which allows to capture the extent to which the farmer “prefers” the agricultural good with respect to the wild one), and a direct indicator of the agroecological productivity effect. We essentially enhance the role of the feedback effects in this static case. We then move to the optimal control problem and perform the typical mathematical treatment, establishing the sufficiency of the first-order Pontryagin conditions, the existence and uniqueness of the steady state and its local (saddlepoint) stability. We end our analysis with a careful analysis of the comparative statics resulting at the optimal steady state with respect to the three key parameters listed above.

Besides the methodological value-added deriving from the comparison between the static and dynamic frames, several highly interesting economic results have been identified. We mention two in this Introduction. First of all, we highlight the importance of the time

discounting rate in presence of the agroecological productivity effect. Remarkably enough, with our canonical specifications, we show that the stock of biodiversity converges to 0 when the discount rate becomes infinitely large. Due to the agroecological feedback effect, the whole economy goes to extinction as the vanishing stock of biodiversity also leads to the collapse of the agricultural sector in this case. Second, we establish several properties at the steady state of the dynamic model, which are quite interesting from the policy point of view. For example we prove that as the agricultural society preferences move away from wild food consumption (maybe following government campaigning), the stock of biodiversity is raised only if the agroecological productivity effect is strong enough, further leading to a virtuous circle. This implies that campaigning with a biodiversity conservation policy is doomed to failure.

The paper is organised as follows. Section 2 presents the model and the associated optimal control problem. Section 3 constructs a static counterpart and studies its qualitative properties at the optimum. Section 4 solves the optimal control problem and derives its qualitative properties at the steady state, with comparison to the static case counterpart, enhancing the interaction between the feedback effects inherent in the model and the intertemporal mechanisms. Section 6 concludes.

2 THE MODEL

We consider a one-sector agricultural economy where the agricultural good is produced according to the following general technology:

$$Y = F(A, T(B)L),$$

where A is the size of the agricultural land, L is the available labor force, and $T(B)$ is labor-saving technology which depends on the amount of biodiversity available. This specification fits several contexts: in the case of agroecology (see for example, Perfecto et al. (2019)), nature services increase the productivity of agriculture: in such a case, $T(\cdot)$ is a non-decreasing function of biodiversity.

Moreover, we normalise initial labor resources and forest land surface to 1. We do not

model space explicitly. Denoting by f the fraction of forest land preserved such that $A = 1 - f$, we assume a Cobb-Douglas agricultural production function:

$$Y = (T(B))^{1-\alpha} (1 - f)^\alpha, \quad (1)$$

with $0 < \alpha < 1$. The agricultural production is entirely consumed at any date, that is:

$$Y = C_A, \quad (2)$$

where C_A is consumption of the agricultural good or domestic food. The individual (or the economy) can also directly consume comestible biodiversity (animals or vegetation; ‘wild food’), denoted C_B , which may or may not lead to further deforestation. Both cases may be considered. In this simple model, we assume that consumption of wild food do not involve any significant cost in terms of deforestation or in terms of labor.³

Biodiversity dynamics We now come to the link between agricultural activity and the evolution of biodiversity over time. We set the following law of motion:

$$\dot{B} = R(f, B) B - C_B,$$

where $R(f, B)$ is the natural regeneration rate of biodiversity, which is here assumed to depend on the extent of forests and the existing stock of biodiversity. $B(0) > 0$ is given. In this paper, we shall choose the following specification:

$$\dot{B} = \beta B^{1-\theta} f^\theta - C_B, \quad (3)$$

³One could legitimately argue that hunting, for example, is time-consuming and diverts labor from agriculture. Adding labor allocation between agriculture and consumption of biodiversity will not alter the main correlations between land use, biodiversity and zoonoses we target in this paper.

with $0 < \theta < 1$. The latter equation can be rewritten in the more meaningful way:

$$\frac{\dot{B}}{B} = \beta \left(\frac{f}{B} \right)^\theta - \frac{C_B}{B}.$$

The optimal control problem The decision maker choose the trajectories of $\{f, C_A, C_B\}$ from $t = 0$ aims at maximising the following intertemporal utility function:

$$\int_0^\infty U(C_A, C_B) e^{-\delta t} dt \quad (4)$$

subject to equations subject to (1), (2) and (3), with $B(0)$ given. $\delta > 0$ is the time discounting rate. We shall consider standard utility functions, namely strictly increasing in each of the arguments, concave in (C_A, C_B) , and checking the Inada conditions for each argument of the function. Using equations (1)- (2), one can substitute for C_A in the objective function, leading to an optimal control problem with two controls $\{f, C_B\}$ and one state variable B . More precisely, the optimal control problem can be finally rewritten as:

$$\max_{f, C_B} \int_0^\infty U \left([T(B)]^{1-\alpha} (1-f)^\alpha, C_B \right) e^{-\delta t} dt$$

subject to

$$\dot{B} = \beta B^{1-\theta} f^\theta - C_B,$$

with $B(0) = B_0$ given and the usual positivity conditions (plus $f \leq 1$).

3 A STATIC COUNTERPART MODEL

3.1 The counterpart static optimisation problem

Consider the same problem within a single period, starting with a biodiversity level, B_0 normalised to 1 for simplicity. Then the biodiversity level within the period is given by (modulo constants inherent in the discretisation step, irrelevant for the qualitative properties of the optimal solutions if any) :

$$B = \beta f^\theta - C_B,$$

leading to following one-period optimisation problem:

$$\max_{f, c_B} U \left(\left[T \left\{ \beta f^\theta - C_B \right\} \right]^{1-\alpha} (1-f)^\alpha, C_B \right) \quad (5)$$

