



**HAL**  
open science

# Dispersive characteristics of non-linear waves propagating and breaking over a mildly sloping laboratory beach

Kévin Martins, Philippe Bonneton, Hervé Michallet

► **To cite this version:**

Kévin Martins, Philippe Bonneton, Hervé Michallet. Dispersive characteristics of non-linear waves propagating and breaking over a mildly sloping laboratory beach. *Coastal Engineering*, 2021, 167, pp.103917. 10.1016/j.coastaleng.2021.103917. hal-03215337

**HAL Id: hal-03215337**

**<https://hal.science/hal-03215337>**

Submitted on 3 May 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Dispersive characteristics of non-linear waves propagating and breaking over a mildly sloping laboratory beach

Kévin Martins<sup>a,\*</sup>, Philippe Bonneton<sup>a</sup>, Hervé Michallet<sup>b</sup>

<sup>a</sup>UMR 5805 EPOC, CNRS - University of Bordeaux, Allée Geoffroy Saint-Hilaire, F-33615 Pessac, France

<sup>b</sup>UMR 5519 LEGI, CNRS - University Grenoble Alpes, CS40700, 38058 Grenoble, France

---

## Abstract

The dispersive characteristics of unidirectional irregular waves propagating and breaking over a mildly sloping beach are examined using a highly-resolved laboratory dataset. Cross-spectral analyses are used to determine the cross-shore evolution of (single-valued) dominant wavenumber  $\kappa$  and phase velocity  $c$  spectra, and lead to the identification of four different regimes of propagation: I - a linear regime where short waves mostly propagate as free components; II - a shoaling regime where non-linear effects at high harmonics are significant but primary components follow the linear wave dispersion relation; III - a shoaling regime near the mean breaking point location, where amplitude dispersion effects at primary components are important; IV - a surf zone regime, where all components propagate slightly faster than non-dispersive shallow water waves. Bispectral analyses performed onshore of the shoaling region show that the presence of forced energy at high harmonics, which originate from non-linear interactions between triads of frequencies, are responsible for the deviations of wavenumber and phase velocity spectral estimates from predictions by the linear dispersion relation, confirming the findings from previous field-based studies. A Boussinesq approximation of the non-linear energy exchanges between triads is then used to quantify the relative amount of forced energy at high harmonics and explain the differences in dispersion properties observed in the shoaling region between broad and narrow-band spectra. Larger relative amounts of forced energy at high frequencies, which suggest more efficient non-linear energy transfers, are found to be associated with larger deviations of dominant  $\kappa$  and  $c$  from predictions by the linear dispersion relation.

---

## 1. Introduction

Wind-generated surface gravity waves (hereafter short waves) are the principal driver of nearshore dynamics. Close to shore, short waves eventually break and through this process, they enhance the vertical and horizontal mixing of the water column (*e.g.*, Ting and Kirby, 1996; Drazen and Melville, 2009; Clark et al., 2012), drive a setup near the shoreline (*e.g.*, Longuet-Higgins and Stewart, 1964; Stive and Wind, 1982) and control the nearshore circulation at various temporal scales (*e.g.*, Svendsen, 1984; Peregrine and Bokhove, 1998; Bühler and Jacobson, 2001; Bonneton et al., 2010; Castelle et al., 2016). At first order, linear wave theory correctly predicts a number of physical processes associated with the propagation of short waves, such as the refraction or shoaling of directionally spread seas (*e.g.* Longuet-Higgins, 1956; Guza and Thornton, 1980; Elgar et al., 1990, and many others). However, as waves shoal and interact

---

\*Corresponding author

Email address: kevin.martins@u-bordeaux.fr (Kévin Martins)

11 with a sloping, mobile bed, non-linear processes become dominant. These are responsible for the changes  
12 observed in wave shape, from nearly symmetric to more (horizontally) skewed in the shoaling region and  
13 more (vertically) asymmetric prior to breaking and in the surf zone (*e.g.* Elgar and Guza, 1985a,b; Doering  
14 and Bowen, 1995; Michallet et al., 2011; Rocha et al., 2017). Non-linear effects not only affect the sea  
15 surface elevation but also the near-bottom wave orbital velocities and thus play a crucial role in short- and  
16 long-term beach morphodynamics (Doering and Bowen, 1986, 1995; Hoefel and Elgar, 2003; Berni et al.,  
17 2013; van der Zanden et al., 2017).

18 Weakly non-linear triad interactions (*e.g.* Phillips, 1960; Freilich et al., 1984; Elgar and Guza, 1985a)  
19 occurring as short waves propagate landward over a sloping bottom are responsible for these changes in  
20 the wave field. The interaction of two primary components of frequencies  $f_1$  and  $f_2$  excite a secondary  
21 component  $f'$  (either sum  $f_1 + f_2$  or difference  $f_1 - f_2$ ), which is bound to the statistically independent  
22 primary components. As such, the bound wave component  $f'$  does not follow the linear wave dispersion  
23 relation (Phillips, 1960; Longuet-Higgins and Stewart, 1962; Freilich et al., 1984). As opposed to non-linear  
24 resonant interactions between quadruplets in deep water (Hasselmann, 1962), which require very large  
25 distances to be effective, non-linear coupling between triads in nearshore areas are non- or near-resonant  
26 and can be very efficient in transforming incident wave spectra over just few typical wavelengths (*e.g.*,  
27 see Freilich et al., 1984, and the references therein). As both forced (or 'bound') and free components  
28 of directionally spread seas can co-exist in a wave field, there is no longer a unique relation between a  
29 frequency and wavenumber (*e.g.* Herbers and Guza, 1994). When forced components dominate over a  
30 region of the spectrum, large deviations from predictions by the linear wave dispersion relation can be  
31 observed in (single-valued) dominant wavenumber and phase velocity spectra (Thornton and Guza, 1982;  
32 Freilich et al., 1984; Elgar and Guza, 1985b). In particular near the breaking point or in the surf zone,  
33 most wave components of a typical sea-surface spectrum travel at the speed of non-dispersive shallow-water  
34 waves (*e.g.*, see Thornton and Guza, 1982; Elgar and Guza, 1985b; Catalán and Haller, 2008; Tissier et al.,  
35 2011), which is due to the dominance of amplitude dispersion effects over frequency ones (Herbers et al.,  
36 2002). As noted by Laing (1986), the deviations of measured wave phase speed from predictions by the  
37 linear dispersion relation, discussed here for nearshore waves, are quite analogous to those observed in  
38 growing seas (*e.g.*, see Ramamonjiarisoa and Coantic, 1976; Mitsuyasu et al., 1979; Crawford et al., 1981;  
39 Donelan et al., 1985). In such conditions, growing short-wave fields are dominated by modulated trains  
40 of finite amplitude waves to which high-frequency components are bound (Lake and Yuen, 1978; Coantic  
41 et al., 1981).

42 In practice, knowledge on the spatial structure of the wave field is generally lacking and the presence of  
43 forced energy is therefore difficult to quantify. As forced components at high harmonics are characterised  
44 by lower wavenumbers than free components of the same frequency, large errors from depth-inversion algo-  
45 rithms based on the linear wave dispersion relation can be expected in regions where non-linear effects are  
46 important (*e.g.*, see Holland, 2001; Brodie et al., 2018). The over-predictions of the dominant wavenumbers  
47 at high frequencies also explain the commonly reported 'blow-up' when reconstructing the free surface ele-  
48 vation from sub-surface pressure measurements with the linear dispersion relation (Bonneton and Lannes,  
49 2017; Bonneton et al., 2018; Mouragues et al., 2019; Martins et al., 2020b). This is related to the fact

50 that forced high harmonics are much less attenuated across the vertical than free components of the same  
51 frequency (e.g. Herbers and Guza, 1991; Herbers et al., 1992). Nonetheless, most field-based studies on  
52 non-linear wave transformation in the shoaling region employed sub-surface hydrodynamic data (whether  
53 pressure or orbital wave velocity), not corrected or corrected for depth-attenuation using the linear wave  
54 dispersion relation. Field-based studies on wave non-linearity also suffer from other limitations such as a  
55 poor spatial resolution and the distance over which waves can be studied. In particular, the cross-shore  
56 location where non-linear effects at high harmonics become predominant remains largely unknown.

57 The present paper uses a high-resolution laboratory dataset (GLOBEX, see Ruessink et al., 2013) to  
58 study the dispersive properties of irregular waves propagating and breaking over a mildly sloping beach.  
59 Besides confirming past findings, the GLOBEX dataset stands out from previously-published field observa-  
60 tions for several reasons. The free surface is directly measured with wave gauges and it is highly-resolved in  
61 space (several points per wavelength at any stage of propagation). The former aspect removes uncertain-  
62 ties currently existing on energy levels at high harmonics as measured in the field by sub-surface pressure  
63 sensors, and where the choice of the surface elevation reconstruction method has a strong influence (Bon-  
64 neton et al., 2018; Mouragues et al., 2019; Martins et al., 2020b). Furthermore, the experiments considered  
65 unidirectional irregular waves, which removes uncertainty about directional effects. In section 2, the high  
66 spatial and temporal resolution experimental dataset collected during GLOBEX is briefly presented. Sec-  
67 tion 3 introduces the cross-spectral and bispectral analysis techniques and describes the weakly non-linear  
68 numerical approach employed here for predicting the cross-shore evolution of energy spectra. In Section 4,  
69 the cross-spectral analysis is performed on the surface elevation data from adjacent wave gauges to extract  
70 dominant wavenumber spectra  $\kappa(f)$ , phase velocity spectra  $c(f)$  and their evolution across the entire wave  
71 flume. From this analysis, we identify four regimes of propagation ranging from a linear up to a surf zone  
72 situation, where wavenumber and phase velocity spectra display specific characteristics. In section 5, the  
73 bispectral analysis is used to quantify non-linear energy transfers towards harmonics, which play a funda-  
74 mental role in the patterns observed at high frequencies in wavenumber and phase velocity spectra. The  
75 dominant wavenumber is shown to vary with the amount of forced energy at a particular frequency, with  
76 larger deviations from the linear wave dispersion expected for higher forced-to-free energy ratios. Finally,  
77 section 6 briefly discusses the results and provides the concluding remarks of this study.

## 78 **2. Experimental dataset**

79 The Gently sLOping Beach Experiment (GLOBEX) project was performed in a 110-m-long, 1-m-wide,  
80 and 1.2-m-high wave flume, located in the Scheldegoet in Delft, the Netherlands (Ruessink et al., 2013).  
81 The experiments aimed at collecting high-resolution data of free surface elevation and current velocities  
82 in order to study infragravity wave dynamics and short-wave propagation and non-linearities (*e.g.*, see  
83 de Bakker et al., 2015; Tissier et al., 2015; Rocha et al., 2017). A combination of 21 capacitance and  
84 resistance-type of wave gauges sampling at 128 Hz were deployed along the low-sloping 1:80 concrete beach  
85 to measure the free surface elevation (see Fig. 1). 18 of these wave gauges were mounted on movable  
86 trolleys, which were repositioned after the repetition of each wave test in order to reach the desired spatial  
87 resolution (10 repetitions per test). Similarly, five electromagnetic current meters were fixed to the trolleys

88 and allowed the collection of current velocities at numerous locations across the wave flume (the data from  
 89 these current meters is not used here). This spatial resolution makes the GLOBEX dataset unique as it  
 90 allows to characterize and quantify non-linearities at various stages of the waves propagation (from a linear  
 91 situation up to the surf zone). The wave paddle steering signals included second-order wave generation and  
 92 the paddle was equipped with an Active Reflection Compensation system to absorb long waves radiated  
 93 from the beach. The water depth at the wave paddle was 0.85 m for all tests.

94 The present study uses free surface elevation measurements from the 70-min-long irregular wave tests  
 95 of the A series (A1, A2, A3, see Ruessink et al., 2013). During this series of tests, JONSWAP spectra  
 96 were imposed, covering moderate to energetic and broad to narrow-banded sea wave conditions, see table  
 97 1. The wave conditions along the flume for these runs are displayed in Fig. 2, through a range of second  
 98 and third-order wave parameters. Fig. 2a first shows the root-mean square wave height  $H_{rms}$  computed  
 99 as  $(8\overline{\zeta^2})^{1/2}$  where  $\zeta$  is the free surface elevation and the overbar denotes the time-averaged operator. Note  
 100 that  $\zeta$  was high-pass filtered using a cutoff frequency at  $0.6 f_p$ , with  $f_p$  the peak wave frequency, so that  
 101 the bulk parameters shown in Fig. 2 are computed on the short-wave frequency band only. As conditions  
 102 were more energetic during A2, the mean breaking point was located farther seaward than during A1 and  
 103 A3. In the inner surf zone, waves were found to be depth-limited during all runs, which is also evidenced by  
 104 the near-constant values reached by the non-linearity parameter  $\epsilon = H_{rms}/\sqrt{2}h$  in this region of the wave  
 105 flume ( $\epsilon \sim 0.3$ , see Fig. 2b), where  $h$  is the mean water depth. Fig. 2c-d show the short-wave skewness  $S_k$   
 106 and asymmetry  $A_s$  computed respectively as

$$S_k = \frac{\overline{(\zeta - \bar{\zeta})^3}}{(\zeta - \bar{\zeta})^2}{}^{3/2}. \quad (1)$$

$$A_s = -\frac{\overline{(\mathcal{H}(\zeta - \bar{\zeta}))^3}}{(\zeta - \bar{\zeta})^2}{}^{3/2}, \quad (2)$$

107 where  $\mathcal{H}\{\cdot\}$  is the Hilbert transform.  $S_k$  and  $A_s$  are measures of wave asymmetry along the horizontal and  
 108 vertical axes respectively and also inform on the energy content at high harmonics (Elgar, 1987; Martins  
 109 et al., 2020b; de Wit et al., 2020). For the same imposed wave height, A1 and A3 differed in the shoaling  
 110 region, with a more intense shoaling process noted during A3 (Fig. 2a and 2c), as expected for longer  
 111 waves. This is also explained by the narrower spectrum imposed during A3, which favoured non-linear  
 112 energy transfers to higher harmonics as compared to A1 (de Bakker et al., 2015). Short waves remain  
 113 (vertically) symmetric ( $A_s \sim 0$ ) until the location where the largest waves start breaking. Short-wave  
 114 asymmetry is maximal in the inner surf zone, where short waves pitch forward and display the commonly  
 115 observed sawtooth-like shape. Finally, the Ursell number  $U_r$  is shown in Fig. 2e. Here,  $U_r$  is defined as the  
 116 ratio between the non-linearity  $\epsilon$  and dispersion  $\mu = (\kappa_p h)^2$  parameters, where  $\kappa_p$  is the peak wavenumber  
 117 given by the linear wave dispersion relation.

