Identification and Mitigation of Intermodulation Products Using a Baseband Polynomial Distortion Model with Instantaneous Frequency Dependence

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Abstract – **The linearization of active electronic components such as the PA or the ADC, is a vast subject. Many issues come into play, including behavioral modeling with the selection of a relevant model in terms of accuracy and complexity, the identification of this model, and the correction of defects by compensation. In this article, we propose a baseband model of nonlinearity defects observed at the digitization output and after IQ demodulation, with a focus on order 3 intermodulation. We then introduce a refinement of this model by adding a dependence on the variation of the instantaneous frequency of the signal. We describe a method suitable for calibration of the model, by identification on a two-tone signal. We finally present a measurement bench adapted to the calibration of the coefficients of the model, followed by some results of linearization.**

I. INTRODUCTION

This paper deals with the baseband modeling of the distortion at the output of a high speed (2 GHz) and high resolution (14 bits) commercially available ADC. This ADC features IQ demodulation, filtering and decimation, making it possible to select and reduce the useful frequency band of the signal thus digitized. We will use these capabilities later, and we are therefore interested in modeling distortion defects on complex signals, at the digitization output and downstream processing stages of the ADC.

The objective is thus to propose a model on a complex analytical signal applicable at the output of the digitization chain (ADC, digital IQ amplitude/phase demodulation, and decimation) suitable for calibration and linearization. Although the targeted application is narrow band here, we propose to take into account the frequency dependence, analogous to a memory effect, with a view to versatility. Finally, these models aim to intervene at the end of the digital chain for practical reasons: limitation in flow due to decimation, and therefore in energy consumption, and ease of implementation. The targeted application of this work is indeed an embedded application. The models envisaged must then present a limited complexity, facilitating their identification and the implementation of the linearization.

In this paper, we propose an extension of a baseband polynomial model by involving the derivative of the sampled analytical signal, and thus by adding an instantaneous frequency dependence to our model. This writing thus makes it possible to model to a certain extent the variation of the non-linearity with the frequency. This method has been patented in 2015 [1].

This paper is organized as follows. We will first develop the distortion model on an analytical signal, and the instantaneous frequency dependence. Afterwards, the identification of these models will be studied, from spectral observations of the distortion of two-tone signals. We will then see a measurement bench allowing the calibration of our ADC, and finally results of identification and linearization by compensation.

II. MODELING

Any active analog or mixed system generates distortion spurs on its output. This is called a non-linear system. The simplest model allowing to illustrate this phenomenon is the polynomial model, or in series of powers, describing a non-linear distortion. A polynomial model links the real input $x(t)$ of a nonlinear system to its output $y(t)$ by the following relation:

$$y(t) = \sum_{n=0}^{N} a_n x^n(t) \approx x(t) + D\left[x(t)\right] = x(t) + \sum_{n=2}^{N} a_n x^n(t) \tag{1}$$

with N the order of non-linearity of the system, and the a_n the coefficients of the model. The 0th order coefficient is an offset ($a_0 \approx 0$), and the 1st order coefficient is the linear gain of the system ($a_1 \approx 1$). The nonlinear distortion is then contained in the higher order terms. This separation in the distorted signal between the input signal and a function (here D) of distortion depending on the input signal allows

a compensation of the defects by subtracting this part of distortion from the signal at the output of the system to be linearized.

A. Baseband distortion modeling

We find an expression for distortion defects on an analytical signal in [2, 3, 4]. This model is described in these papers as coming from the polynomial expansion of the complex envelope of an RF signal: $x_{RF}(t) = 2\Re\left[x(t)e^{j\omega_0 t}\right]$, where ω_0 is the pulse of the RF carrier frequency of the signal (using the notation of [2]).