The static optimisation problem (5) is highly nontrivial for general functions $T(\cdot)$ and $U(\cdot, \cdot)$. The first-order necessary optimality conditions write as follows (with $U_i(\cdot, \cdot)$ the partial derivative of U with respect to its i th argument, $i = 1, 2$):

$$U_2(\cdot, \cdot) = (1 - \alpha)U_1(\cdot, \cdot)T'(\cdot) [T(\cdot)]^{-\alpha} (1 - f)^\alpha,$$

and

$$(1 - \alpha)\beta\theta U_1(\cdot, \cdot)T'^{-\alpha}(1 - f)^\alpha f^{\theta-1} = \alpha U_1(\cdot, \cdot)T^{1-\alpha}(1 - f)^{\alpha-1},$$

where U_1 and U_2 are evaluated at $\left(\left[T \left\{ \beta f^\theta - C_B \right\} \right]^{1-\alpha} (1-f)^\alpha, C_B \right)$ and T and T' are evaluated at $\beta f^\theta - C_B$. The first equation is the necessary optimality condition with respect to C_B : it equalises the marginal benefit from harvesting biodiversity (U_2) to the corresponding marginal cost (the right-hand side of the equation) reflecting the marginal loss in biodiversity impacting productivity in the agricultural sector, which in turn affects negatively consumption of the agricultural good. The second equation is the necessary optimality condition with respect to f : again it equalises the marginal benefit from increasing the forest surface through increasing biodiversity (thus productivity and ultimately the consumption of the agricultural good) to the corresponding marginal cost, which is simply the one decreasing the latter consumption as the resulting agricultural land shrinks. Both conditions make perfect sense as they accurately reflect the economic trade-offs at work in our model. The two equations are quite intricate algebraically in the general case, even after simplification of the second equation, we shall refer to these final formulations below:

$$U_2(\cdot, \cdot) = (1 - \alpha)U_1(\cdot, \cdot)T'(\cdot) [T(\cdot)]^{-\alpha} (1 - f)^\alpha, \quad (6)$$

and

$$\frac{T'(\cdot)}{T(\cdot)} = \frac{\alpha}{(1 - \alpha)\beta\theta} \frac{f^{1-\theta}}{1 - f}. \quad (7)$$

Equation (7) is interesting: it gives the growth rate of agricultural productivity as a highly nonlinear increasing function of forest land in the agroecological context we are studying.

Given the objectives of this paper, we shall concentrate on functional specifications which deliver closed-form solutions for the static problem and the dynamic extension (at the steady state). The analytical case specifications given below will therefore also serve for the latter in Section 4.

An analytically tractable case A full analytical solution to the static problem is obtained with the following specifications:

1. $U(x, y) = x^\gamma y^{1-\gamma}$, with $0 < \gamma < 1$,
2. $T(x) = x$.

Clearly the two specifications are highly “stylised”: the agricultural and non-agricultural goods may show in some practical cases a strong form of complementarity, and the relationship between productivity and the amount of biodiversity is probably nonlinear. In our model, the amount of biodiversity will remain bounded over optimal trajectories, so the postulated linearity of productivity is not so problematic. With these specifications, the problem (5) becomes

$$\max_{f, C_B} \left[\beta f^\theta - C_B \right]^{\gamma(1-\alpha)} (1-f)^{\gamma\alpha} C_B^{1-\gamma} \quad (8)$$

Using the first-order optimality conditions (6)-(7) with the specifications above yields:

$$C_B = \frac{(1-\gamma)}{\gamma(1-\alpha)} \left(\beta f^\theta - C_B \right), \quad (9)$$

and

$$\frac{1}{\beta f^\theta - C_B} = \frac{\alpha}{(1-\alpha)\beta\theta} \frac{f^{1-\theta}}{1-f}. \quad (10)$$

The following proposition summarises the main results.

Proposition 1 *The maximisation problem (8) admits a unique solution (f^*, C_B^*) such that:*

- $f^* = \frac{(1-\alpha\gamma)\theta}{\gamma\alpha + \theta(1-\alpha\gamma)}$ (so $f^* < 1$),

- $C_B^* = \frac{(1-\gamma)\beta}{1-\alpha\gamma} (f^*)^\theta$
- Consequently, one gets: $B^* = \beta \frac{\gamma(1-\alpha)}{1-\alpha\gamma} (f^*)^\theta$, and $C_A^* = (B^*)^{1-\alpha} (1-f^*)^\alpha$.

Proof. To prove this latter point, first order derivatives are given as follows.

$$\begin{aligned}\frac{\partial U}{\partial C_B} &= \left(-\gamma(1-\alpha) C_B + (1-\gamma) [\beta f^\theta - C_B] \right) (1-f)^\alpha [\beta f^\theta - C_B]^{\gamma(1-\alpha)-1} C_B^{-\gamma} \\ \frac{\partial U}{\partial f} &= \left((1-\alpha)\beta\theta f^{\theta-1} (1-f) - \alpha [\beta f^\theta - C_B] \right) \gamma [\beta f^\theta - C_B]^{\gamma(1-\alpha)-1} (1-f)^{\gamma\alpha-1} C_B^{1-\gamma}\end{aligned}$$

Consistently the optimum yields:

$$\begin{aligned}C_B &= \frac{(1-\gamma)}{\gamma(1-\alpha)} [\beta f^\theta - C_B] \\ \frac{1}{\beta f^\theta - C_B} &= \frac{\alpha f^{1-\theta}}{(1-\alpha)\beta\theta (1-f)}\end{aligned}$$

$$\begin{aligned}C_B &= \frac{1-\gamma}{1-\alpha\gamma} \beta f^\theta \\ \frac{(1-\alpha\gamma)\theta}{\gamma\alpha + \theta(1-\alpha\gamma)} &= f\end{aligned}$$

Let us denote $\pi_c = (1-f^*)^\alpha [\beta f^{*\theta} - C_B^*]^{\gamma(1-\alpha)-1} C_B^{*-\gamma}$, $\pi_f = \gamma [\beta f^{*\theta} - C_B^*]^{\gamma(1-\alpha)-1} (1-f^*)^{\gamma\alpha-1} C_B^{*1-\gamma}$. The evaluation of the Hessian at steady state is given as follows

$$\begin{bmatrix} (\alpha\gamma - 1) \pi_c & -\beta\theta \left(\frac{-\theta(\alpha\gamma-1)}{\alpha\gamma-\theta(\alpha\gamma-1)} \right)^{(\theta-1)} (\gamma-1) \pi_c \\ \alpha\pi_f & -\beta\theta \left(\frac{-\theta(\alpha\gamma-1)}{\alpha\gamma-\theta(\alpha\gamma-1)} \right)^{\theta-2} \frac{\alpha^2\gamma\theta - \alpha^2\gamma - 2\alpha\gamma\theta + \alpha\gamma + \theta}{\alpha\gamma - \theta(\alpha\gamma-1)} \pi_f \end{bmatrix}$$