### 118 3. Methods

#### 119 3.1. Computation of wavenumber and phase velocity spectra

120 Cross-spectral analysis between adjacent wave gauges is used to compute the dominant wavenumber  
 121 and phase velocity spectra across the wave flume. As this approach provides phase differences (or delay)

122 between two signals in the frequency domain (*e.g.*, see Ochi, 1998), it has been successfully used in the past  
 123 to study, in both the laboratory and field, the dispersive properties of ocean waves propagating in deep  
 124 (*e.g.*, see Ramamonjiarisoa and Coantic, 1976; Mitsuyasu et al., 1979) and intermediate to shallow waters  
 125 (Thornton and Guza, 1982; Freilich et al., 1984; Elgar and Guza, 1985b).

126 Let  $C_{x_1, x_2}$  denote the cross-spectrum computed with surface elevation timeseries from two gauges  
 127 located at positions  $x_1$  and  $x_2$ . The coherence  $\text{coh}(f)$  and phase  $\phi(f)$  spectra computed between  $x_1$  and  $x_2$   
 128 are then respectively given by

$$\text{coh}_{x_1, x_2}(f) = \left[ \frac{C_{x_1, x_2}(f)C_{x_1, x_2}^*(f)}{C_{x_1, x_1}(f)C_{x_2, x_2}(f)} \right]^{1/2} \quad (3)$$

$$\phi_{x_1, x_2}(f) = \arctan \left[ \frac{\text{Im}\{C_{x_1, x_2}(f)\}}{\text{Re}\{C_{x_1, x_2}(f)\}} \right], \quad (4)$$

129 where  $\text{Re}\{\cdot\}$  and  $\text{Im}\{\cdot\}$  are the real and imaginary parts of the cross-spectra respectively and  $*$  denotes the  
 130 complex conjugate. As shown in Fig. 3a with an example of cross-spectral analysis performed during A2 in  
 131 the deepest region of the flume ( $x \sim 10$  m), the phase spectra  $\phi(f)$  provides a phase lag per frequency that  
 132 is bounded between  $-\pi/2$  and  $\pi/2$ . The time delay (in sec) per frequency is obtained from the unwrapped  
 133 phase  $\phi^{unw}$  which, in the case of progressive waves propagating in one dimension, is easily retrieved from  
 134 the phase jumps (*e.g.*, see Fig. 3a). The wavenumber  $\kappa(f)$  and (cross-shore) phase velocity  $c(f)$  spectra  
 135 are then readily computed as

$$\kappa(f) = \phi_{x_1, x_2}^{unw}(f) / \Delta x \quad (5)$$

$$c(f) = 2\pi f \Delta x / \phi_{x_1, x_2}^{unw}(f), \quad (6)$$

136 where  $\Delta x$  is the spacing between the two wave gauges. As in Herbers et al. (2002),  $\kappa$  refers to the  
 137 single-valued wavenumber modulus, representative of a mixed sea-state composed of both free and forced  
 138 components (*i.e.*, for a given angular frequency  $\omega = 2\pi f$ , energy in the  $(\omega, k)$  space is spread across several  
 139 wavenumbers  $k$ ). In practice,  $\kappa$  and  $c$  provide estimates at  $x = (x_1 + x_2)/2$  of the dominant wavenumber (in  
 140 an energy-averaged sense) and the corresponding propagation speed respectively, as shown by the analysis  
 141 on synthetic data in Appendix A. This aspect is also discussed in Section 5. Cross-spectra were computed  
 142 using Welch's method and 63 Hann-windowed records of 128 seconds, which were overlapping by 50%. This  
 143 resulted in each spectral estimate having approximately 70 equivalent degrees of freedom and a spectral  
 144 resolution of 0.008 Hz.

145 As observed by many authors in the past, the coherence spectra at high harmonics were found to be  
 146 quite sensitive to the spacing between the two adjacent wave gauges. This is especially true in the deepest  
 147 parts of the wave flume, where energy levels at these frequencies are quite low and the spacing between  
 148 the two gauges can represent several wavelengths. An example is provided in Fig. 3b, which shows the  
 149 coherence spectra computed over the flat section of the flume during A2 with  $\Delta x = 0.93L_p$ ,  $L_p$  being the  
 150 peak wavelength given by linear wave theory. The coherence remains very high ( $\text{coh} > 0.95$ ) at frequencies  
 151 between 0.6 and 1.5  $f_p$ , which explain over 86% of the short-wave variance at this location. However, valleys  
 152 in the coherence can be observed around 2.5  $f_p$  for this particular spacing configuration and the coherence  
 153 weakens quickly after 2.8  $f_p$  (Fig. 3b). Since using a different spacing slightly shifts the coherence 'valleys',  
 154 several combinations of wave gauges and spacing were used to obtain spectral estimates at a single location

155 with coherence higher than typically 0.5 at all frequencies (similar idea as that used by Herbers et al.,  
 156 2002, with their field observations). After the removal of data associated with a coherence inferior to 0.5  
 157 ( $\text{coh}^2 \gtrsim 0.25$  in Fig. 3b), spectral estimates were ensemble-averaged. When non-linearities are strong,  
 158 such as near the breaking point or in the surf zone, the coherence remains high up to  $4-5 f_p$ , as long as  
 159  $\Delta x$  is taken sufficiently small (typically  $0.2-0.4 L_p$  in those regions). In such cases the ensemble-averaging  
 160 procedure is not really necessary but it was used all along the flume for consistency in the analysis.

### 161 3.2. Computation of bispectra

162 The power bispectrum of discretely sampled data corresponds to a representation in the frequency  
 163 domain of its third-order cumulant or moment. As it provides information on the strength of the phase  
 164 coupling between triads of frequencies  $f_1$ ,  $f_2$  and  $f_1 + f_2$ , the bispectrum is a useful and powerful tool for  
 165 studying non-linear phenomena in ocean waves (Hasselmann et al., 1963; Elgar and Guza, 1985a). Here,  
 166 we use the definition given by Kim and Powers (1979):

$$B(f_1, f_2) = \mathcal{E} [A(f_1) A(f_2) A^*(f_1 + f_2)], \quad (7)$$

167 where  $A(f)$  are the complex Fourier coefficients and  $\mathcal{E}$  is an expected, or ensemble-average, value. Similar  
 168 to the cross-spectrum phase, the biphase is obtained from the bispectrum as

$$\beta(f_1, f_2) = \arctan \left[ \frac{\text{Im}\{B(f_1, f_2)\}}{\text{Re}\{B(f_1, f_2)\}} \right]. \quad (8)$$

169 It is particularly insightful to recast the biphase as a function of the Fourier coefficients phases  $\theta(f)$ :  
 170  $\beta(f_1, f_2) = \theta(f_1) + \theta(f_2) - \theta(f_1 + f_2)$  (*e.g.*, see Kim et al., 1980; Elgar and Guza, 1985a). The amount of  
 171 energy transfer between near-resonant components depends on their relative phases, whose information is  
 172 contained in the biphase (Hasselmann et al., 1963; Kim et al., 1980).

173 Bispectra were computed on the free surface elevation signals down-sampled to 16 Hz by averaging  
 174 estimates from 126 Hann-windowed records of 128 seconds, which were overlapping by 75%. Statistical  
 175 stability was increased by merging estimates over three frequencies (*e.g.*, see Elgar and Guza, 1985a) yielding  
 176 approximately 205 equivalent degrees of freedom, and a spectral resolution of 0.023 Hz. A validation of the  
 177 bispectra computation is provided in Appendix B, which compares surface elevation third-order moments  
 178 computed across the whole wave flume via bispectral (Elgar and Guza, 1985a; Elgar, 1987) and statistical  
 179 definitions (Eq. 1 and 2).

### 180 3.3. Non-linear energy transfers

181 In the present study, we are principally interested in the growth of high harmonics due to non-linear  
 182 energy exchanges between triads and the effect of these forced components on the dispersive properties of  
 183 shoaling waves. To quantify the relative importance of forced energy at high harmonics, we represent these  
 184 exchanges with a Boussinesq approximation in a spectral energy balance equation, following a similar ap-  
 185 proach as developed in Herbers et al. (2000). Such Boussinesq approximation has already demonstrated its  
 186 potential to predict, in a very computationally efficient manner, the cross-shore evolution of spectral com-  
 187 ponents in the nearshore (Herbers and Burton, 1997; Norheim et al., 1998; Herbers et al., 2000; de Bakker  
 188 et al., 2015; Padilla and Alsina, 2017). Note that the Boussinesq approximation (detailed below) restricts  
 189 the present modelling approach to A2 and A3 since conditions during A1 are too dispersive.

190 For unidirectional waves propagating shoreward on an alongshore-uniform beach and assuming a weak  
 191 reflection at the shoreline, a balance between the cross-shore gradient of the energy flux spectrum  $F(f)$ ,  
 192 a source term  $S_{nl}(f)$  quantifying the non-linear energy exchanges between triads, and a dissipation term  
 193  $S_{dis}(f)$  reads (*e.g.*, Eq. 1 of Herbers et al., 2000)

$$\frac{\partial F(f)}{\partial x} = S_{nl}(f) + S_{dis}(f). \quad (9)$$

194 Breaking processes are ignored so that the dissipation term  $S_{dis}$  reduces to the energy losses by bottom  
 195 friction  $S_{fr}$ , here simply modelled after Thornton and Guza (1983):

$$S_{fr}(f) = \rho c_f \frac{1}{6\pi} \left( \frac{2\pi f}{\sinh |k|h} H \right)^3, \quad (10)$$

196 where  $h$  is the mean water depth and  $|k|$  is the wavenumber modulus obtained from the linear wave  
 197 dispersion relation (hereafter denoted  $\kappa_L$ , see Section 4). The friction coefficient  $c_f$  was set to 0.0055 after  
 198 calibration in the deepest section of the wave flume.

199 Assuming that the wave field is weakly non-linear, weakly dispersive, and that these effects are of similar  
 200 order ( $U_r \sim O(1)$ , *i.e.* Boussinesq regime), the non-linear source term  $S_{nl}$  can be approximated with an  
 201 integral of the bispectrum as follows (Herbers and Burton, 1997; Norheim et al., 1998; Herbers et al., 2000):

$$S_{nl}(f) = \frac{3\pi f}{h} \int_{-\infty}^{\infty} \text{Im}\{B^*(f', f - f')\} df'. \quad (11)$$

202 Eq. 11 differs from the expression of Herbers et al. (2000) (their Eq. 2) in several points: the conjugate  
 203 of  $B$  is taken in order to be consistent with their definition of the bispectrum (conjugate of the present  
 204 definition);  $S_{nl}$  is here defined as a function of  $f$  (not  $\omega$ ) and the definition with the full integral is kept  
 205 (symmetric properties of the bispectrum are not used to decompose it). Previous studies proposed evolution  
 206 equations for the energy and bispectra (*e.g.*, the stochastic model of Herbers and Burton, 1997) to simulate  
 207 the propagation and transformation of directional seas. As in Herbers et al. (2000), we take advantage of  
 208 the spatial resolution of the present dataset and directly evaluate  $S_{nl}$  and  $S_{fr}$  at each available cross-shore  
 209 location using trapezoidal rules for approximating the integrals. Besides removing uncertainties associated  
 210 with its cross-shore prediction, using measured bispectra has for advantage to relax the limitations of the  
 211 stochastic Boussinesq model of Herbers and Burton (1997) (such as the distance over which bispectra can  
 212 be propagated, see Freilich et al., 1984). In order to be consistent with the original equation derived by  
 213 Herbers and Burton (1997, their Eq. 22a), the energy in Eq. 9 is assumed to propagate at the shallow  
 214 water wave speed so that  $F(f) = \rho g E(f) \sqrt{gh}$ , with  $\rho$  the water density and  $g$  the acceleration of gravity.  
 215 After these considerations, Eq. 9 simplifies to

$$\frac{\partial E(f) \sqrt{gh(x)}}{\partial x} = \frac{1}{\rho g} (S_{nl}(f) + S_{fr}(f)). \quad (12)$$

216 Integrating this equation (in space) between the location of the first gauge  $x_0$  and any location  $x_1$  prior  
 217 to the mean breaking point location yields the following expression for the energy density spectrum at  $x_1$   
 218  $E(x_1, f)$ :

$$E(x_1, f) = E(x_0, f) \frac{\sqrt{gh(x_0)}}{\sqrt{gh(x_1)}} + \frac{1}{\rho g \sqrt{gh(x_1)}} \int_{x_0}^{x_1} (S_{nl} + S_{fr}) dx. \quad (13)$$



219 Finally, the ratio  $\int_{x_0}^{x_1} S_{nl} dx / F(x_1)$ , which represents the energy flux received (or lost) via non-linear  
 220 coupling between  $x_0$  and  $x_1$  over the total energy flux at  $x_1$ , is used in the following as an approximation  
 221 of the relative amount of forced energy at  $x_1$ . Outside the surf zone, where dissipative processes dominate,  
 222 this estimation was found to be more reliable than the bicoherence, which is often used as a proxy for such  
 223 an estimation but lacks general consensus upon its definition, see Appendix C for more details.