Our distortion model is based on the Order 3 Volterra Base Band Series model, with no memory effect [4]:

$$\tilde{D}^{(3)}(x) = x\alpha\,|x|^2 \tag{2}$$

This model can be extended to the order $2N + 1$ as follows:

$$\tilde{D}^{(2N+1)}(x) = x\sum_{k=1}^{N}\alpha_k|x|^{2k} \tag{3}$$

The model presented so far does not present a frequency dependence, i.e. it does not allow to model a memory effect, being a variation of the non-linear distortion as a function of frequency. This point will be dealt with in the next subsection.

B. Adding and instataneous frequency dependence

We here propose to add a dependence of the distortions on the instantaneous frequency of the signal. This instantaneous frequency will be noted f_i. It is expressed as follows for a sinusoidal signal of natural frequency f_0:

$$x(t) = ae^{j\psi(t)} \qquad \text{with } \psi(t) = 2\pi f_0 t + \phi \tag{4}$$

$$f_i(t) = \frac{1}{2\pi}\frac{\partial\psi(t)}{\partial t} = \frac{1}{2\pi}\frac{\partial}{\partial t}\left(2\pi f_0 t + \phi\right) = f_0 \tag{5}$$

The instantaneous frequency of a sinusoidal signal therefore corresponds to its natural frequency, the instantaneous frequency of a signal made up of two tones of the same level, to the average of the frequencies of the two tones.

We have,

$$\frac{x'}{x} = \frac{\frac{\partial x}{\partial t}}{x} = j\frac{a2\pi fe^{j(2\pi f_0 t+\phi)}}{ae^{j(2\pi f_0 t+\phi)}} = j2\pi f_0$$

Thus :

$$f_i(t) = \frac{1}{2\pi}\Im\left[\frac{x'}{x}\right] = \frac{1}{2\pi}\frac{\Im\left[x'x^*\right]}{|x|^2} \tag{6}$$

We therefore propose the following refinement to the 3rd order distortion model:

$$\tilde{D}_i^{(3)}(x) = x\left(\alpha + \beta f_i\right)|x|^2 \tag{7}$$

i.e.

$$\tilde{D}_i^{(3)}(x) = x\left[\alpha|x|^2 + \frac{1}{2\pi}\beta\Im\left(x'x^*\right)\right] \tag{8}$$

C. Synoptic view in blocks of the model

This model can be represented by a block diagram view (see Fig. 1). The differentiator filter is called h_D and its delay is written τ_D. It is compensated on the other channels so that the following operations are synchronous. We can also see here linearization by compensation, by reconstructing the distortions from the input signal and then subtracting them from the signal. This point will be discussed in section v..

III. IDENTIFICATION

The identification of this model is done here from frequency observations of a well-known two-tone CW (Continuous Wave) signal, i.e. whose frequencies are known. The frequency identification of this model follows the work of [5]. This method is suitable for a model calibration phase.

A. Two-tone reference signal

In this paper, we will focus on the response of a commercially available ADC to the excitation of a two-tone signal. This signal used for the calibration of the model, indeed makes it possible to reveal the defects which interest us here, namely the near-carrier inter-modulation. A two-tone signal also has several interesting advantages, an ease in the practical implementation of its generation, and a good coverage of the phase space allowing the excitation of the full dynamic range of the ADC [6, 7, 8, 9].

For the calibration of the model, we look at the excitation of the ADC by a two-tone signal of the form:

$$x(t) = a_1e^{j2\pi f_1 t} + a_2e^{j2\pi f_2 t}$$

with a_1 and a_2 the complex amplitudes of the tones of frequencies f_1 and f_2 respectively.