$$\begin{aligned}\Delta &= \left[(\alpha\gamma - 1) \left(-\beta\theta \left(\frac{-\theta(\alpha\gamma-1)}{\alpha\gamma-\theta(\alpha\gamma-1)} \right)^{\theta-2} \frac{\alpha^2\gamma\theta - \alpha^2\gamma - 2\alpha\gamma\theta + \alpha\gamma + \theta}{\alpha\gamma - \theta(\alpha\gamma-1)} \right) + \beta\theta \left(\frac{-\theta(\alpha\gamma-1)}{\alpha\gamma-\theta(\alpha\gamma-1)} \right)^{(\theta-1)} (\gamma-1) \alpha \right] (\pi_c \pi_f) \\ &= [(\alpha\gamma - 1) (- (\alpha^2\gamma\theta - \alpha^2\gamma - 2\alpha\gamma\theta + \alpha\gamma + \theta)) - \theta(\alpha\gamma - 1)(\gamma - 1)\alpha] \left(\frac{-\theta(\alpha\gamma-1)}{\alpha\gamma-\theta(\alpha\gamma-1)} \right)^{\theta-2} \frac{\beta\theta}{\alpha\gamma - \theta(\alpha\gamma-1)} (\pi_c \pi_f)^2\end{aligned}$$

Δ has the same sign as d , given by:

$$\begin{aligned} d &= [(\alpha\gamma - 1) (-(\alpha^2\gamma\theta - \alpha^2\gamma - 2\alpha\gamma\theta + \alpha\gamma + \theta)) + (-\theta(\alpha\gamma - 1))(\gamma - 1)\alpha] \\ &= (1 - \alpha)(1 - \alpha\gamma)(\theta(1 - \alpha\gamma) + \alpha\gamma) \end{aligned}$$

Thus $\Delta > 0$. Moreover, $(\alpha\gamma - 1) < 0$, thus the optimum is a maximum. ■

We study the economic implications of Proposition 1 just below.

3.2 Economic properties

To explore the economic properties of the optimal static solution, we will concentrate on three key parameters: the regeneration rate of biodiversity, β ; the elasticity of instantaneous utility with respect to the agricultural good, γ , and a direct indicator of the agroecological productivity effect, $1 - \alpha$. Indeed, $1 - \alpha$ is the elasticity of the agricultural output with respect to biodiversity. The three parameters above “quantify” the three important ingredients of the paper: the biodiversity growth rate, the agroecological engine of agricultural production, and the arbitrage between the agricultural good and wild food (inherent in the preferences of the consumers).

The proposition below gives the comparative statics of the optimal solution for three key variables: the agricultural land, the biodiversity stock and the amount of biodiversity harvested (to be consumed). The three variables show up in the biodiversity equation (static and dynamic), which is the central one for the purpose of our analysis as outlined above. We get the following economic picture.

Proposition 2 *The following comparative statics hold:*

- $\frac{\partial f^*}{\partial \beta} = 0$, $\frac{\partial f^*}{\partial \alpha} < 0$, and $\frac{\partial f^*}{\partial \gamma} < 0$
- $\frac{\partial C_B^*}{\partial \beta} > 0$, $\frac{\partial C_B^*}{\partial \gamma} < 0$, and $\frac{\partial C_B^*}{\partial \alpha} > 0$.
- It follows that $\frac{\partial B^*}{\partial \beta} > 0$, $\frac{\partial B^*}{\partial \alpha} < 0$ and $\frac{\partial B^*}{\partial \gamma} > 0$

The computations deliver indeed the following results for the partial derivatives needed

and the resulting signs. For forest land surface, one gets the following direct results:

$$\begin{aligned}\frac{\partial f^*}{\partial \alpha} &= -\frac{\gamma\theta}{(\gamma\alpha + \theta(1 - \alpha\gamma))^2} < 0, \\ \frac{\partial f^*}{\partial \gamma} &= -\frac{\alpha\theta}{(\gamma\alpha + \theta(1 - \alpha\gamma))^2} < 0\end{aligned}$$

Things are more involved for the stock of biodiversity, in particular for parameter γ :

$$\begin{aligned}\frac{\partial B^*}{B^*\partial \alpha} &= -\frac{(1 - \gamma)}{(1 - \alpha)(1 - \alpha\gamma)} - \frac{\gamma}{(1 - \alpha\gamma)(\gamma\alpha + \theta(1 - \alpha\gamma))} < 0, \\ \frac{\partial B^*}{B^*\partial \beta} &= \frac{1}{\beta} > 0, \\ \frac{\partial B^*}{B^*\partial \gamma} &= \frac{1}{\gamma(1 - \alpha\gamma)} - \frac{\alpha\theta}{(1 - \alpha\gamma)(\gamma\alpha + \theta(1 - \alpha\gamma))} \\ &= (\gamma\alpha + \theta(1 - \alpha\gamma) - \gamma\alpha\theta) \frac{1}{\gamma(1 - \alpha\gamma)(\gamma\alpha + \theta(1 - \alpha\gamma))} \\ &= (\gamma\alpha + \theta(1 - 2\alpha\gamma)) \frac{1}{\gamma(1 - \alpha\gamma)(\gamma\alpha + \theta(1 - \alpha\gamma))},\end{aligned}$$

However, it can be seen that $\gamma\alpha + \theta(1 - 2\alpha\gamma) > 0$. Thus, $\frac{\partial B^*}{B^*\partial \gamma} > 0$.