#### 224 4. Wavenumber and phase velocity spectra of shoaling and surf zone waves

225 In this section, we present and describe the main results from the cross-spectral and bispectral analyses.  
 226 The computation of dominant wavenumber  $\kappa$  and phase velocity  $c$  spectra for varying degrees of non-  
 227 linearity resulted in the identification of four different regimes of propagation, which broadly consist of: a  
 228 linear regime (stage I), a shoaling regime relatively far from the mean breaking point (stage II), a shoaling  
 229 regime near the mean breaking point (stage III) and a surf zone regime (stage IV). The Ursell number  $U_r$   
 230 (Fig. 2e) was used as a reference to define these regimes of propagation, which are characterized by similar  
 231 patterns - between all wave tests considered here - in wavenumber and phase velocity spectra. For all  
 232 stages,  $\kappa(f)$  and  $c(f)$  spectra are compared to predictions from the linear wave dispersion relation, which  
 233 links the spatial and temporal information of a linear wave field:

$$\omega^2 = g|k| \tanh(|k|h). \quad (14)$$

234 In the following, the subscript 'L' is used throughout the manuscript to refer to  $\kappa$  or  $c$  values that are solution  
 235 to Eq. 14 (*i.e.*  $\kappa_L$  and  $c_L = \omega/\kappa_L$ ). For conciseness, the propagation as linear waves is discussed using  
 236 the results from the most non-linear case (A2), already shown in Fig. 3. The other stages of propagation  
 237 focus on the differences between broad and narrow spectra, *i.e.* between A2 and A3. Note that  $\kappa$  and  $c$   
 238 characteristics during A1 are very similar to those obtained during A2 for similar  $U_r$  numbers. Finally,  
 239 given the relatively high spatial resolution, frequency-wavenumber power spectra  $P(\omega, k)$  (*e.g.*, see Redor  
 240 et al., 2019) were computed using 2D Fourier analysis at four different cross-shore sections corresponding  
 241 to stages I, II, III and IV, by averaging over 63 Hann-windowed time-records of 128 seconds overlapping by  
 242 50% (Fig. 4). The chosen length of the cross-shore section (width of the data window for computing the  
 243 2D Fast-Fourier transform) is a best compromise for reaching a proper resolution in wavenumber  $k$  and still  
 244 assuming little change in wave type over the section. The 2D Fourier analysis shown in Fig. 4 provides a  
 245 qualitative information on energy spreading in  $(\omega, k)$  space and is particularly useful to illustrate what the  
 246 single-valued dominant wavenumber represents in a mixed sea-state. Fig. 4 will be used in the present and  
 247 following sections to analyse the link between non-linear energy exchanges and the dispersive properties of  
 248 the wave field.

##### 249 4.1. Propagation as free, linear waves (I)

250 Fig. 3 shows for the most non-linear case A2 an almost perfect match for  $f > 0.6 f_p$  between the  
 251 measured wave phases, wavenumbers and phase velocity spectral estimates with the predictions from the  
 252 linear wave dispersion relation (Fig. 3a, 3c and 3d respectively). The decrease of coherence at high  
 253 frequencies (Fig. 3b) can be explained by the relatively low energy content in the spectrum tail ( $> 86\%$  of  
 254 the variance is contained between  $0.6 f_p$  and  $1.5 f_p$  during A2). It also provides an explanation for the slight

255 deviations of measured  $\kappa$  and  $c$  from predictions by the linear wave dispersion relation (Fig. 3c-d). However,  
 256 the 2D Fourier analysis performed over the flat section of the flume (Fig. 4a-b) reveals the presence of  
 257 energy along the dispersion relation for bound high-frequency components (*e.g.*, see at  $f > 2.5f_p$  for A2  
 258 and around  $2f_p$  for A3), suggesting that energy transfers towards high harmonics already occur in this  
 259 region. Nonetheless, the energy is mainly distributed along the linear dispersion relation, which indicates  
 260 that short-wave components were mostly propagating as free waves in the deepest regions of the wave flume  
 261 ( $U_r \sim O(0.01 - 0.1)$ ).

262 Data points in the infragravity frequency band ( $f < 0.6f_p$ ) correspond to wave components generated  
 263 via difference interactions and that are bound to short-wave groups. As such, bound infragravity waves  
 264 propagate at a speed close to the corresponding group velocity, which is lower than their intrinsic phase  
 265 velocity. This is confirmed in Fig. 3d with a good correspondence between measured  $c$  values with short-  
 266 wave groups propagation speed ( $c_{env}$ , shown as the orange line), estimated via cross-spectral analysis of  
 267 short-wave groups envelop. The latter was computed following Janssen et al. (2003), as the absolute value  
 268 of the analytical signal of  $\zeta$ :  $|\zeta + i\mathcal{H}\{\zeta\}|$  (no low-pass filtering is needed for the present application). Note  
 269 that the discrepancies observed at  $f \in [0.3f_p; 0.6f_p]$  can be explained by the lower levels of energy at these  
 270 frequencies:  $E(f) \sim O(10^{-5}) \text{ m}^2/\text{Hz}$  as opposed to  $E(f) \sim O(10^{-4}) \text{ m}^2/\text{Hz}$  around  $f = 0.1f_p$ .

#### 271 4.2. Shoaling of weakly non-linear waves (II)

272 At higher Ursell number ( $U_r \sim 0.3$ ), wave non-linearities become significant and non-linear energy  
 273 transfers to high harmonics promote the dominance of bound high harmonics (Fig. 4c-d). This is observed  
 274 in the considerable deviation of dominant wavenumbers and phase velocity spectral estimates from the  
 275 predictions of the linear dispersion relation for  $f > 1.5f_p$  (see Fig. 5c-f). During A3, the phase velocity  
 276 values at  $2f_p$  and  $3f_p$  are equal to that of the peak frequency (difference  $< 1\%$ ,  $c(f_p) = 1.95 \text{ m/s}$ ), while  
 277 during A2 they lie between the value at  $f_p$  and the predictions of the linear dispersion relation ( $\sim 7\%$   
 278 difference from  $c(f_p) = 2.18 \text{ m/s}$ ). At the second harmonic  $2f_p$ , most of the non-linear energy transfers  
 279 occur via sum interactions of components around  $f_p$  (self-self interactions) as evidenced by the positive  
 280 imaginary part of  $B^*$  around  $(f_p, f_p)$ , see Fig. 5g and 5h for tests A2 and A3 respectively. Although less  
 281 evident in the bispectra, the normalised bispectra shown in Fig. C1 indicate that strong coupling involving  
 282  $f_p$  and higher harmonics also exist in the shoaling region. These are particularly strong around  $(2f_p, f_p)$   
 283 for A3, which explains the significant growth of the third harmonic  $3f_p$  observed for that test (Fig. 5b).  
 284 Consistent with field observations (*e.g.* Elgar and Guza, 1985a; Norheim et al., 1998), relatively strong non-  
 285 linear exchanges by sum interactions are observed during A2, despite the broader spectrum conditions (note  
 286 the scale difference between the A2 and A3 cases). Short-wave frequencies distant from  $f_p$  (*e.g.*  $\sim 1.5f_p$ )  
 287 show a strong coupling with  $f_p$ , which is the consequence of a broader spectrum for A2 compared to A3.  
 288 A striking result in this regime of propagation is the fact that, despite non-linearities becoming important  
 289 ( $\epsilon \sim 0.12 - 0.2$ ,  $S_k \sim 1$ ), the principal wave components follow the linear wave dispersion relation. The  
 290 spatial structure of the wave field for  $f \in [0.6f_p; 1.5f_p]$  is therefore well-described by the linear wave theory  
 291 (Fig. 5c and 5d). However, this is not the case for  $f > 1.5f_p$ , where the overestimation of wavenumbers by  
 292 the linear dispersion relation increases with  $f$  (up to a factor 2.5 at  $3.5f_p$ ). As the biphase at both  $(f_p, f_p)$   
 293 and  $(f_p, 2f_p)$  are close to 0, this regime of propagation is consistent with Stokes-like non-linearities (*e.g.*,

294 see Elgar and Guza, 1985a).

295 In the infragravity band, wave components are still bound to short-wave groups as indicated by the  
296 good match between spectral estimates of phase velocity and the propagation speed of short-wave groups.  
297 Energy transfers towards the infragravity band concentrate at frequencies around  $0.1-0.2f_p$  (Fig. 5a and  
298 5b) and principally originate from strong difference interactions which transfer energy from  $f_p$  towards that  
299 infragravity frequency and components at frequency slightly lower than  $f_p$  (see the negative imaginary part  
300 of  $B^*$  along the  $f_p$  anti-diagonal, Fig. 5g and 5h).

#### 301 4.3. Shoaling of non-linear waves in the vicinity of the breaking point (III)

302 The next regime analysed in Fig. 6 corresponds to non-linear waves approaching the mean breaking  
303 point location ( $U_r \sim 0.7$ ). At this Ursell number, wavenumber and phase velocity spectra become less  
304 frequency-dependent at high frequencies. This can be explained by the fact that most of the energy at  
305 those frequencies is bound to primary components in the range  $[0.6f_p; 1.5f_p]$ , and that  $c$  values display  
306 little variations at these frequencies ( $c/c_L(f_p)$  is within 5% of  $c(f_p)/c_L(f_p)$ ). As a result, wavenumbers at  
307 high frequencies are relatively well described by a simple dispersion relation given by  $\kappa(f) = 2\pi f/c(f_p)$   
308 (Fig. 6c-d). Note that at these depths ( $\kappa_p h \sim 0.57$  and  $0.47$  for A2 and A3 respectively), this approximately  
309 corresponds to the shallow water dispersion relation  $\kappa_{sw}(f) = 2\pi f/\sqrt{gh}$ , which explains why most of the  
310 energy is spread around this dispersion relation in Fig 4e-f. Non-linear amplitude effects become increasingly  
311 important ( $\epsilon = 0.25$  and  $S_k = 1.4-1.5$ ) and deviations between  $c(f_p)$  and predictions by the linear dispersion  
312 relation are observed (Fig. 6e-f). As noted and observed by Herbers et al. (2002), this is the result of non-  
313 linear interactions, which alter the wavenumber of all three wave components involved. Indeed, bound high  
314 harmonics contribute to an increase in skewness and height of principal wave crests, which has for effect  
315 to enhance their propagation speed. Variations in the strength of non-linear interactions thus explain the  
316 varying magnitude of the deviations of  $c(f_p)$  from predictions by the linear dispersion relation as observed  
317 in both field and laboratory settings (Thornton and Guza, 1982; Herbers et al., 2002; Tissier et al., 2011).  
318 For this range of Ursell number, overestimations of  $\kappa$  by the linear dispersion relation remain close to a  
319 factor 2 at  $f = 3.5f_p$ .

320 In these regions of the wave flume, surface elevation bispectra are still dominated by real values (note the  
321 x2 in imaginary values, Fig. 6g-h), typical of (horizontally) skewed and nearly symmetric waves ( $A_s \sim 0$ , see  
322 Fig. 2c-d and also Masuda and Kuo, 1981; Elgar and Guza, 1985a). The strong coupling between first and  
323 second harmonics, which for instance explains the energy peak at  $3f_p$  in A3 (Fig. 6b) is here evident in the  
324 imaginary part of the bispectra at  $(f_p, 2f_p)$  (see also the bicoherence in Fig. C1). Infragravity frequencies  
325 between  $0.1-0.2f_p$  keep receiving energy via relatively strong difference interactions. As a consequence,  
326 prior to short-wave breaking infragravity waves are still bound to short-wave groups during both tests (see  
327 Fig. 6e-f, and also de Bakker et al., 2015).