B. Identification of the model

The response of our distortion model to the excitation of a two-tone signal develops as follows:
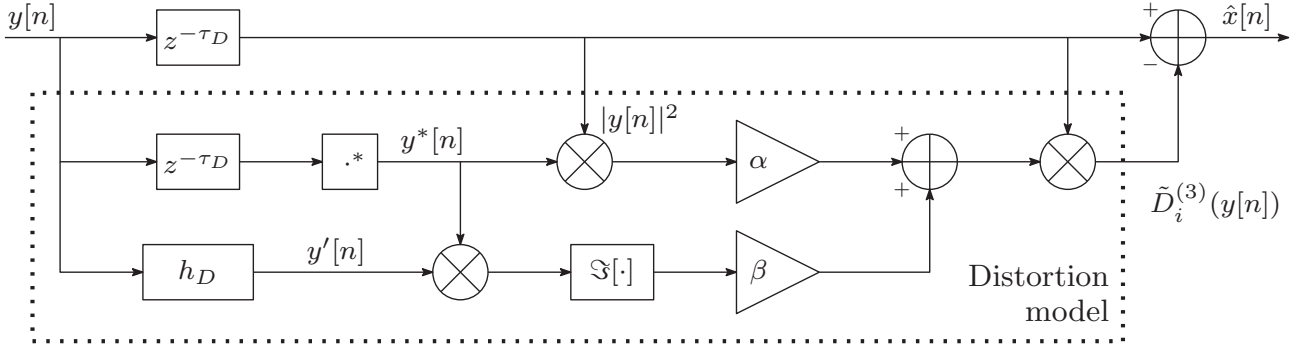
Fig. 1. Synoptic model (order 3 with instantaneous frequency dependence) of reconstruction of distortion and compensation

$$\begin{aligned}
y(t) &= x(t) + \tilde{D}_i^{(3)}(x(t)) \\
&= x(t) + x(t)\left(\alpha + \beta f_i\right)|x(t)|^2 \\
&= a_1 e^{j2\pi f_1 t} + a_2 e^{j2\pi f_2 t} \\
&+ \left(\alpha + \beta f_i\right)\Big[\left(a_1^3 + 2a_2^2 a_1\right)e^{j2\pi f_1 t} \\
&\qquad + \left(a_2^3 + 2a_1 a_2\right)e^{j2\pi f_2 t} \\
&\qquad + a_1^2 a_2^* e^{j2\pi(2f_1 - f_2)t} \\
&\qquad + a_1^* a_2^2 e^{j2\pi(2f_2 - f_1)t}\Big]
\end{aligned}$$

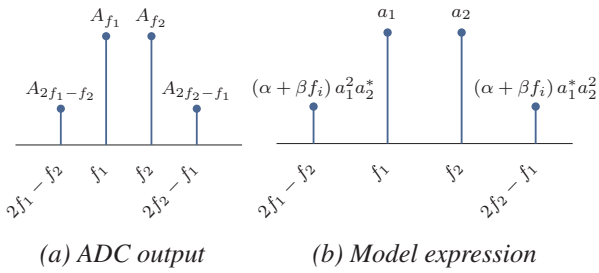We then observe the following tones at the output of the ADC, and on the model (cf. Fig. 2).

(a) ADC output   *(b) Model expression*

Fig. 2. Model identification
from spectral observation

For the identification of the coefficients α and β, we will use the results of two measurements, around the instantaneous frequencies f_{i_a} and f_{i_b}. These two frequencies will be chosen as the extremities of the band in which the ADC will be modeled.

In our case, the modeling will be performed in a band B of a few hundreds kHz to a few MHz, around the RF carrier f_0. So we will have $f_{i_a} = f_0 - \frac{B}{2}$ and $f_{i_b} = f_0 + \frac{B}{2}$.