The comparative statics of wild goods consumption is more straightforward:

$$\begin{aligned}\frac{\partial C^*}{C^*\partial \alpha} &= \frac{\gamma^2\alpha(1 - \theta)}{(1 - \alpha\gamma)(\gamma\alpha + \theta(1 - \alpha\gamma))} > 0 \\ \frac{\partial C^*}{C^*\partial \beta} &= \frac{1}{\beta} > 0, \\ \frac{\partial C^*}{C^*\partial \gamma} &= \frac{1}{(1 - \alpha\gamma)} \left(\frac{-(1 - \alpha)}{(1 - \gamma)} - \frac{\alpha}{(\gamma\alpha + \theta(1 - \alpha\gamma))} \right) < 0\end{aligned}$$

The Proposition yields a number of nontrivial results given the feedback loops involved in our model. Let's start with optimal land use, namely with the forest land surface, f^* . A bigger regeneration rate β raises the level of biodiversity as a direct effect but it does also induce an original second-round effect. Indeed through the agroecological productivity upward shift, it increases the agricultural output for given agricultural land, $1 - f^*$. This may either lead to increase agricultural land (decreasing f^*) to take advantage of this higher productivity or to lower it (if the productivity increment is large enough) to raise the forest

land, which would generate more biodiversity without penalising consumption of the agricultural good. Subsequent posterior round effects could occur according to the same opposite mechanisms. A third possibility is that optimal agricultural land remains overall unchanged by the shock on the regeneration rate β as the direct and indirect effects may cancel out. Due to our functional specifications (linearity of the agroecological productivity effect and Cobb-Douglas utility function mainly), this is what happens here. Things are different when either α or γ goes up. An increase in γ increases the weight of the agricultural good in the instantaneous utility function leading to more priority to the consumption of this good, which tends to increase agricultural land (or decrease f^*). This is the principal mechanism generating the decline of forest surface as the elasticity parameter γ rises, it eventually dominates indirect effects originating in the subsequent biodiversity decline and the posterior weakening of the agroecological productivity effect. A similar picture occurs with α : if this parameter is augmented, the strength of the agroecological productivity boost drops, which reduces the incentives to preserve and/or increase biodiversity, leading to the erosion of forest land. Again in this case, feedback loop effects arise that mitigate the latter effect but they end up dominated.

As to biodiversity consumption, C_B , the comparative statics obtained, while quite intuitive, are indeed far from trivial except for β . A larger regeneration rate of biodiversity, β , leaves room for more harvesting as it makes biodiversity more abundant. This seems natural and indeed it is so in our analytical frame: by Proposition 1, one gets : $C_B^* = \frac{(1-\gamma)\beta}{1-\alpha\gamma} (f^*)^\theta$, the comparative static derives automatically from the fact that the optimal forest land f^* is independent of β , which is, as argued above, due to our chosen functional specifications. This need not be the case in general if the optimal forest land surface does depend on β . A larger β indeed also favors the agroecological productivity effect since this increases directly the biodiversity amount, and may further lead to less land for agricultural production if the productivity boost is big enough. Therefore, everything can happen in the general case depending on the function specifications adopted. Equally intuitive is the comparative static with respect to γ : the larger the elasticity of instantaneous utility with respect to this parameter, more weight is given to the consumption of the agricultural good, which eventually

leads to a drop in biodiversity harvesting. More intricate, as the elasticity of the agricultural good production with respect to agricultural land, α , goes up, more land will be devoted to agriculture (and more agricultural goods will be consumed). This in turn leads to less biodiversity and subsequent wild good consumption drop. However, as α increases, the fraction of biodiversity produced by Nature and going to optimal wild food consumption, that is $\frac{1-\gamma}{1-\alpha\gamma}$,⁴ goes up. The latter effect dominates the former, and wild food consumption goes actually up when α rises, which is far from trivial. Again as for comparative statics above, these are the dominant effects which reflect the qualitative results obtained in Proposition 2. Feedback effects may mitigate the latter but end up dominated.

Let us finish this section with the comparative statics of the biodiversity stock, which is the central variable of our model being the unique dynamic variable, that's the one which drives all the dynamics of the model (given decisions/controls) in the canonical intertemporal problem. Recall that:

$$B^* = \beta (f^*)^\theta - C_B^* = \beta \frac{\gamma(1-\alpha)}{1-\alpha\gamma} (f^*)^\theta.$$

An increase in β naturally increases the stock of biodiversity despite it also raises wild good consumption. This property is very likely to be robust to changes in the functional specifications of the utility function and the agroecological productivity effect. Another easy property can be found using the results of some of the comparative statics characterised above: since an increase in α decreases the surface of the forest land and raises wild food consumption, this leads to a non-ambiguous fall in the biodiversity stock. Inversely, if the strength of the agroecological productivity boost is raised as $1-\alpha$ increases, the stock of biodiversity increases unambiguously pushed by the two latter mechanisms (operated in opposite direction). The comparative statics of B^* with respect to γ is a bit more involved. Indeed, a larger elasticity of utility to the agricultural good decreases both the optimal forest land and consumption of the wild goods, resulting in an ambiguous effect on B^* . Proposition 2 states that **whatever the parameter values** (θ , α , β and γ), the magnitude of the decrease in wild goods consumption is larger than the reduction in biodiversity originating in optimal forest land shrinking. We

⁴This results from $C_B^* = \frac{(1-\gamma)\beta}{1-\alpha\gamma} (f^*)^\theta$ in Proposition 1.

shall see that such a property cannot hold in the dynamic counterpart of the model, at least not in the long-term.

4 EXPLORING THE DYNAMICS

We now come back to our optimal control problem, reformulated with the functional specification choices made in the static problem in the previous section:

$$\max_{f, c_B} \int_0^{\infty} B^{(1-\alpha)\gamma} (1-f)^{\alpha\gamma} C_B^{1-\gamma} e^{-\delta t} dt \quad (11)$$

subject to

$$\dot{B} = \beta B^{1-\theta} f^\theta - C_B,$$

with $B(0) = B_0 > 0$ given.