#### 328 4.4. Surf zone waves (IV)

329 In the surf zone ( $U_r$  taken at 3.2), wavenumber and phase velocity spectra are frequency-independent  
330 (Fig. 7e-f). The large differences observed between the measured phase velocity at peak frequency and that  
331 predicted with the linear dispersion relation demonstrate the dominance of amplitude effects over dispersive

332 ones (Herbers et al., 2002). All components travel at a speed slightly larger than that of shallow water  
 333 waves ( $\sim 10 - 20\%$  depending on the wave test), which is consistent with many past field and laboratory  
 334 observations of wave dispersion in the surf zone (Inman et al., 1971; Svendsen et al., 1978; Stive, 1980;  
 335 Thornton and Guza, 1982, and many others). For both A2 and A3 tests, the relation  $c = \sqrt{gh(1 + \epsilon)}$  is  
 336 shown in Fig. 7e-f (see also Fig 4g-h, for the corresponding frequency-wavenumber surface elevation power  
 337 spectra). This formula is asymptotic in shallow water to the non-linear dispersion relation given by Hedges  
 338 (1976) with the adjustment of Booij (1981) (see also Catalán and Haller, 2008, for an assessment of these  
 339 relations for depth-inversion applications). In the inner surf zone, where  $\epsilon \sim 0.3$  during all tests (Fig. 2b),  
 340 this yields  $c \sim 1.14\sqrt{gh}$ , a relation also found in the field by Tissier et al. (2011). This relation appears  
 341 as an upper bound, in an average sense (*e.g.* over several wave groups), for the propagation speed of wave  
 342 components in the inner surf zone.

343 In the short-wave frequency range, bispectra display relatively weak and nearly equal real and imaginary  
 344 parts, characteristics of asymmetric (pitched forward) broken waves, with biphases approaching  $-\pi/2$  (Elgar  
 345 and Guza, 1985a; Doering and Bowen, 1987): for instance during A3,  $\beta(f_p, f_p) \sim \beta(f_p, 2f_p) \sim -\pi/3$ . Since  
 346 these values were still close to 0 during stage III, this indicates that bound high-harmonic components (here  
 347 only shown for  $2f_p$  and  $3f_p$ ) slowly drift out of phase from the principal component starting from the mean  
 348 breaking point location. This is in contrast with short-wave group-forced bound infragravity waves that  
 349 slowly drift out of phase with the short-wave envelope in the shoaling region (*e.g.*, see Elgar and Guza,  
 350 1985a). Since this process occurs over long distances, the differences in phase velocities between  $f_p$ ,  $2f_p$   
 351 and  $3f_p$  are not sufficiently large to be observed in the present dataset. Finally, in the inner surf zone,  
 352 intense and complex energy transfers occur within the infragravity band and also between infragravity and  
 353 short-wave components. For more information on these processes during the GLOBEX experiments, the  
 354 reader is referred to the study of de Bakker et al. (2015).

## 355 5. Role of non-linear energy transfers on $\kappa$ and $c$

356 Most differences observed in wavenumber and phase velocity spectra between broad and narrow-banded  
 357 wave conditions concentrate in the shoaling region (stages II and III). In stage II,  $c$  values at  $2f_p$  and  $3f_p$   
 358 during A2 lie between  $c(f_p)$  and the values predicted by linear wave theory while  $c(f_p)$ ,  $c(2f_p)$  and  $c(3f_p)$  are  
 359 all equal during A3 (Fig. 5e-f). As these differences between broad versus narrow-band spectra are likely  
 360 explained by the relative importance of forced energy at those frequencies (see the cross-spectral analysis  
 361 performed on synthetic data and discussed in Appendix A and also Herbers and Guza, 1992; Herbers et al.,  
 362 2000), we analyse non-linear energy transfers to  $2f_p$  and  $3f_p$  in more details here.

363 Wave amplitudes at  $f_p$ ,  $2f_p$  and  $3f_p$  ( $a_{f_p}$ ,  $a_{2f_p}$  and  $a_{3f_p}$  respectively) computed from energy spectra  
 364 modelled with Eq. 13 are compared against observations in Fig. 8. Overall, the Boussinesq approach  
 365 of Herbers and Burton (1997) for the non-linear energy transfers between triads accurately predicts the  
 366 growth of second and third harmonics across the shoaling zone for both broad and narrow-band wave tests.  
 367 The cross-shore evolution of  $a_{f_p}$ ,  $a_{2f_p}$  and  $a_{3f_p}$  are very well described up to the mean breaking point, with  
 368 mean absolute percentage errors (MAPE) lower than 5% and 13% for  $a_{2f_p}$  and  $a_{3f_p}$  respectively. With  
 369 respect to the amount of (free) energy imposed at the paddle, the narrow-banded conditions during A3

370 promote more efficient energy transfers, with growths of  $a_{2f_p}$  and  $a_{3f_p}$  by a factor 3 and 10 respectively,  
 371 while these factors are only between 1.5 and 2 during A2.

372 A closer look at the cross-shore evolution of the source terms (Fig. 9) shows that friction effects are  
 373 negligible at high frequencies but not around the peak frequency where the energy dissipated through  
 374 bottom friction is of similar order than the energy lost via non-linear coupling. During A3 (Fig. 9b), the  
 375 steady increase of  $S_{nl}(2f_p)$  indicates a gradual growth of  $a_{2f_p}$  in the shoaling region (see also Fig. 8b). This  
 376 growth is most intense right before the mean breaking point location, around the location corresponding  
 377 to stage III. Computations of the ratio  $\int_{x_0}^{x_1} S_{nl} dx / F(x_1)$  at the location  $x_1$  corresponding to stage II (resp.  
 378 III) suggest that approximately 70% (resp. 80%) of the energy at  $2f_p$  is forced during A3. At  $3f_p$ , these  
 379 estimations range between 90% and 100% during stage II and III respectively. Although the resolution in  
 380 wavenumbers does not allow a clear separation of forced and free energy at  $2f_p$ , the frequency-wavenumber  
 381 power spectra shown in Fig. 4d (II) and 4f (III) corroborate these numbers and clearly indicate that most of  
 382 the energy at  $3f_p$  is forced. In contrast, non-linear energy exchanges between triads display a more complex  
 383 picture for the broader conditions of A2.  $S_{nl}(2f_p)$  oscillates around 0 so that when spatially integrated,  
 384 non-linear energy exchanges explain only 20% (30%) and 30% (50%) of the total energy at  $2f_p$  and  $3f_p$   
 385 respectively during stage II (III). These numbers are quite consistent with the bicoherence values shown  
 386 in Appendix C. For both wave tests,  $b^2$  values are found to vary little between stage II and III: for A2,  
 387  $b^2(f_p, f_p) \sim 0.3$  and  $b^2(f_p, 2f_p) \sim 0.2$ , while these values oscillate between 0.55 and 0.65 for A3.  $b^2(f_p, f_p)$   
 388 and  $b^2(f_p, 2f_p)$  can be considered as crude estimates of the relative amount of forced energy at  $2f_p$  and  $3f_p$   
 389 respectively (Kim and Powers, 1979).

390 The effect of varying relative amounts of forced energy not only explain the differences observed in  $\kappa$  and  
 391  $c$  spectra at high frequencies between A2 and A3 but also their variation across frequencies. During A3 for  
 392 instance, non-linear energy transfers in the short-wave frequency band were predominantly towards  $2f_p$  and  
 393  $3f_p$  (Fig. 5h and 6h). At these harmonics, energy is predominantly forced ( $> 70\%$  around  $2f_p$  and  $> 90\%$   
 394 around  $3f_p$ ), thus, the dominant  $\kappa$  and corresponding  $c$  values are directly related to the values at the peak  
 395 frequency. In contrast, frequencies located in valleys between harmonics (*i.e.*  $1.5f_p$  and  $2.5f_p$ ) receive very  
 396 little energy through the coupling of triads. This eventually leads to the patterns in  $c$  observed in Fig. 5f,  
 397 which are very similar to those obtained by Crawford et al. (1981) with their model for modulated wave  
 398 trains. During A2, lower relative amounts of forced energy are found at high harmonics and dominant  
 399 wavenumbers lie between  $\kappa(f_p)$  and the intrinsic value at that frequency predicted by the linear dispersion  
 400 relation. Furthermore, due to the broader spectrum conditions, this forced energy is more spread across  
 401 frequencies, which explains the weaker variations of  $\kappa$  and  $c$  across frequencies as compared to A3 (*e.g.*, see  
 402 Fig. 5e-f).

403 The present study considers unidirectional wave fields transforming across a mildly sloping beach. A  
 404 typical field situation where incident wave spectra exhibit varying degrees of directional spreading is unclear  
 405 as far as non-linear energy transfers are concerned. According to Boussinesq theory (*e.g.*, Herbers and  
 406 Burton, 1997), directional spreading has only a weak influence on the efficiency of non-linear energy transfers  
 407 by sum interactions, as opposed to difference interactions. For instance, these authors predicted a weaker  
 408 growth of high-harmonic bound wave by 10-20% for very large directional spreads ( $60^\circ$ ). For more realistic

409 spreading angles in the nearshore, de Wit et al. (2020) also recently noted variations of approximately 10%.  
 410 Although small, such a decrease in efficiency of non-linear energy transfer towards high harmonics due to  
 411 an increasing directional spreading might lead to lower relative amounts of forced energy, subsequently  
 412 leading to slightly less important deviations of  $\kappa$  and  $c$  spectral estimates from predictions by the linear  
 413 wave dispersion relation (see also Appendix A). The larger spectral bandwidth generally associated with  
 414 wave spectra exhibiting a large directional spreading is, however, likely to be the principal reason explaining  
 415 weaker couplings between triads and thus weaker relative amounts of forced energy at high harmonics.

## 416 6. Discussion and concluding remarks

417 Cross-spectral and bispectral analyses were employed on a highly-resolved surface elevation dataset  
 418 to study the dispersive properties of waves shoaling and breaking over a mildly sloping beach. For all  
 419 wave tests considered here, four regimes of propagation (I to IV) with specific characteristics in dominant  
 420 wavenumber and phase velocity spectra could be defined using a local Ursell number. Stage II ( $U_r \sim 0.3$ ) is  
 421 particularly interesting as it simultaneously shows significant non-linear effects at high harmonics (typically  
 422  $f > 1.5f_p$ ), which are evidenced by the large deviations of  $\kappa$  and  $c$  spectra from predictions by the linear  
 423 wave dispersion relation, and a clearly linear spatial field for the primary components. Although less  
 424 energy was imposed during A3 as compared to A2, the narrow-banded conditions promoted relatively more  
 425 important non-linear energy transfers towards high harmonics such that phase velocity at  $2f_p$  and  $3f_p$  were  
 426 found equal to those at the peak frequency. For A2, with a broader spectrum imposed, forced energy at  
 427 high frequencies was found in relatively smaller proportion and  $c$  spectral estimates lied between values at  
 428 the peak frequency and that given by the linear dispersion relation. Closer to the mean breaking point  
 429 (stage III), non-linear energy transfers were found more intense, which, together with the less dispersive  
 430 conditions compared to stage II explain that  $\kappa$  and  $c$  spectra become less frequency dependent. In the surf  
 431 zone (stage IV), wavenumber and phase velocity spectra essentially showed the same characteristics for all  
 432 tests and a modified shallow water dispersion that accounts for non-linearity ( $c = \sqrt{gh(1 + \epsilon)}$ ) accurately  
 433 describes the dispersive properties of the wave field. Some of the results presented here for shoaling and  
 434 breaking waves confirm previous findings obtained in the field (Thornton and Guza, 1982; Elgar and Guza,  
 435 1985b; Herbers et al., 2002, among others), yet the unprecedented level of accuracy obtained with the  
 436 GLOBEX dataset allowed to highlight and quantify the effect of varying levels of forced energy at high  
 437 harmonics on the dispersive properties of the wave field.

438 The detailed analysis of the different regimes of propagation highlighted here indirectly provides an  
 439 assessment of the validity of the linear wave dispersion relation at several locations in the nearshore and for  
 440 a range of wave conditions. Two specific regions in the short-wave frequency band can be defined: primary  
 441 components ( $f \in [0.6f_p, 1.5f_p]$ ) and high harmonics ( $f > 1.5f_p$ ). For the primary components, deviations  
 442 from the linear wave dispersion relation are due to non-linear amplitude effects and concentrate in a region  
 443 near the mean breaking point and in the surf zone. These effects can cause deviations of  $\kappa$  and  $c$  from  
 444 predictions by the linear wave dispersion of the order of 10-20%. At high frequencies, deviations are much  
 445 larger ( $O(100\%)$ ) and also occur much farther seaward. For instance, stage II corresponds to  $\kappa_p h = 0.71$ ,  
 446 0.70 and 0.62 for wave tests A1, A2 and A3 respectively. The fact that such deviations from predictions

447 by the linear dispersion relation are found in  $\kappa$  and  $c$  spectra so far seaward of the surf zone for different  
448 wave conditions is expected to have strong implications for a wide range of applications, two of which are  
449 briefly discussed below.