We then have to solve the following system. The coefficients are obtained by averaging a redundant observation of the model:

$$\left(\alpha + \beta f_{i_a}\right) = \frac{1}{2}\left[\frac{A_{2f_{1_a} - f_{2_a}}}{A_{f_{1_a}}^2 A_{f_{2_a}}^*} + \frac{A_{2f_{2_a} - f_{1_a}}}{A_{f_{2_a}}^2 A_{f_{1_a}}^*}\right] = R_a \quad (9)$$

$$\left(\alpha + \beta f_{i_b}\right) = \frac{1}{2}\left[\frac{A_{2f_{1_b} - f_{2_b}}}{A_{f_{1_b}}^2 A_{f_{2_b}}^*} + \frac{A_{2f_{2_b} - f_{1_b}}}{A_{f_{2_b}}^2 A_{f_{1_b}}^*}\right] = R_b \quad (10)$$

This system then resolves to:

$$\alpha = \frac{f_{i_a} R_b - f_{i_b} R_a}{f_{i_a} - f_{i_b}} \quad (11)$$

$$\beta = \frac{R_a - R_b}{f_{i_a} - f_{i_b}}; \quad (12)$$

During this identification phase, the frequencies of the two tones being known, the instantaneous frequencies f_{i_a} and f_{i_b} are obtained as follows:

$$f_{i_a} = \frac{a_1^2 f_{1_a} + a_2^2 f_{2_a}}{a_1^2 + a_2^2} \quad (13)$$

$$f_{i_b} = \frac{a_1^2 f_{1_b} + a_2^2 f_{2_b}}{a_1^2 + a_2^2} \quad (14)$$

with f_{1_a} and f_{2_a} the frequencies of the two-tones of the first measurement, around f_{i_a}, and f_{1_b} and f_{2_b} the frequencies of the two-tones of the second measurement, around of f_{i_b}.

IV. EXPERIMENTAL SETUP

To validate the theoretical concepts developed in the previous sections, we build the following experimental setup (see Fig. 3).
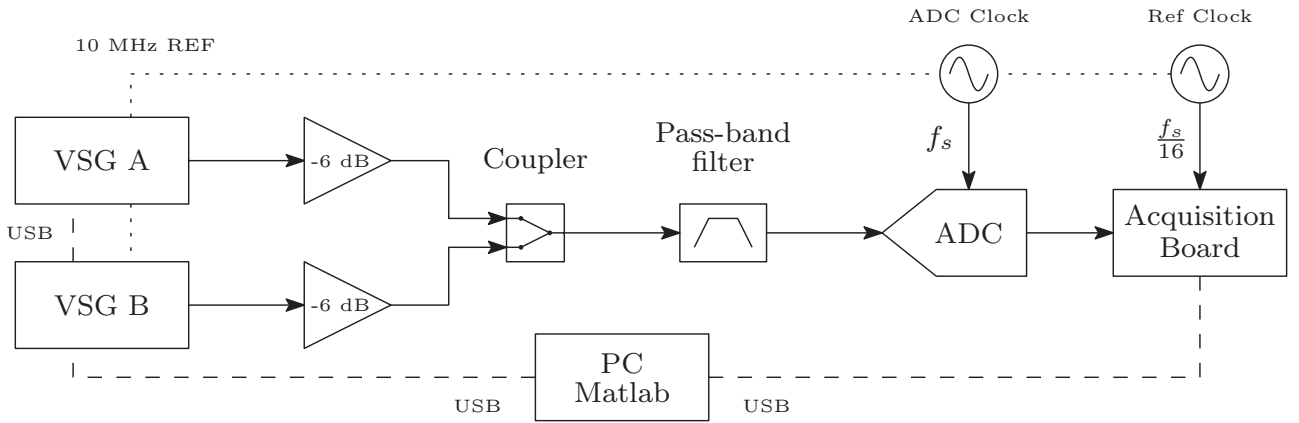
Fig. 3. Experimental setup synoptic

The two-tone signal used as a reference for the calibration is generated by two vector signal generators (VSG A and VSG B). Each sends a CW signal, one at frequency f_1, and the other at frequency f_2, and the two-tone signal is then assembled by a coupler. This signal could be generated by only one of these instruments, but the linearity of the input signal would not be sufficient for the precision required by our measurements. The presence of attenuation in the assembly, between the VSGs and the coupler, also makes it possible to improve the linearity of the reference signal during calibration, by attenuating twice (due to the goings and comings routes), any bounces in the assembly.