4.1 Solving the optimal control problem

The current value Hamiltonian writes as follows

$$\tilde{H} = B^{(1-\alpha)\gamma} (1-f)^{\alpha\gamma} C_B^{1-\gamma} + \mu \left(\beta B^{1-\theta} f^\theta - C_B \right),$$

where μ is the current-valued shadow price of the biodiversity stock, B . The corresponding first-order Pontryagin conditions are (with $U \equiv B^{(1-\alpha)\gamma} (1-f)^{\alpha\gamma} C_B^{1-\gamma}$ is the instantaneous utility):

$$(1-\gamma) \frac{U}{C_B} = \mu, \quad (12)$$

$$\alpha\gamma \frac{U}{1-f} = \mu\theta\beta B^{1-\theta} f^{\theta-1} \quad (13)$$

$$(1-\alpha)\gamma \frac{U}{B} + \mu(1-\theta)\beta B^{-\theta} f^\theta = -\dot{\mu} + \delta\mu, \quad (14)$$

with the transversality condition: $\lim_{t \rightarrow +\infty} e^{-\rho t} \mu B = 0$. Equation (12) gives the necessary

optimality condition for C_B : it equalizes the marginal utility from one additional unit of biodiversity harvesting to biodiversity shadow price, μ . Equation (13) is the necessary optimality condition for f : the marginal disutility from a one unit additional unit of forest land (left-hand side of the equation) should be equal to the marginal benefit in terms of biodiversity increment evaluated at the biodiversity shadow price. Equation (14) is the co-state equation, which delivers the time variation of the (current-valued) shadow price of biodiversity resulting from time discounting (term $\delta\mu$ in the right-hand side), the marginal utility gained from one additional unit of biodiversity (through the agorecological productivity effect leading to more consumption of the agricultural good) and the benefit from increasing the biodiversity stock evaluated at the corresponding shadow price. The transversality condition is standard, it stated that the marginal discounted value of biodiversity evaluated at the current-valued shadow price should be exhausted asymptotically. The second-order (sufficiency) optimality conditions, being quite intricate, are proved and reported in the Appendix

Before, we study the existence of stationary states for the system of necessary optimality conditions uncovered above. We first define a stationary state accurately.

The 4-uple $\{f^s, C_B^s, B^s, \mu^s\} \in \mathbb{R}_+^4$ with $f^s < 1$ is stationary state of the optimal control problem (11). It solves the system of equations (3), (12), (13) and (14), plus the transversality condition, under the stationarity conditions: $\dot{B} = \dot{\mu} = 0$.

The corresponding system of equations solving for stationary states is therefore given by equations (12)-(13) plus the stationarised state and co-state equations:

$$C_B^s = \beta (B^s)^{1-\theta} (f^s)^\theta, \quad (15)$$

$$(1 - \alpha)\gamma \frac{U^s}{B^s} + \mu^s(1 - \theta)\beta (B^s)^{-\theta} (f^s)^\theta = \delta\mu^s, \quad (16)$$

where U^s stands for instantaneous utility evaluated at the steady state. Notice that if such a state exists, the transversality conditions is automatically checked. The next proposition proves existence and uniqueness of such a stationary state and explores local stability .

Proposition 3 *The 4-uple $\{f^s, C_B^s, B^s, \mu^s\}$ check the following properties:*

- *It exists and is unique. It's given by:*

$$f^s = \frac{(1-\gamma)\theta}{\alpha\gamma + (1-\gamma)\theta},$$

$$B^s = \left[\frac{\beta(1-\theta + \gamma(\theta - \alpha))}{\delta(1-\gamma)} \right]^{\frac{1}{\theta}} f^s,$$

$$C_B^s = \beta (B^s)^{1-\theta} (f^s)^\theta,$$

and

$$\mu^s = (1-\alpha) \frac{U^s}{C_B^s}.$$

- *The stationary state is saddlepoint stable.*

We skip the algebraic details of the computation of the steady state, it's easy. A remarkable property of the dynamic problem at steady state with respect to the static counterpart is the role of the time discounting parameter (that's the so-called impatience rate), δ , which of course does not show up in static settings. Remarkably enough, with our canonical specifications, **the stock of biodiversity converges to 0 when δ becomes infinitely large. Indeed, the whole economy goes to extinction as the vanishing stock of biodiversity also leads to the collapse of the agricultural sector in our case.** That's what also makes this model useful as a benchmark frame.

We develop now the analysis of the stationary state local stability before moving to the economic exploration. To study the local stability, we need to write the dynamics. Using that μ appears in both Equations 12 and 13,

$$C_B = \chi \beta B^{1-\theta} (1-f) f^{\theta-1}, \quad (17)$$

where $\chi = \frac{(1-\gamma)\theta}{\alpha\gamma}$. As a consequence, the state dynamics is given as follows.

$$\dot{B} = \beta B^{1-\theta} f^\theta \left(1 - \chi \frac{1-f}{f} \right). \quad (18)$$

Moreover, using Equation 12 in Equation 14 yields

$$(1 - \alpha)\gamma \frac{\chi \beta B^{-\theta} (1 - f) f^{\theta-1}}{(1 - \gamma)} + (1 - \theta)\beta B^{-\theta} f^\theta = -\frac{\dot{\mu}}{\mu} + \delta,$$

which can be written as

$$\frac{\dot{\mu}}{\mu} = \delta - \beta B^{-\theta} f^\theta \left((1 - \alpha) \frac{\theta}{\alpha} \frac{1 - f}{f} + (1 - \theta) \right).$$

As, according to Equation 12, it can be obtained

$$(1 - \gamma) (\chi \beta)^{-\gamma} B^{(\theta-\alpha)\gamma} (1 - f)^{(\alpha-1)\gamma} f^{(1-\theta)\gamma} = \mu.$$

As a consequence,

$$(\theta - \alpha) \gamma \frac{\dot{B}}{B} - (\alpha - 1) \gamma \frac{\dot{f}}{1 - f} + (1 - \theta) \gamma \frac{\dot{f}}{f} = \frac{\dot{\mu}}{\mu},$$

from which we can write the dynamics of f .

$$\left(-(\alpha - 1) \frac{1}{1 - f} + (1 - \theta) \frac{1}{f} \right) \gamma \dot{f} = \delta - \beta B^{-\theta} f^\theta \left(\frac{\theta}{\alpha} ((1 - \alpha) - (\theta - \alpha) (1 - \gamma)) \frac{1 - f}{f} + (1 - \theta) + (\theta - \alpha) \gamma \right)$$

$$\left(\frac{1 - \alpha}{1 - f} + \frac{1 - \theta}{f} \right) \gamma \dot{f} = \delta - \beta B^{-\theta} f^\theta ((1 - \theta) + (\theta - \alpha) \gamma) \left(\frac{\theta}{\alpha} \frac{1 - f}{f} + 1 \right) \quad (19)$$

It can be noticed that $\frac{1-\theta}{f} + \frac{1-\alpha}{1-f} \neq 0$, as $f \neq \frac{\theta-1}{\theta-\alpha}$.