450 Depth-inversion algorithms such as *cBathy* (Holman et al., 2013) estimate the water depth by applying  
451 the linear wave dispersion relation to the most coherent pairs of wave frequency and wavenumber extracted  
452 from timeseries of video images (Stockdon and Holman, 2000; Plant et al., 2008). However, due to the  
453 working principles of video cameras, video data can be more coherent at high harmonics and frequencies  
454 greater than  $1.5 f_p$  are often picked to invert the water depth (Stockdon and Holman, 2000; Bergsma and  
455 Almar, 2018). In such cases, large deviations of the dominant wave phase velocity from linear wave theory  
456 are expected, which likely explains why Brodie et al. (2018) noted an overestimation in the detected phase  
457 velocities by 20-100% up to 50 m seaward of the mean breaking point location. In the absence of knowledge  
458 on the relative amounts of forced energy at high harmonics, pairs of frequencies and wavenumbers around  
459 the peak frequency should be preferred in regions where non-linearities are expected. Similarly, the large  
460 overestimation of  $\kappa$  by the linear dispersion relation causes the widely reported blow-up at high harmonics  
461 when correcting sub-surface pressure signals for depth attenuation (Bonneton and Lannes, 2017; Mouragues  
462 et al., 2019; Martins et al., 2020b). The use of a cutoff frequency prevents for instance the accurate  
463 description of third-order parameters and wave height distributions (*e.g.*, see Martins et al., 2020a). In  
464 shallow water ( $\mu \lesssim 0.3$ ), a weakly dispersive formula recently developed allows for an accurate correction of  
465 the energy levels at high harmonics both seaward of (Bonneton et al., 2018; Mouragues et al., 2019) and in  
466 the surf zone (Martins et al., 2020b). However in deeper water, a non-linear fully dispersive reconstruction  
467 requires knowledge on the spatial structure of the wave field, which is generally lacking.

## 468 **Acknowledgments**

469 Kévin Martins greatly acknowledges the financial support from the University of Bordeaux, through an  
470 International Postdoctoral Grant (I dex, nb. 1024R-5030). The GLOBEX project was supported by the  
471 European Community’s Seventh Framework Programme through the Hydralab IV project, EC Contract  
472 261520. The GLOBEX data used in this research can be accessed on Zenodo at <https://zenodo.org/record/4009405>  
473 or from the authors, and can be used under the Creative Commons Attribution 4.0 International licence.  
474 The bispectral analysis tools developed and used in this study are accessible from the first author github  
475 repository at <https://github.com/ke-martins/bispectral-analysis>. Fruitful exchanges with Steve Elgar re-  
476 garding the bispectral analysis were greatly appreciated. The authors thank two anonymous reviewers for  
477 their helpful comments and suggestions.

## 478 **Appendix A. Sensitivity analysis of the cross-spectral technique to the presence of forced** 479 **energy**

480 The situation where both free and forced components exist in a wave field is not entirely clear as far  
481 as the cross-spectral analysis is concerned. Some studies consider the cross-spectral estimates to be biased  
482 towards bound harmonics (*e.g.*, see Lake and Yuen, 1978; Thornton and Guza, 1982), arguing that the  
483 travelling distance between two gauges is reduced for bound components compared to free ones. This is

484 evidently exacerbated when using sub-surface pressure sensors or current meters since bound harmonics  
 485 are much less attenuated and will dominate the spectrum at some depth.

486 To analyse the sensitivity of the cross-spectral analysis to different relative levels of forced energy, some  
 487 tests are performed here on synthetic data. Starting at a position  $x_1$ , a surface elevation timeseries following  
 488 a JONSWAP spectrum with similar parameters to the A3 test is generated (Fig. A1). A Gaussian-shaped  
 489 perturbation around  $2f_p$  is added to represent forced harmonics: for any  $f$  around  $2f_p$ , the forced harmonic  
 490 wavenumber is set at  $\kappa_{forced} = 2\kappa(f/2)$ , so that we have  $c_{forced}(f) = c(f/2)$ . For the phases of forced  
 491 harmonics, we choose to impose  $\theta_{forced}(f) = \theta(f/2)$  for simplicity. The maximum ratio of forced to free  
 492 harmonic amplitudes  $a_{2f_p, forced}/a_{2f_p, free}$  is set to 6, which corresponds to the maximum ratio of energy  
 493  $E(f_p)/E(2f_p)$  of approximately 0.4 that was observed in the flume outside the surf zone. Each wave  
 494 component is then propagated using its corresponding wavenumber to a second location  $x_2$ .

495 The results from the cross-spectral analysis performed on the synthetic timeseries at  $x_1$  and  $x_2$  are pro-  
 496 vided in Fig. A2 as the wavenumber and phase speed at  $2f_p$  shown as a function of  $a_{2f_p, forced}/a_{2f_p, free}$ .  
 497 In the absence of forced components ( $a_{2f_p, forced}/a_{2f_p, free} = 0$ ), components around  $2f_p$  follow the dis-  
 498 persion relation given by linear wave theory, *i.e.*  $\kappa(2f_p) = \kappa_L(2f_p)$  and  $c(2f_p) = c_L(2f_p)$ . Interestingly,  
 499 when forced and free components are found in equal proportion ( $a_{2f_p, forced}/a_{2f_p, free} = 1$ ), the relation  
 500  $\kappa(2f_p) \sim (\kappa_L(2f_p) + 2\kappa_L(f_p))/2$  is verified. For higher ratio of forced to free energy,  $\kappa(2f_p)$  and  $c(2f_p)$   
 501 values rapidly converge towards the values corresponding to the peak frequency. Overall, these results  
 502 suggest that the cross-spectral analysis is not biased towards forced components, but simply provides a  
 503 dominant wavenumber (or averaged in terms of energy) and its corresponding phase velocity.

## 504 **Appendix B. Validation of the bispectrum computations using the skewness and asymmetry** 505 **test**

506 The wave skewness and asymmetry are third-order moments characterizing the wave shape. These  
 507 parameters are directly related to the energy content at high frequencies of the spectra and bispectra  
 508 (Elgar, 1987). The fact that the statistical (see Eq. 1 and 2) and the bispectrum-based definitions are  
 509 theoretically equivalent provide a means to validate the bispectrum's calculations.

510 Due to its symmetry properties, the bispectrum can be uniquely defined in a single octant in the  
 511 frequency-space (Hasselmann et al., 1963; Elgar and Guza, 1985a). If we denote by  $f_N$  the Nyquist fre-  
 512 quency, the octant with positive frequencies is bounded by the vertices at  $(0,0)$ ,  $(f_{N/2}, f_{N/2})$  and  $(f_N, 0)$ .  
 513 The wave skewness and asymmetry are defined by the sum of the real and imaginary parts of the bispec-  
 514 trum over this octant respectively, normalized by the cube of the free surface elevation standard deviation  
 515 (Elgar, 1987; Elgar and Guza, 1985a). This reads:

$$S_k = \left[ 12 \sum_n \sum_l \operatorname{Re}\{B(f_n, f_l)\} + 6 \sum_n \operatorname{Re}\{B(f_n, f_n)\} \right] / (\overline{(\zeta - \bar{\zeta})^2})^{3/2} \quad (\text{B.1})$$

$$A_s = \left[ 12 \sum_n \sum_l \operatorname{Im}\{B(f_n, f_l)\} + 6 \sum_n \operatorname{Im}\{B(f_n, f_n)\} \right] / (\overline{(\zeta - \bar{\zeta})^2})^{3/2} \quad (\text{B.2})$$

516 with  $n > l$  and  $n + l < N$ .

517 The cross-comparison between statistical and bispectrum-based definitions of the wave skewness and  
 518 asymmetry computed for A3 is provided in Fig. B1 and shows a perfect match between both definitions,



519 thus validating the present computations of the bispectrum. Note that for these comparisons, blocks were  
 520 not tapered and rectangular windows were used for computing the Fast Fourier Transforms. Using any  
 521 other types of windows resulted in small differences between the two definitions of both third-order wave  
 522 parameters.

### 523 **Appendix C. Normalised bispectra (bicoherence)**

524 We generally seek a normalisation of the bispectrum so that it takes 1 as value when there is a full  
 525 coupling between the three components involved and 0 when there is none. In the case of surface gravity  
 526 waves, this would correspond to a situation with the only presence of forced or free energy respectively.  
 527 At present, there does not seem to be a consensus in the literature on what definition of the bicoherence  
 528 should be used to robustly quantify or at least characterize the relative dominance of forced energy (*e.g.*  
 529 Haubrich, 1965; Kim and Powers, 1979; Elgar and Guza, 1985a; Hinich and Wolinsky, 2005; de Bakker  
 530 et al., 2015, and many others). The normalisation proposed by Haubrich (1965) is frequently used for  
 531 studying non-linearities in the nearshore area but was shown to lead to values greater than 2 (*e.g.* Elgar  
 532 and Guza, 1985a). In the present study, this was also the case, but only when merging across frequencies  
 533 was performed beforehand. Nonetheless, a slightly different normalisation that was proposed in Hagihira  
 534 et al. (2001) is used in the present case as it seems the most appropriate normalisation (see their Appendix):

$$b(f_1, f_2) = \frac{|B(f_1, f_2)|}{\mathcal{E}[|A(f_1)| \cdot |A(f_2)| \cdot |A^*(f_1 + f_2)|]} \quad (\text{C.1})$$

535 As explained in Hagihira et al. (2001), the bicoherence is 1 only when phase angles of all triple products  
 536 are equal.

537 Squared bicoherence values  $b^2$  for both II and III regimes of propagation are shown in Fig. C1. As  
 538 frequency merging is inappropriate with this definition of the bicoherence (due to the expected value in the  
 539 denominator),  $b$  values were averaged over a 3x3 square in the frequency space.

### 540 **References**

- 541 de Bakker, A.T.M., Herbers, T.H.C., Smit, P.B., Tissier, M.F.S., Ruessink, B.G., 2015. Nonlinear  
 542 infragravity-wave interactions on a gently sloping laboratory beach. *Journal of Physical Oceanography*  
 543 45, 589–605. doi:10.1175/JPO-D-14-0186.1.
- 544 Bergsma, E.W.J., Almar, R., 2018. Video-based depth inversion techniques, a method comparison with  
 545 synthetic cases. *Coastal Engineering* 138, 199–209. doi:10.1016/j.coastaleng.2018.04.025.
- 546 Berni, C., Barthélemy, E., Michallet, H., 2013. Surf zone cross-shore boundary layer velocity asymmetry  
 547 and skewness: An experimental study on a mobile bed. *Journal of Geophysical Research: Oceans* 118,  
 548 2188–2200. doi:10.1002/jgrc.20125.
- 549 Bonneton, P., Bruneau, N., Castelle, B., Marche, F., 2010. Large-scale vorticity generation due to  
 550 dissipating waves in the surf zone. *Discrete & Continuous Dynamical Systems - B* 13, 729–738.  
 551 doi:10.3934/dcdsb.2010.13.729.

- 552 Bonneton, P., Lannes, D., 2017. Recovering water wave elevation from pressure measurements. *Journal of*  
553 *Fluid Mechanics* 833, 399–429. doi:10.1017/jfm.2017.666.
- 554 Bonneton, P., Lannes, D., Martins, K., Michallet, H., 2018. A nonlinear weakly dispersive method for  
555 recovering the elevation of irrotational surface waves from pressure measurements. *Coastal Engineering*  
556 138, 1–8. doi:10.1016/j.coastaleng.2018.04.005.
- 557 Booi, N., 1981. Gravity waves on water with non-uniform depth and current. Ph.D. thesis. Technische  
558 Hogeschool, Delft (Netherlands).
- 559 Brodie, K.L., Palmsten, M.L., Hesser, T.J., Dickhudt, P.J., Raubenheimer, B., Ladner, H., Elgar, S.,  
560 2018. Evaluation of video-based linear depth inversion performance and applications using altimeters  
561 and hydrographic surveys in a wide range of environmental conditions. *Coastal Engineering* 136, 147–  
562 160. doi:10.1016/j.coastaleng.2018.01.003.
- 563 Bühler, O., Jacobson, T.E., 2001. Wave-driven currents and vortex dynamics on barred beaches. *Journal*  
564 *of Fluid Mechanics* 449, 313–339. doi:10.1017/S0022112001006322.
- 565 Castelle, B., Scott, T., Brander, R.W., McCarroll, R.J., 2016. Rip current types, circulation and hazard.  
566 *Earth-Science Reviews* 163, 1–21. doi:10.1016/j.earscirev.2016.09.008.
- 567 Catalán, P.A., Haller, M.C., 2008. Remote sensing of breaking wave phase speeds with application to  
568 non-linear depth inversions. *Coastal Engineering* 55, 93–111. doi:10.1016/j.coastaleng.2007.09.010.
- 569 Clark, D.B., Elgar, S., Raubenheimer, B., 2012. Vorticity generation by short-crested wave breaking.  
570 *Geophysical Research Letters* 39. doi:10.1029/2012GL054034.
- 571 Coantic, M., Ramamonjjarisoa, A., Mestayer, P., Resch, F., Favre, A., 1981. Wind-water tunnel simulation  
572 of small-scale ocean-atmosphere interactions. *Journal of Geophysical Research: Oceans* 86, 6607–6626.  
573 doi:10.1029/JC086iC07p06607.
- 574 Crawford, D.R., Lake, B.M., Yuen, H.C., 1981. Effects of nonlinearity and spectral bandwidth on the  
575 dispersion relation and component phase speeds of surface gravity waves. *Journal of Fluid Mechanics*  
576 112, 1–32. doi:10.1017/S0022112081000281.
- 577 Doering, J.C., Bowen, A.J., 1986. Shoaling surface gravity waves: A bispectral analysis, in:  
578 *Proceedings of the 20th Conference on Coastal Engineering, Taipei, Taiwan*, pp. 150–162.  
579 doi:10.1061/9780872626003.012.
- 580 Doering, J.C., Bowen, A.J., 1987. Skewness in the nearshore zone: A comparison of estimates from marsh-  
581 mcbirney current meters and colocated pressure sensors. *Journal of Geophysical Research: Oceans* 92,  
582 13173–13183. doi:10.1029/JC092iC12p13173.
- 583 Doering, J.C., Bowen, A.J., 1995. Parametrization of orbital velocity asymmetries of shoaling and breaking  
584 waves using bispectral analysis. *Coastal Engineering* 26, 15–33. doi:10.1016/0378-3839(95)00007-X.