A band-pass filter is then used to remove out-of-band noise that can fall back during digitization. Finally, the ADC studied is a commercially available ADC (AD9689, 14 bit, 2 Gsps), mounted on its acquisition card. The latter is used to configure the ADC and to recover the points captured at the PC level. Two clock generators are used, one for the ADC clock, and the other for the acquisition card reference clock, for data transfer.

The various measurement and generation instruments are controlled from MATLAB via USB. Finally, all the instruments share the same reference (10 MHz) which allows them to operate synchronously.

V. LINEARIZATION

We present in this section linearization results of an ADC using the model, the identification method, and the measurement bench, described in the previous sections.

To linearize the signal at the output of the ADC from the proposed model, the distortions are reconstructed from the distorted signal itself (cf. Fig. 1). Indeed, the signal is assumed to be weakly non-linear, that is to say that
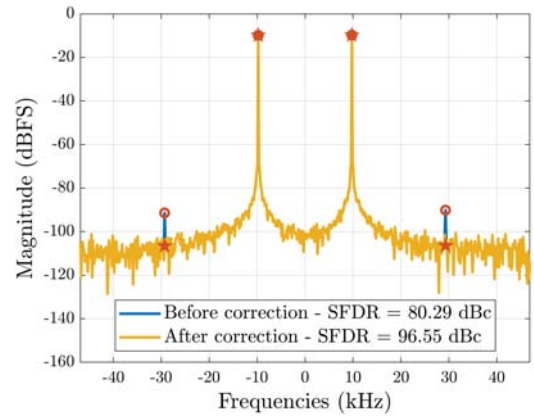

Fig. 4. Linearization results using model $\tilde{D}_i^{(3)}$ with fundamentals both at -10 dBFS
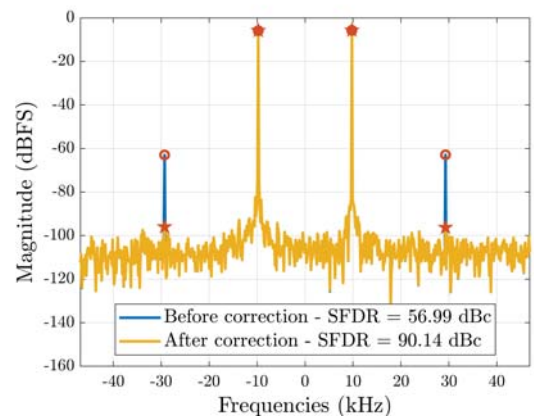

Fig. 5. Linearization results using model $\tilde{D}_i^{(3)}$ with fundamentals both at -6 dBFS

the parasitic spurs are at a sufficiently low level for their contribution in the reconstruction of the distortions to be

negligible.

We thus observe some linearization results in Fig. 4 and 5 using our 3rd order model with instantaneous frequency dependence. In these figures, the amplitudes of the fundamentals and of the IMD3 products are characterized by cirle markers on the signal before correction, and by stars on the signal after correction.

These figures demonstrate the effectiveness of the compensation with the proposed model. The SFDR (Spurious Free Dynamic Range) is indeed pushed to the noise floor (15 to 24 dB amelioration).

To illustrate the relevance and efficiency of the model depending on the instantaneous frequency, it is necessary to observe the variation of IMD3 (Order 3 intermodulation) in a frequency band. It is then necessary to calibrate the compensation model (here simply to order 3) and note that a simple coefficient α alone cannot translate a variation in frequency of the IMD3, i.e. a memory effect. The instantaneous frequency model is then calibrated on this same band, so as to reproduce this behaviour. The calibrated coefficients can be seen in Fig. 6 and the compensation results in Fig. 7.
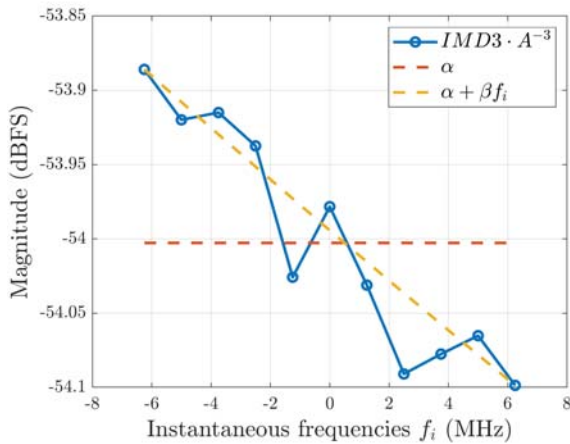