The Jacobian Matrix of system 18 -19 at steady state is given as follows.

$$J = \begin{bmatrix} 0 & \frac{\beta \eta^{\frac{1-\theta}{\theta}} \chi}{f^s} \\ \frac{\beta \eta^{-(1/\theta)} \delta f^s}{\alpha \gamma \left(\frac{1-\theta}{f} - \frac{\alpha-1}{1-f} \right)} & \frac{\delta ((\alpha-\theta)\gamma + \theta(1-\alpha))}{\alpha f^s \gamma \left(\frac{1-\theta}{f} - \frac{\alpha-1}{1-f} \right)} \end{bmatrix}$$

where $\frac{\beta(1-\theta+\gamma(\theta-\alpha))}{\delta(1-\gamma)} = \eta$. The eigenvalues are the solution of $\Delta(\lambda) = 0$, where the characteristic polynomial is given by:

$$\Delta(\lambda) = \lambda^2 - \text{tr}(J)\lambda + \det(J),$$

where $\det(J) > 0$ and $\text{tr}(J) > 0$ are respectively determinant and trace of J . As a consequence, eigenvalues are real, with opposite sign. So the stationary state is saddle point stable. After validating the optimality (in Appendix) and (local) stability of the steady state, we explore here below its economic outcomes and compare them with those of the static counterpart.

4.2 Economic properties

Before getting to the comparison task, we start with the comparative statics of the stationary state. As we shall see, they are more involved than in the static counterpart.

Proposition 4 *The following comparative statics hold:*

1. $\frac{\partial f^*}{\partial \beta} = 0$, $\frac{\partial f^*}{\partial \alpha} < 0$, $\frac{\partial f^*}{\partial \gamma} < 0$, and $\frac{\partial f^*}{\partial \delta} = 0$.
2. It follows that $\frac{\partial B^*}{\partial \beta} > 0$, $\frac{\partial B^*}{\partial \alpha} < 0$ and $\frac{\partial B^*}{\partial \delta} < 0$.
3. The sign of $\frac{\partial B^*}{\partial \gamma}$ is ambiguous, it depends on the parameters' values. *Ceteris paribus*, B^* increases with γ if the strength of the agroecological productivity effect is strong enough (i.e α is small enough), and it's a decreasing function of γ when the latter productivity effect is weak enough (i.e α is large enough).
4. $\frac{\partial C_B^*}{\partial \beta} > 0$, $\frac{\partial C_B^*}{\partial \alpha} < 0$, and $\frac{\partial C_B^*}{\partial \delta} < 0$.
5. $\frac{\partial C_B^*}{\partial \gamma}$ is ambiguous, it depends on the parameters' values. Similarly to B^* , C_B^* increases with γ if α is small enough, and it's a decreasing function of γ when α is large enough.

A few observations are worth doing before discussing the economic mechanisms involved in these comparative statics. First of all, and as mentioned above, the stationary states of dynamic systems do depend on the way the optimizer/planner value future utility flows compared to the current ones, that's on the time discounting rate, δ . A very large time discounting brings the whole economy close to extinction asymptotically as demonstrated in the previous section. A larger time discounting reduces the stock of biodiversity and

the consumption of the wild food.⁵ Second, some comparative results are reversed with respect to the static counterpart. This is clear in the fourth comparative static: wild food consumption is a decreasing function of the strength of the agroecological productivity effect in the optimal static solution (see Proposition 1), it's increasing the dynamic case at the stationary equilibrium. Third, and more frequently, a new type of results with respect to the static case may emerge as the dynamic interaction of direct and feedback effects may lead to much more complex pictures in the dynamic settings, even at the steady state, yielding non-monotonic relationships and the like. In our case, as featured in comparative statics 3. and 5.. Here we enhance the role of the parameter (that's α) which somehow measures, as repeatedly invoked above, the strength of the feedback mechanism from the amount of biodiversity to the productivity of the agricultural sector.

We shall concentrate on the comparative statics 3. to 5., those which differ from those of the static case, precisely those with respect to the key parameters α and γ . We skip most of the algebraic details. Let's start with the stock variable, B^s (comparative static 3.) Recall that it's given by

$$B^s = \left[\frac{\beta(1 - \theta + \gamma(\theta - \alpha))}{\delta(1 - \gamma)} \right]^{\frac{1}{\theta}} f^s,$$

It follows that

$$\frac{\partial B^s}{\partial \alpha} = \left[\frac{\beta(1 - \theta + \gamma(\theta - \alpha))}{\delta(1 - \gamma)} \right]^{\frac{1}{\theta} - 1} \left(\frac{\partial f^s}{\partial \alpha} \left[\frac{\beta(1 - \theta + \gamma(\theta - \alpha))}{\delta(1 - \gamma)} \right] - f^s \frac{\beta\gamma}{\theta\delta(1 - \gamma)} \right) < 0,$$

and

$$\begin{aligned} \frac{\frac{\partial B^s}{\partial \gamma}}{B^s} &= \frac{1}{\theta} \frac{\frac{\beta(1-\alpha)}{\delta(1-\gamma)^2}}{\frac{\beta(1-\theta+\gamma(\theta-\alpha))}{\delta(1-\gamma)}} + \frac{\frac{\partial f^s}{\partial \gamma}}{f^s} \\ &= \frac{1}{(1-\gamma)} \left(\frac{1}{\theta} \frac{(1-\alpha)}{1-\theta+\gamma(\theta-\alpha)} - \frac{\alpha}{(\alpha-\theta)\gamma+\theta} \right) \end{aligned}$$

⁵It could be also shown that the same property holds for the consumption of the agricultural good due to the agricultural sector productivity feedback effect of falling biodiversity similarly to the extinction story told in the previous section. We skip computations for lack of space.