- 585 Donelan, M.A., Hamilton, J., Hui, W.H., Stewart, R.W., 1985. Directional spectra of wind-generated ocean  
586 waves. *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical*  
587 *Sciences* 315, 509–562. doi:10.1098/rsta.1985.0054.
- 588 Drazen, D.A., Melville, W.K., 2009. Turbulence and mixing in unsteady breaking surface waves. *Journal*  
589 *of Fluid Mechanics* 628, 85–119. doi:10.1017/S0022112009006120.
- 590 Elgar, S., 1987. Relationships involving third moments and bispectra of a harmonic process. *IEEE Trans-*  
591 *actions on Acoustics, Speech, and Signal Processing* 35, 1725–1726. doi:10.1109/TASSP.1987.1165090.
- 592 Elgar, S., Freilich, M.H., Guza, R.T., 1990. Model-data comparisons of moments of nonbreak-  
593 ing shoaling surface gravity waves. *Journal of Geophysical Research: Oceans* 95, 16055–16063.  
594 doi:10.1029/JC095iC09p16055.
- 595 Elgar, S., Guza, R.T., 1985a. Observations of bispectra of shoaling surface gravity waves. *Journal of Fluid*  
596 *Mechanics* 161, 425–448. doi:10.1017/S0022112085003007.
- 597 Elgar, S., Guza, R.T., 1985b. Shoaling gravity waves: comparisons between field observations, linear theory,  
598 and a nonlinear model. *Journal of Fluid Mechanics* 158, 47–70. doi:10.1017/S0022112085002543.
- 599 Freilich, M.H., Guza, R.T., Whitham, G.B., 1984. Nonlinear effects on shoaling surface gravity waves.  
600 *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*  
601 311, 1–41. doi:10.1098/rsta.1984.0019.
- 602 Guza, R.T., Thornton, E.B., 1980. Local and shoaled comparisons of sea surface elevations, pressures, and  
603 velocities. *Journal of Geophysical Research: Oceans* 85, 1524–1530. doi:10.1029/JC085iC03p01524.
- 604 Hagihira, S., Takashina, M., Mori, T., Mashimo, T., Yoshiya, I., 2001. Practical issues in bispectral  
605 analysis of electroencephalographic signals. *Anesthesia & Analgesia* 93, 966–970. doi:10.1097/00000539-  
606 200110000-00032.
- 607 Hasselmann, K., 1962. On the non-linear energy transfer in a gravity-wave spectrum. Part 1. General  
608 theory. *Journal of Fluid Mechanics* 12, 481–500. doi:10.1017/S0022112062000373.
- 609 Hasselmann, K., Munk, W., MacDonald, G., 1963. Bispectra of ocean waves, in: *Proceedings of the*  
610 *Symposium on Time Series Analysis*, M. Rosenblatt, Ed., New York: Wiley. pp. 125–139.
- 611 Haubrich, R.A., 1965. Earth noise, 5 to 500 millicycles per second: 1. spectral stationarity, normality, and  
612 nonlinearity. *Journal of Geophysical Research (1896-1977)* 70, 1415–1427. doi:10.1029/JZ070i006p01415.
- 613 Hedges, T.S., 1976. An empirical modification to linear wave theory. *Proceedings of the Institution of Civil*  
614 *Engineers* 61, 575–579. doi:10.1680/iicep.1976.3408.
- 615 Herbers, T.H.C., Burton, M.C., 1997. Nonlinear shoaling of directionally spread waves on a beach. *Journal*  
616 *of Geophysical Research: Oceans* 102, 21101–21114. doi:10.1029/97JC01581.

- 617 Herbers, T.H.C., Elgar, S., Guza, R.T., 1994. Infragravity-frequency (0.005-0.05 Hz) motions on the  
618 shelf. Part I: Forced waves. *Journal of Physical Oceanography* 24, 917–927. doi:10.1175/1520-  
619 0485(1994)024;0917:IFHMOT;2.0.CO;2.
- 620 Herbers, T.H.C., Elgar, S., Sarap, N.A., Guza, R.T., 2002. Nonlinear dispersion of surface grav-  
621 ity waves in shallow water. *Journal of Physical Oceanography* 32, 1181–1193. doi:10.1175/1520-  
622 0485(2002)032;1181:NDOSGW;2.0.CO;2.
- 623 Herbers, T.H.C., Guza, R.T., 1991. Wind-wave nonlinearity observed at the sea floor. Part  
624 I: Forced-wave energy. *Journal of Physical Oceanography* 21, 1740–1761. doi:10.1175/1520-  
625 0485(1991)021;1740:WWNOAT;2.0.CO;2.
- 626 Herbers, T.H.C., Guza, R.T., 1992. Wind-wave nonlinearity observed at the sea floor. Part II: Wavenum-  
627 bers and third-order statistics. *Journal of Physical Oceanography* 22, 489–504. doi:10.1175/1520-  
628 0485(1992)022;0489:WWNOAT;2.0.CO;2.
- 629 Herbers, T.H.C., Guza, R.T., 1994. Nonlinear wave interactions and high-frequency seafloor pressure.  
630 *Journal of Geophysical Research: Oceans* 99, 10035–10048. doi:10.1029/94JC00054.
- 631 Herbers, T.H.C., Lowe, R.L., Guza, R.T., 1992. Field observations of orbital velocities and pres-  
632 sure in weakly nonlinear surface gravity waves. *Journal of Fluid Mechanics* 245, 413–435.  
633 doi:10.1017/S0022112092000521.
- 634 Herbers, T.H.C., Russnogle, N.R., Elgar, S., 2000. Spectral energy balance of breaking waves  
635 within the surf zone. *Journal of Physical Oceanography* 30, 2723–2737. doi:10.1175/1520-  
636 0485(2000)030;2723:SEBOBW;2.0.CO;2.
- 637 Hinich, M.J., Wolinsky, M., 2005. Normalizing bispectra. *Journal of Statistical Planning and Inference*  
638 130, 405–411. doi:10.1016/j.jspi.2003.12.022.
- 639 Hoefel, F., Elgar, S., 2003. Wave-induced sediment transport and sandbar migration. *Science* 299, 1885–  
640 1887. doi:10.1126/science.1081448.
- 641 Holland, T.K., 2001. Application of the linear dispersion relation with respect to depth inversion and  
642 remotely sensed imagery. *IEEE Transactions on Geoscience and Remote Sensing* 39, 2060–2072.  
643 doi:10.1109/36.951097.
- 644 Holman, R., Plant, N., Holland, T., 2013. cBathy: A robust algorithm for estimating nearshore bathymetry.  
645 *Journal of Geophysical Research: Oceans* 118, 2595–2609. doi:10.1002/jgrc.20199.
- 646 Inman, D.L., Tait, R.J., Nordstrom, C.E., 1971. Mixing in the surf zone. *Journal of Geophysical Research*  
647 (1896-1977) 76, 3493–3514. doi:10.1029/JC076i015p03493.
- 648 Janssen, T.T., Battjes, J.A., van Dongeren, A.R., 2003. Long waves induced by short-wave groups over a  
649 sloping bottom. *Journal of Geophysical Research: Oceans* 108. doi:10.1029/2002JC001515.

- 650 Kim, Y.C., Beall, J.M., Powers, E.J., Miksad, R.W., 1980. Bispectrum and nonlinear wave coupling. *The*  
651 *Physics of Fluids* 23, 258–263. doi:10.1063/1.862966.
- 652 Kim, Y.C., Powers, E.J., 1979. Digital Bispectral Analysis and Its Applications to Nonlinear Wave Inter-  
653 actions. *IEEE Transactions on Plasma Science* 7, 120–131. doi:10.1109/TPS.1979.4317207.
- 654 Laing, A.K., 1986. Nonlinear properties of random gravity waves in water of finite depth. *Journal of*  
655 *Physical Oceanography* 16, 2013–2030. doi:10.1175/1520-0485(1986)016;2013:NPORGW;2.0.CO;2.
- 656 Lake, B.M., Yuen, H.C., 1978. A new model for nonlinear wind waves. Part 1. Physical model and experi-  
657 mental evidence. *Journal of Fluid Mechanics* 88, 33–62. doi:10.1017/S0022112078001974.
- 658 Longuet-Higgins, M.S., 1956. The refraction of sea waves in shallow water. *Journal of Fluid Mechanics* 1,  
659 163–176. doi:10.1017/S0022112056000111.
- 660 Longuet-Higgins, M.S., Stewart, R.W., 1962. Radiation stress and mass transport in gravity waves, with  
661 application to 'surf beats'. *Journal of Fluid Mechanics* 13, 481–504. doi:10.1017/S0022112062000877.
- 662 Longuet-Higgins, M.S., Stewart, R.W., 1964. Radiation stresses in water waves; a physical discussion,  
663 with applications. *Deep Sea Research and Oceanographic Abstracts* 11, 529–562. doi:10.1016/0011-  
664 7471(64)90001-4.
- 665 Martins, K., Bonneton, P., Bayle, P.M., Blenkinsopp, C.E., Mouragues, A., Michallet, H., 2020a. Surf  
666 zone wave measurements from lidar scanners: Analysis of non-hydrostatic processes. *Journal of Coastal*  
667 *Research* 95, 1189–1194. doi:10.2112/SI95-231.1.
- 668 Martins, K., Bonneton, P., Mouragues, A., Castelle, B., 2020b. Non-hydrostatic, non-linear processes in the  
669 surf zone. *Journal of Geophysical Research: Oceans* 125, e2019JC015521. doi:10.1029/2019JC015521.
- 670 Masuda, A., Kuo, Y.Y., 1981. A note on the imaginary part of bispectra. *Deep Sea Research Part A.*  
671 *Oceanographic Research Papers* 28, 213–222. doi:10.1016/0198-0149(81)90063-7.
- 672 Michallet, H., Cienfuegos, R., Barthélemy, E., Grasso, F., 2011. Kinematics of waves propagat-  
673 ing and breaking on a barred beach. *European Journal of Mechanics - B/Fluids* 30, 624–634.  
674 doi:10.1016/j.euromechflu.2010.12.004.
- 675 Mitsuyasu, H., Kuo, Y.Y., Masuda, A., 1979. On the dispersion relation of random gravity waves. Part 2.  
676 An experiment. *Journal of Fluid Mechanics* 92, 731–749. doi:10.1017/S0022112079000859.
- 677 Mouragues, A., Bonneton, P., Lannes, D., Castelle, B., Mariou, V., 2019. Field data-based evaluation of  
678 methods for recovering surface wave elevation from pressure measurements. *Coastal Engineering* 150,  
679 147–159. doi:10.1016/j.coastaleng.2019.04.006.
- 680 Norheim, C.A., Herbers, T.H.C., Elgar, S., 1998. Nonlinear evolution of surface wave spec-  
681 tra on a beach. *Journal of Physical Oceanography* 28, 1534–1551. doi:10.1175/1520-  
682 0485(1998)028;1534:NEOSWS;2.0.CO;2.

- 683 Ochi, M.K., 1998. Spectral analysis. Cambridge University Press. Cambridge Ocean Technology Series, pp.  
684 13–57. doi:10.1017/CBO9780511529559.003.
- 685 Padilla, E.M., Alsina, J.M., 2017. Transfer and dissipation of energy during wave group prop-  
686 agation on a gentle beach slope. *Journal of Geophysical Research: Oceans* 122, 6773–6794.  
687 doi:10.1002/2017JC012703.
- 688 Peregrine, D.H., Bokhove, O., 1998. Vorticity and surf zone currents, in: *Proceedings of the 26th Conference*  
689 *on Coastal Engineering*, Copenhagen, Denmark, pp. 745–748.
- 690 Phillips, O.M., 1960. On the dynamics of unsteady gravity waves of finite amplitude. Part 1. The elementary  
691 interactions. *Journal of Fluid Mechanics* 9, 193–217. doi:10.1017/S0022112060001043.
- 692 Plant, N.G., Holland, K.T., Haller, M.C., 2008. Ocean wavenumber estimation from wave-resolving  
693 time series imagery. *IEEE Transactions on Geoscience and Remote Sensing* 46, 2644–2658.  
694 doi:10.1109/TGRS.2008.919821.
- 695 Ramamonjiarisoa, A., Coantic, M., 1976. Loi experimentale de dispersion des vagues produites par le vent  
696 sur une faible longueur d’action. *Comptes rendus hebdomadaires des séances de l’Académie des sciences,*  
697 *Série B* 282, 111–113.
- 698 Redor, I., Bartélemy, E., Michallet, H., Onorato, M., Mordant, N., 2019. Experimental evidence of a  
699 hydrodynamic soliton gas. *Physical Review Letters* 122, 214502. doi:10.1103/PhysRevLett.122.214502.
- 700 Rocha, M.V.L., Michallet, H., Silva, P.A., 2017. Improving the parameterization of wave nonlinearities –  
701 the importance of wave steepness, spectral bandwidth and beach slope. *Coastal Engineering* 121, 77–89.  
702 doi:10.1016/j.coastaleng.2016.11.012.
- 703 Ruessink, G.B., Michallet, H., Bonneton, P., Mouazé, D., Lara, J.L., Silva, P.A., Wellens, P., 2013.  
704 GLOBEX: wave dynamics on a gently sloping laboratory beach, in: *Coastal Dynamics ’13: Proceed-*  
705 *ings of the Seventh Conference on Coastal Dynamics*, Arcachon, France.
- 706 Stive, M.J.F., 1980. Velocity and pressure field of spilling breakers, in: *Proceedings of the 17th Conference*  
707 *on Coastal Engineering*, Sydney, Australia, pp. 547–566.
- 708 Stive, M.J.F., Wind, H.G., 1982. A study of radiation stress and set-up in the nearshore region. *Coastal*  
709 *Engineering* 6, 1–25. doi:10.1016/0378-3839(82)90012-6.
- 710 Stockdon, H.F., Holman, R.A., 2000. Estimation of wave phase speed and nearshore bathymetry from video  
711 imagery. *Journal of Geophysical Research: Oceans* 105, 22015–22033. doi:10.1029/1999JC000124.
- 712 Svendsen, I.A., 1984. Mass flux and undertow in a surf zone. *Coastal Engineering* 8, 347–365.  
713 doi:10.1016/0378-3839(84)90030-9.
- 714 Svendsen, I.A., Madsen, P.A., Buhr Hansen, J., 1978. Wave characteristics in the surf zone, in: *Proceedings*  
715 *of the 16th Conference on Coastal Engineering*, Hamburg, Germany, pp. 520–539.