Fig. 6. *Coefficients of order 3 compensation models. A is the magnitude of the fundamentals (assuming they are at the same level)*

This model is limited here to a linear variation of the IMD3 as a function of the instantaneous frequency, which can be associated with a narrow band behaviour of the ADC. To model more complex behaviours of the IMD3 with regards to frequency, this model would then have to be extended to a polynomial frequency dependence.
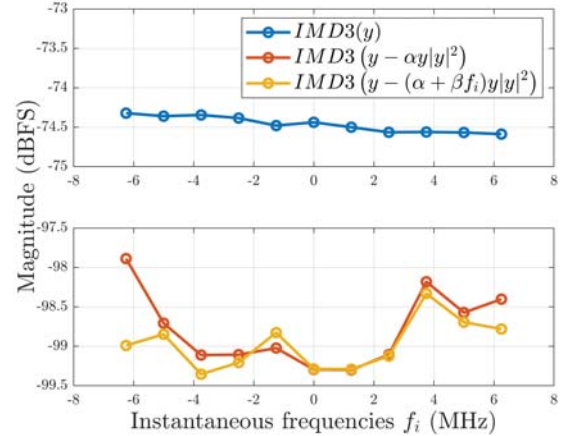
We finally note that although the studied ADC presents

Fig. 7. *Linearization results using order 3 compensation models with and without instantaneous frequency dependence*

an IMD3 varying with the instantaneous frequency, this variation is here too weak in the studied band so that the gain brought by this new contribution is significant during the linearization (less than 1 dB of improvement compared to a simpler model without memory effect). This approach nevertheless seems promising for systems with more frequency dependence of nonlinearities, or for a study on a wider band.

VI. CONCLUSION

In this paper, we have presented a baseband distortion model, with an instantaneous frequency dependence, in order to model a nonlinear frequency evolving behavior. A method for identifying this model, based on a frequency observation of the application of a two-tone signal, is then presented. We finally built a measurement bench adapted to the implementation of the identification of this model.

The results of modeling and linearization present effective mitigation of intermodulation products. The frequency variation of the distortion is modeled by the instantaneous frequency (f_i) dependence.

When trying to improve the accuracy of the modeling or identifying models going to higher orders, in order to acount for higher order intermodulation products, limiting dynamic range, it quickly appears crucial to get out of the noise floor. Indeed, the higher the distortion order observed, the lower the amplitude of the intermodulation products will be, approaching the noise floor. The solution is then to observe on longer time intervals, in order to integrate this measurement noise and to make the noise floor go down spectrally.

To go further, the model with a dependence in instantaneous frequency could be studied for signals more complex than a two-tone signal, not allowing to lift the correlation of the model on a single measurement. One could be inspired by the identification of Volterra kernels from white noise [10].

Finally, the model presented has certain limitations inherent in its structure. A distortion model going to order 3 with a simple instantaneous frequency dependence is in fact only able to model defects that vary linearly with the level, and with the instantaneous frequency. This model is therefore initially suitable for narrow-band modeling of components with relatively basic behavior. Subsequently, a wider-bandwidth behavior can be modeled by a polynomial variation of the instantaneous frequency. Similarly, identifying a model at higher orders of non-linearities can also allow modeling of a polynomial variation of the intermodulation with respect to the level.

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