Let $z = (\alpha - \theta)\gamma + \theta$

$$\frac{\frac{\partial B^s}{\partial \gamma}}{B^s} = \frac{1}{\theta(1-\gamma)} \left(\frac{z(1-\alpha) - \alpha\theta(1-z)}{(1-z)z} \right)$$

The sign of $\frac{\partial B^s}{\partial \gamma}$ is the same as the sign of $z(1-\alpha) - \alpha\theta(1-z)$, which is ambiguous. For $\alpha = 0.9, \theta = 0.5, \gamma = 0.1$, $\frac{\partial B^s}{\partial \gamma} < 0$, whereas for $\alpha = 0.1, \theta = 0.5, \gamma = 0.1$, $\frac{\partial B^s}{\partial \gamma} > 0$. It's easy to show that the derivative $\frac{\partial B^s}{\partial \gamma}$ is decreasing with respect to α , given the magnitude of α and γ , both strictly lower than 1. An even more direct way to see this property is to study the limit cases of $\frac{\partial B^s}{\partial \gamma}$ when α goes respectively to 1 and 0.

We finally turn to C_B^s . As $C_B^s = \beta(B^s)^{1-\theta}(f^s)^\theta$, we get

$$\begin{aligned} \frac{\partial C_B^s / \partial \alpha}{C_B^s} &= (1-\theta) \left(\frac{\partial B^s / \partial \alpha}{B^s} \right) + \theta \left(\frac{\partial f^s / \partial \alpha}{f^s} \right) < 0 \\ \frac{\partial C_B^s / \partial \beta}{C_B^s} &= (1-\theta) \left(\frac{\partial B^s / \partial \beta}{B^s} \right) > 0 \\ \frac{\partial C_B^s / \partial \gamma}{C_B^s} &= (1-\theta) \left(\frac{\partial B^s / \partial \gamma}{B^s} \right) + \theta \left(\frac{\partial f^s / \partial \gamma}{f^s} \right) \\ &= (1-\theta) \left(\frac{\partial B^s / \partial \gamma}{B^s} \right) - \frac{\alpha\theta}{(1-\gamma)((\alpha-\theta)\gamma + \theta)} \end{aligned}$$

We can observe that:

$$\begin{aligned} \frac{\partial C_B^s / \partial \gamma}{C_B^s} &= \frac{1}{(1-\gamma)z} \left[\frac{(1-\theta)}{\theta} \left(\frac{z(1-\alpha) - \alpha\theta(1-z)}{(1-z)} \right) - \alpha\theta \right] \\ &= \frac{1}{\theta(1-z)(1-\gamma)z} ((1-\theta)z(1-\alpha) - \alpha\theta(1-z)) \end{aligned}$$

However, the sign of $(1-\theta)z(1-\alpha) - \alpha\theta(1-z)$ is ambiguous. Indeed, $\alpha = 0.9, \theta = 0.5, \gamma = 0.9$, then $a < 0$ but for $\alpha = 0.1$, then $a > 0$. Similarly to the comparative static of B^s , we obtain property 5. in Proposition 5.

Let us now comment on the economic mechanisms involved and compare with the static counterpart outcomes. First of all, we note that the property that the biodiversity stock increases with the strength of the agroecological productivity effect (or decreases with α) also holds in the dynamic case at the stationary equilibrium. In the static case, this relationship is

clear-cut: a drop in α raises the surface of the forest land and decreases wild food consumption, which eventually yields a non-ambiguous (and potentially strong) increment in the biodiversity stock. While we get the same corresponding qualitative relationship in the dynamic case (which is indeed an indicator of the well-posedness of our problem), the mechanisms are not the same. As stated in the property 4. of Proposition 4, the impact of a decrease in α on the biodiversity stock is much more involved in the dynamic case: while the positive forest land effect is still present, consumption of wild food goes in the opposite direction, it increases when α drops. However, the former positive effect dominates for any values of the parameters of the model. The fact that the comparative static for C_B^s with respect to α is reversed in the dynamic case at the stationary equilibrium is not surprising *per se* considering the richness of feedback effects in our model, not speaking about the forces governing convergence to the steady state (including time discounting): while a rising stock of biodiversity will push upward its harvesting in first place, it also rises the incentives to produce the agricultural good due to the agroecological productivity effect, which goes in opposite direction relatively to the first effect. Not speaking about the dynamics of convergence to the steady state which in all cases alter the size of the different involved effects over time, especially in the medium and long run.

Let's move now to the comparative static of B^s with respect to γ , that's comparative static 3., it's similar to the comparative static 5. concerning C_B^s with respect to the same parameter. We shall concentrate on the B^s , similar arguments can be used for C_B^s using the relationship relating both variables and f^s given in Proposition 3. As explained in Section 3.2, the comparative statics of B^* with respect to γ are *a priori* intricate even in the static case: in such a case, an agricultural economy which exhibits a larger elasticity of utility to the agricultural good, that's with a stronger preference to the latter good, would initially decrease both the optimal forest land and consumption of the wild goods, resulting in an ambiguous direct effect on B^* . However in the static case, we have found that **whatever the parameter values** (θ , α , β and γ), the magnitude of the decrease in wild goods consumption is larger than the reduction in biodiversity originating in optimal forest land shrinking, accounting for all the feedback effects generated. In the dynamic case at the stationary equi-

librium, everything depends on the parameters' values. To simplify our discussion, we focus on the key parameter in the genesis of feedback effects, α . Proposition 4 shows that when the agroecological productivity effect is strong enough (α small enough), we get the static picture, and we get the reverse in the opposite case. This is rather intuitive: in a dynamic model, the feedback effects play longer over time (though constrained by the convergence to steady state forces as time advances), this leads to a bigger role for the forest land surface effect, which is inherent in the agroecological effect, compared to the one channeled through wild food consumption. This yields in particular to the (long-term) property that as the agricultural society preferences move away from wild food consumption (γ increases), the stock of biodiversity is raised only if the agroecological productivity effect is strong enough, which is a quite interesting result.

5 CONCLUDING REMARKS

We have shown both in the static and dynamic cases how the feedback effect generated by the agroecological productivity boost shapes the qualitative properties within a canonical agricultural model with land use control and consumption (optimal) arbitrage between agricultural and wild goods. The interaction between the feedback effect and the mechanisms inherent in intertemporal optimisation has been shown to deliver several highly interesting results (for the long-term equilibrium), with some policy relevance. Several methodological lessons have been drawn along the way.