- 716 Thornton, E.B., Guza, R.T., 1982. Energy saturation and phase speeds measured on a natural beach.  
717 *Journal of Geophysical Research* 87, 9499–9508. doi:10.1029/JC087iC12p09499.
- 718 Thornton, E.B., Guza, R.T., 1983. Transformation of wave height distribution. *Journal of Geophysical*  
719 *Research: Oceans* 88, 5925–5938. doi:10.1029/JC088iC10p05925.
- 720 Ting, F.C., Kirby, J.T., 1996. Dynamics of surf-zone turbulence in a spilling breaker. *Coastal Engineering*  
721 27, 131–160. doi:10.1016/0378-3839(95)00037-2.
- 722 Tissier, M., Bonneton, P., Almar, R., Castelle, B., Bonneton, N., Nahon, A., 2011. Field measurements  
723 and non-linear prediction of wave celerity in the surf zone. *European Journal of Mechanics - B/Fluids*  
724 30, 635–641. doi:10.1016/j.euromechflu.2010.11.003.
- 725 Tissier, M., Bonneton, P., Michallet, H., Ruessink, B.G., 2015. Infragravity-wave modulation of  
726 short-wave celerity in the surf zone. *Journal of Geophysical Research: Oceans* 120, 6799–6814.  
727 doi:10.1002/2015JC010708.
- 728 van der Zanden, J., van der A, D.A., Hurther, D., Cáceres, I., O’Donoghue, T., Hulscher, S.J.M.H.,  
729 Ribberink, J.S., 2017. Bedload and suspended load contributions to breaker bar morphodynamics. *Coastal*  
730 *Engineering* 129, 74–92. doi:10.1016/j.coastaleng.2017.09.005.
- 731 de Wit, F., Tissier, M., Reniers, A., 2020. The relationship between sea-swell bound wave height and wave  
732 shape. *Journal of Marine Science and Engineering* 8, 643. doi:10.3390/jmse8090643.

Test	$H_s$ [m]	$f_p$ [Hz]	$\gamma$
A1	0.10	0.633	3.3
A2	0.20	0.444	3.3
A3	0.10	0.444	20

Table 1: Significant wave height ( $H_s$ ) and peak wave frequency ( $f_p$ ) for the three tests considered in this study. The peak enhancement  $\gamma$  of the JONSWAP spectra characterizes its spectral bandwidth. A value of 3.3 corresponds to broad spectra while a narrow-band spectra is imposed with  $\gamma = 20$ .

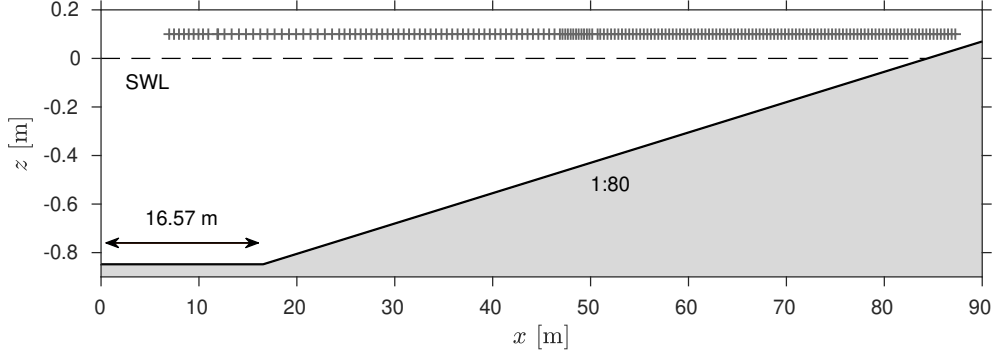


Figure 1: Elevation  $z$  of the 1:80 concrete beach against the cross-shore distance  $x$  in the Scheldegoot flume during the GLOBEX project. The wave paddle is located at  $x = 0$  m. The grey '+' symbols show the position of the wave gauges across the wave flume.

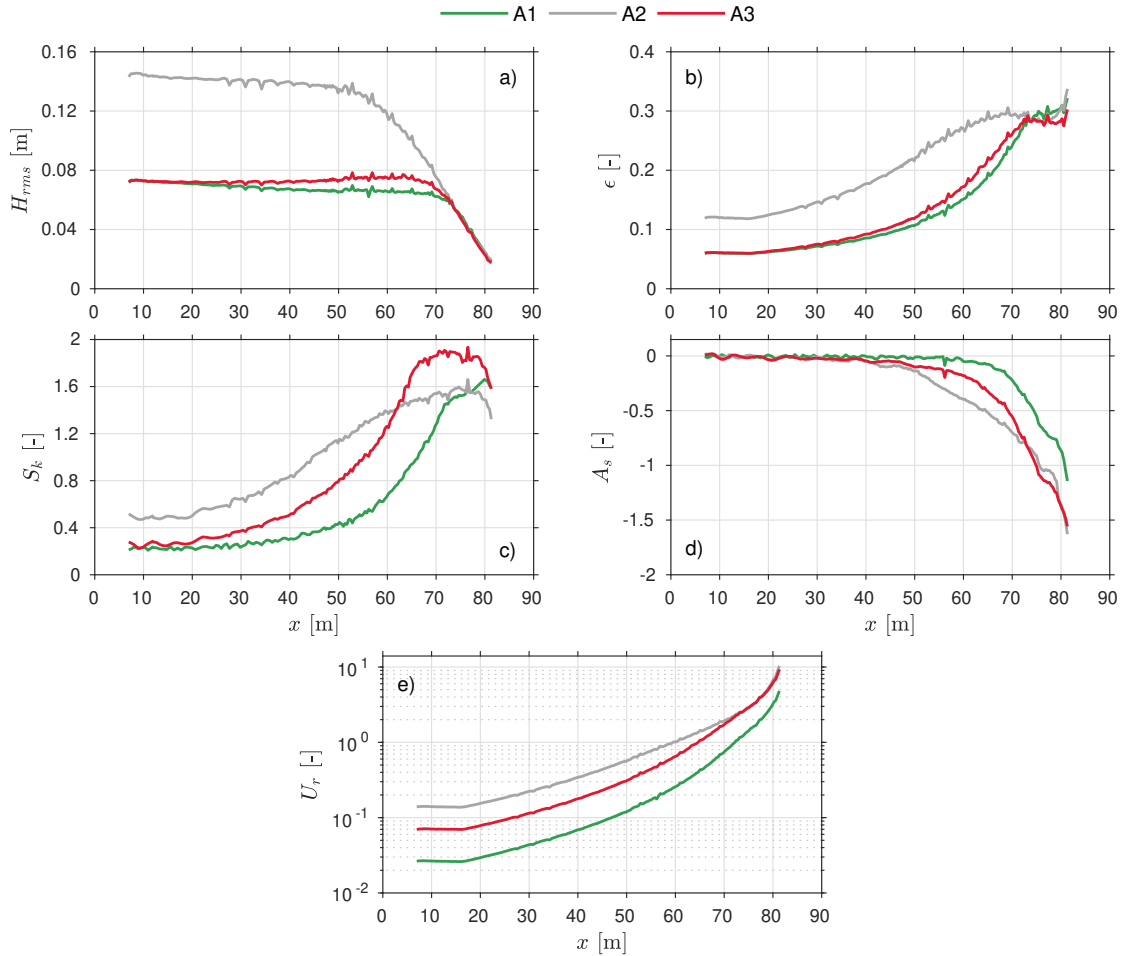


Figure 2: Second and third-order short-wave parameters during tests A1, A2 and A3: a) root-mean square wave height  $H_{rms}$  computed as  $(8\zeta^2)^{1/2}$ ; b) corresponding wave amplitude to water depth ratio  $\epsilon = H_{rms}/\sqrt{2}h$ ; c) wave skewness  $S_k$ ; d) wave asymmetry  $A_s$  and e) Ursell number  $U_r$  computed as  $\epsilon/\mu$ . Note that these are short-wave parameters, computed using the high-pass filtered surface elevation signal (frequency cutoff at  $0.6f_p$ ).



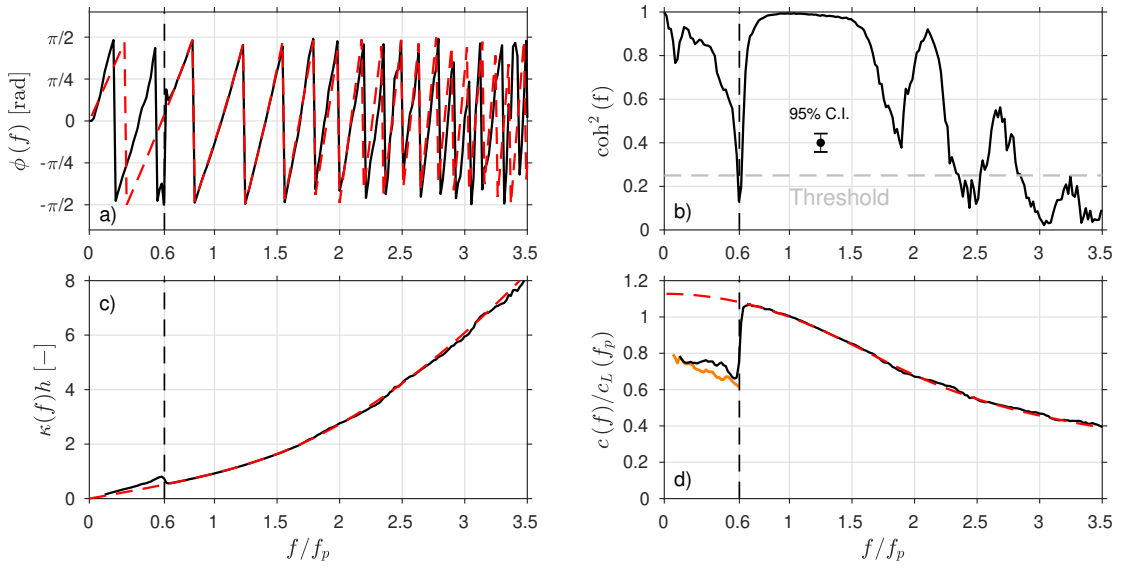


Figure 3: Phase (a), coherence-squared (b), dimensionless wavenumber (c) and phase velocity (d) spectra computed at  $x = 10.65$  m for case A2. The spacing between the adjacent gauges is  $\Delta x = 0.93L_p$ , with  $L_p$  the peak wavelength given by the linear wave dispersion relation (Eq. 14). The phase velocity spectrum is normalised by  $c_L(f_p)$ , the phase velocity predicted by linear wave theory at the peak frequency. The red dashed lines in panels a, c and d correspond to the values given by the linear wave dispersion relation. The separation between infragravity and short wave frequencies ( $0.6 f_p$ ) is shown as dashed black line. In panel b, the gray dashed line corresponds to the coherence-squared threshold used for computing ensemble average spectral estimated of  $\kappa$  and  $c$ . In panel d, the orange line corresponds to the short-wave envelop propagation speed.