We believe that this model is a useful benchmark, and we have explained why in several places in the main text. The fact remains that it's a benchmark. Several extensions are worth doing starting with the analysis (possibly numerical) of the general model with nonlinear feedback effects for example. Also, more general consumption preferences are interesting to incorporate, possibly with endogenous cultural dynamics moving societies away from wild food. Last but not least, more convincing specifications of biodiversity dynamics are needed starting with the endogenisation of the regeneration rate.

APPENDIX: SUFFICIENCY ANALYSIS OF THE OPTIMAL CONTROL PROBLEM

We now study the sufficient optimality conditions. We consider \tilde{H} as a function of B and f .

Indeed, $\tilde{H} = B^{(1-\alpha)\gamma} (1-f)^{\alpha\gamma} C_B^{1-\gamma} + \mu (\beta B^{1-\theta} f^\theta - C_B)$. According to Equation 17,

$$\tilde{H} = B^{(1-\alpha)\gamma+(1-\theta)(1-\gamma)} (1-f)^{\alpha\gamma+1-\gamma} (\chi\beta)^{1-\gamma} f^{(\theta-1)(1-\gamma)} + \mu\beta B^{1-\theta} f^{\theta-1} (f - \chi(1-f)).$$

Moreover, according to Equation 12

$$(1-\gamma) B^{(1-\alpha)\gamma} (1-f)^{\alpha\gamma} C_B^{-\gamma} = \mu,$$

Using 17, it can be written as

$$(\chi\beta)^{-\gamma} (1-\gamma) B^{(1-\alpha)\gamma-\gamma(1-\theta)} (1-f)^{(\alpha-1)\gamma} f^{\gamma(1-\theta)} = \mu,$$

$$\tilde{H} = \frac{1}{\theta} (\chi\beta)^{1-\gamma} B^{(1-\alpha)\gamma+(1-\gamma)(1-\theta)} f^{(\theta-1)(1-\gamma)} (1-f)^{(\alpha-1)\gamma} \gamma [\theta + (\alpha-\theta)f].$$

As $\theta + (\alpha-\theta)f = \theta(1-f) + \alpha f > 0$, letting $h = \frac{\gamma}{\theta} (\chi\beta)^{1-\gamma}$,

$$\tilde{H} = h B^{1-\theta+\gamma(\theta-\alpha)} f^{(\theta-1)(1-\gamma)} (1-f)^{(\alpha-1)\gamma} (\theta + f(\alpha-\theta)) > 0$$

In order to compute the Hessian Matrix of \tilde{H} , let us notice that

$$\begin{aligned} \frac{\partial \tilde{H}}{\partial B} &= (1-\theta + \gamma(\theta-\alpha)) \frac{\tilde{H}}{B}, \\ \frac{\partial^2 \tilde{H}}{\partial B^2} &= (1-\theta + \gamma(\theta-\alpha)) (-\theta + \gamma(\theta-\alpha)) \frac{\tilde{H}}{B^2}, \\ &= -(1-\theta + \gamma(\theta-\alpha)) ((1-\gamma)\theta + \alpha\gamma) \frac{\tilde{H}}{B^2}. \end{aligned}$$

As $1-\theta + \gamma(\theta-\alpha) = (1-\alpha)\gamma + (1-\theta)(1-\gamma)$, then $1-\theta + \gamma(\theta-\alpha) > 0$. Thus as a

consequence,

$$\frac{\partial^2 \tilde{H}}{\partial B^2} < 0.$$

Moreover

$$\frac{\partial \tilde{H}}{\partial B \partial f} = (1 - \theta + \gamma(\theta - \alpha)) \frac{\partial \tilde{H} / \partial f}{B}$$

,and

$$\begin{aligned} \frac{\partial \tilde{H}}{\tilde{H} \partial f} &= \frac{(\theta - 1)(1 - \gamma)}{f} + \frac{(1 - \alpha)\gamma}{1 - f} + \frac{\alpha - \theta}{\theta + f(\alpha - \theta)} \\ &= \frac{-(1 - \theta + f(\theta - \alpha))(- (1 - f)\theta(1 - \gamma) + f\alpha\gamma)}{f(1 - f)(\theta + f(\alpha - \theta))}. \end{aligned}$$

Let $g(f) = \frac{-(1 - \theta + f(\theta - \alpha))(- (1 - f)\theta(1 - \gamma) + f\alpha\gamma)}{f(1 - f)(\theta + f(\alpha - \theta))}$, thus the following equation holds.

$$\frac{\partial^2 \tilde{H}}{\partial f^2} = \left(\frac{\partial \tilde{H}}{\partial f} \right) g(f) + \tilde{H} \left(\frac{(1 - \theta)(1 - \gamma)}{f^2} + \frac{(1 - \alpha)\gamma}{(1 - f)^2} - \frac{(\alpha - \theta)^2}{(\theta + f(\alpha - \theta))^2} \right)$$

.

The Hessian Matrix of \tilde{H} as a function of B and μ is given as follows.

$$\mathcal{H}(B, f) = \begin{bmatrix} -(1 - \theta + \gamma(\theta - \alpha))((1 - \gamma)\theta + \alpha\gamma) \frac{\tilde{H}}{B^2} & (1 - \theta + \gamma(\theta - \alpha)) \frac{\partial \tilde{H} / \partial f}{B} \\ (1 - \theta + \gamma(\theta - \alpha)) \frac{\partial \tilde{H} / \partial f}{B} & \frac{\partial^2 \tilde{H}}{\partial f^2} \end{bmatrix}$$

,

and the determinant of Hessian Matrix of \tilde{H} is

$$\det(\mathcal{H}(B, f)) = (1 - \theta + \gamma(\theta - \alpha)) \frac{\tilde{H}^2}{B^2} [-((1 - \gamma)\theta + \alpha\gamma) g'(f) - g^2(f)].$$

Moreover, in the neighborhood of the steady state $f^s = \frac{(1 - \gamma)\theta}{\alpha\gamma + (1 - \gamma)\theta}$, $g(f) \simeq 0$ and $g'(f) < 0$. As $(1 - \theta + \gamma(\theta - \alpha)) > 0$, then $\det(\mathcal{H}(B, f)) > 0$. Moreover, as $\frac{\partial^2 \tilde{H}}{\partial B^2} < 0$, then the optimum is a maximum.

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