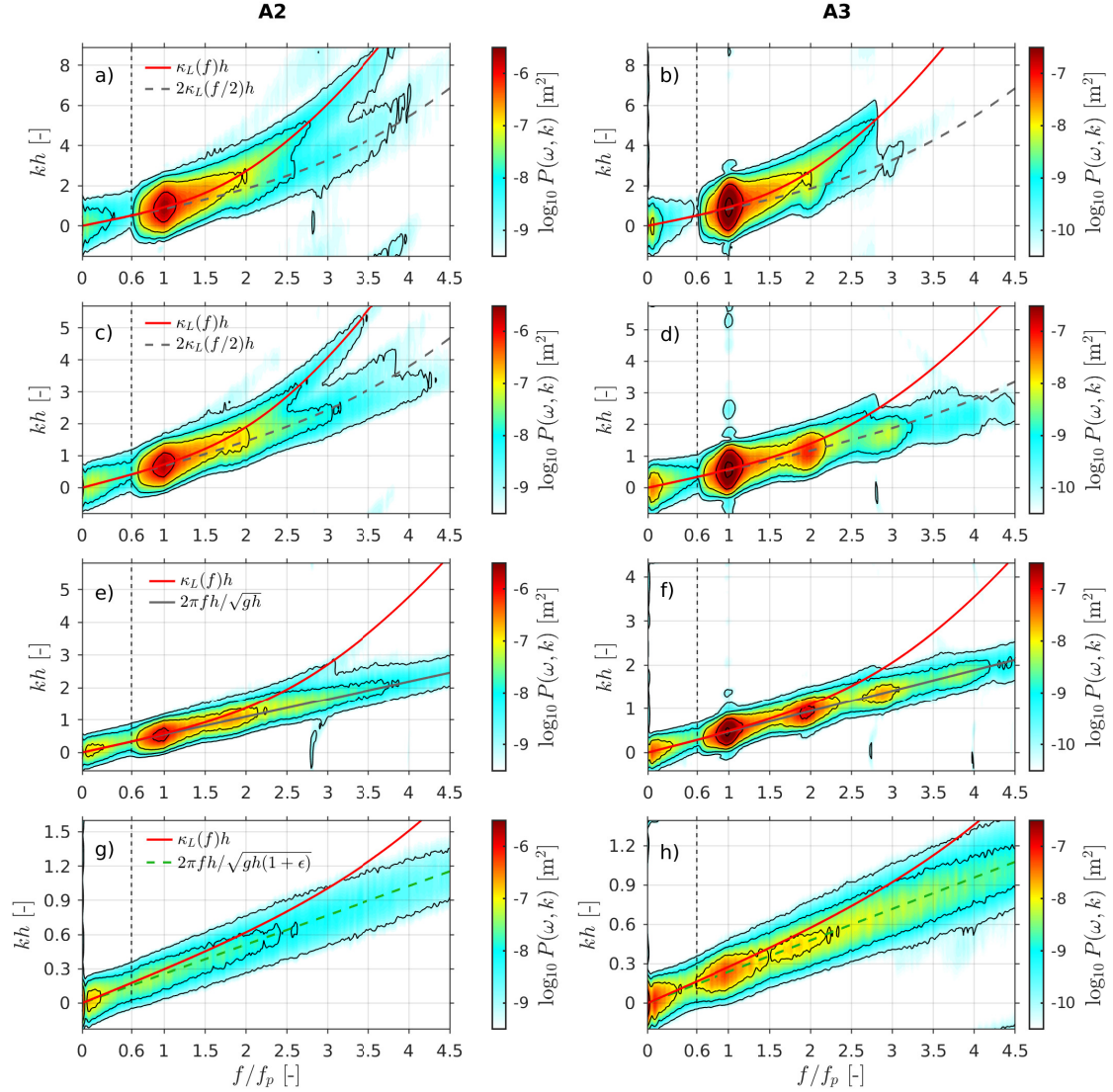


Figure 4: Frequency-wavenumber surface elevation power spectra  $P(\omega, k)$  computed for A2 (left) and A3 (right) at the four stages of propagation described in the paper (I: panels a-b; II: panels c-d; III: panels e-f and IV: panels g-h). For A2, these power spectra were computed at I:  $x = 9.8 \pm 2.8$  m; II:  $x = 38.9 \pm 3.6$  m; III:  $x = 54.5 \pm 3.5$  m and IV:  $x = 76 \pm 2.4$  m. For A3, the cross-shore locations were I:  $x = 9.8 \pm 2.8$  m; II:  $x = 53.4 \pm 2.4$  m; III:  $x = 62.3 \pm 3.5$  m and IV:  $x = 77 \pm 2.4$  m. Each black contour line correspond to a power of 10. The separation between infragravity and short-wave frequencies ( $0.6 f_p$ ) is shown as the dashed black line. In all panels, the red curves correspond to the linear wave dispersion relation. In panels a-d, the dashed grey curves correspond to the dispersion relation for the second harmonic, and was computed based on the assumption that it travels at a similar speed as its principal component. In panels e-f, the grey curve corresponds to the shallow water wave dispersion relation ( $c = \sqrt{gh}$ ) while the dashed green line in panels g-h refer to the modified one ( $c = \sqrt{gh(1 + \epsilon)}$ ).

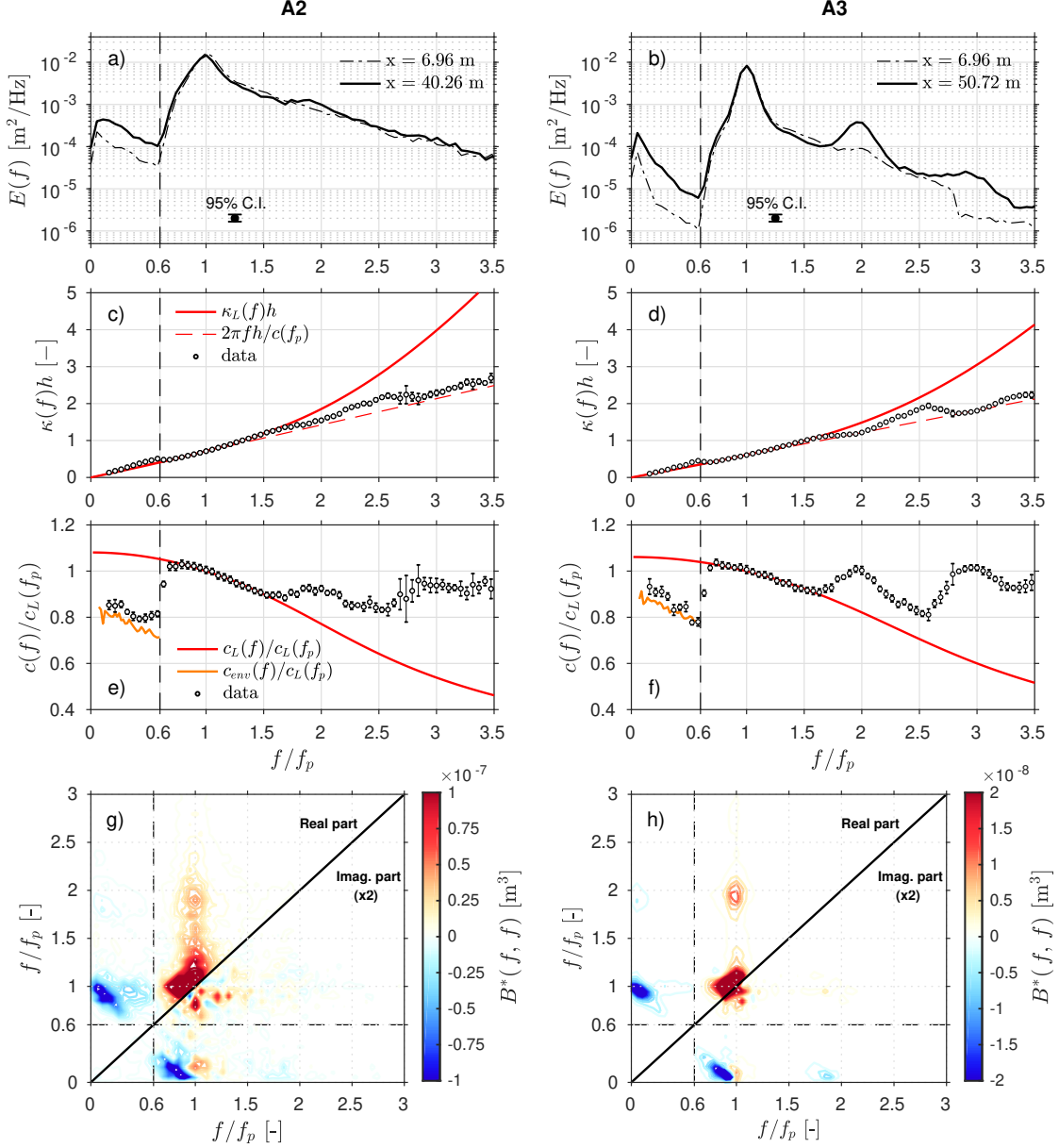


Figure 5: From top to bottom: surface elevation energy density spectra, dimensionless wavenumber spectra, phase velocity spectra normalised by  $c_L(f_p)$  and bispectra computed at  $U_r \sim 0.3$  (stage II) for wave tests A2 (left panels) and A3 (right panels). The cross-shore location corresponding to this stage is indicated in panels a and b (*cf* figure 2e). The wavenumber and phase velocity spectra were computed with five spacing configurations: each point corresponds to the ensemble-averaged value and the error bars correspond to the standard deviation. For readability, only one on three data points are shown. Red lines in panels c-f correspond to values given by the linear wave dispersion relation. The separation between infragravity and short-wave frequencies ( $0.6 f_p$ ) is shown as the dashed black line. In panels c-d, the red dashed line corresponds to  $\kappa(f) = 2\pi f/c(f_p)$ . In panels e-f, the orange line corresponds to the short-wave envelop propagation speed denoted  $c_{env}$ .

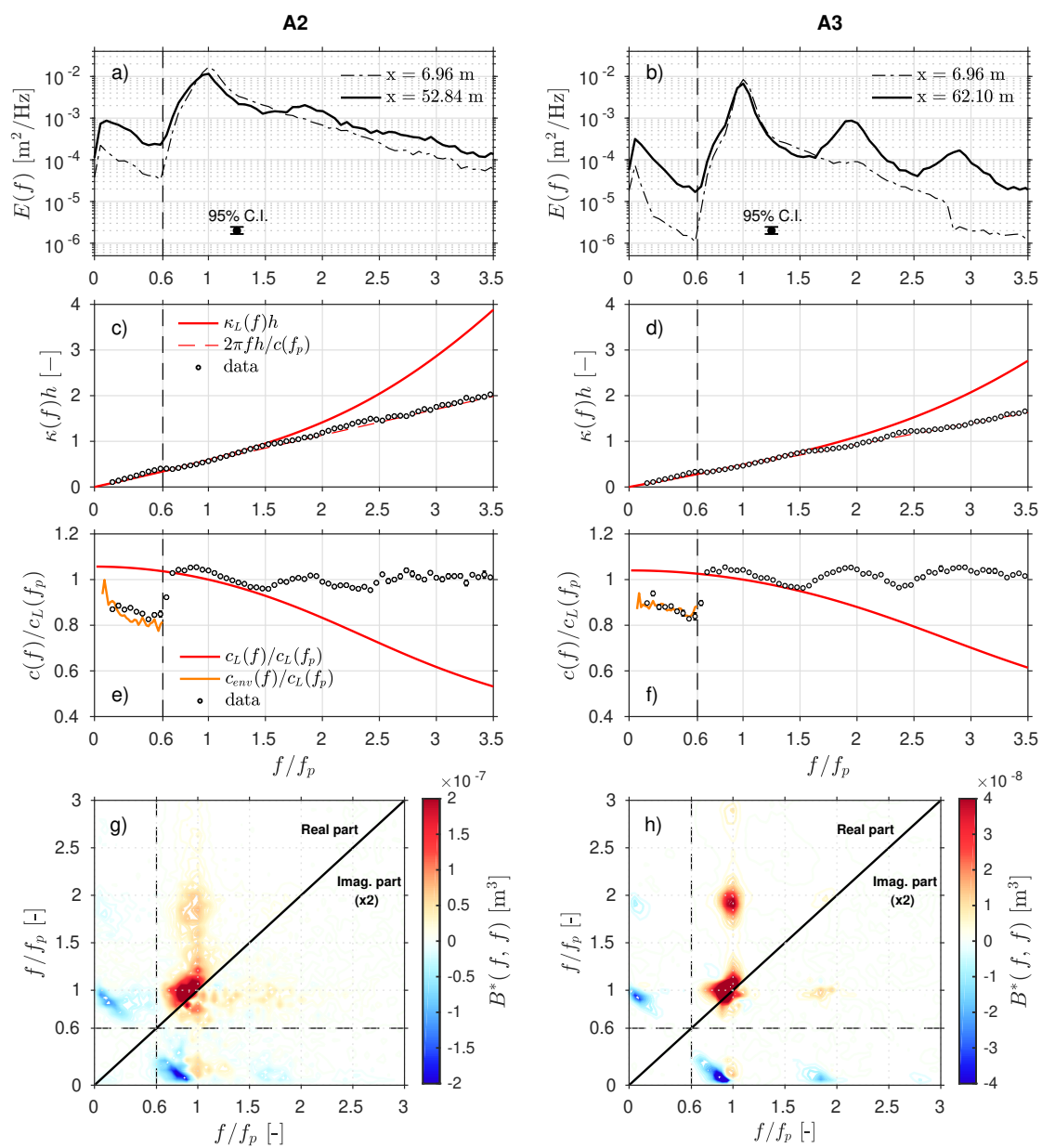


Figure 6: Similar to figure 5, but with  $U_r \sim 0.7$  (stage III).

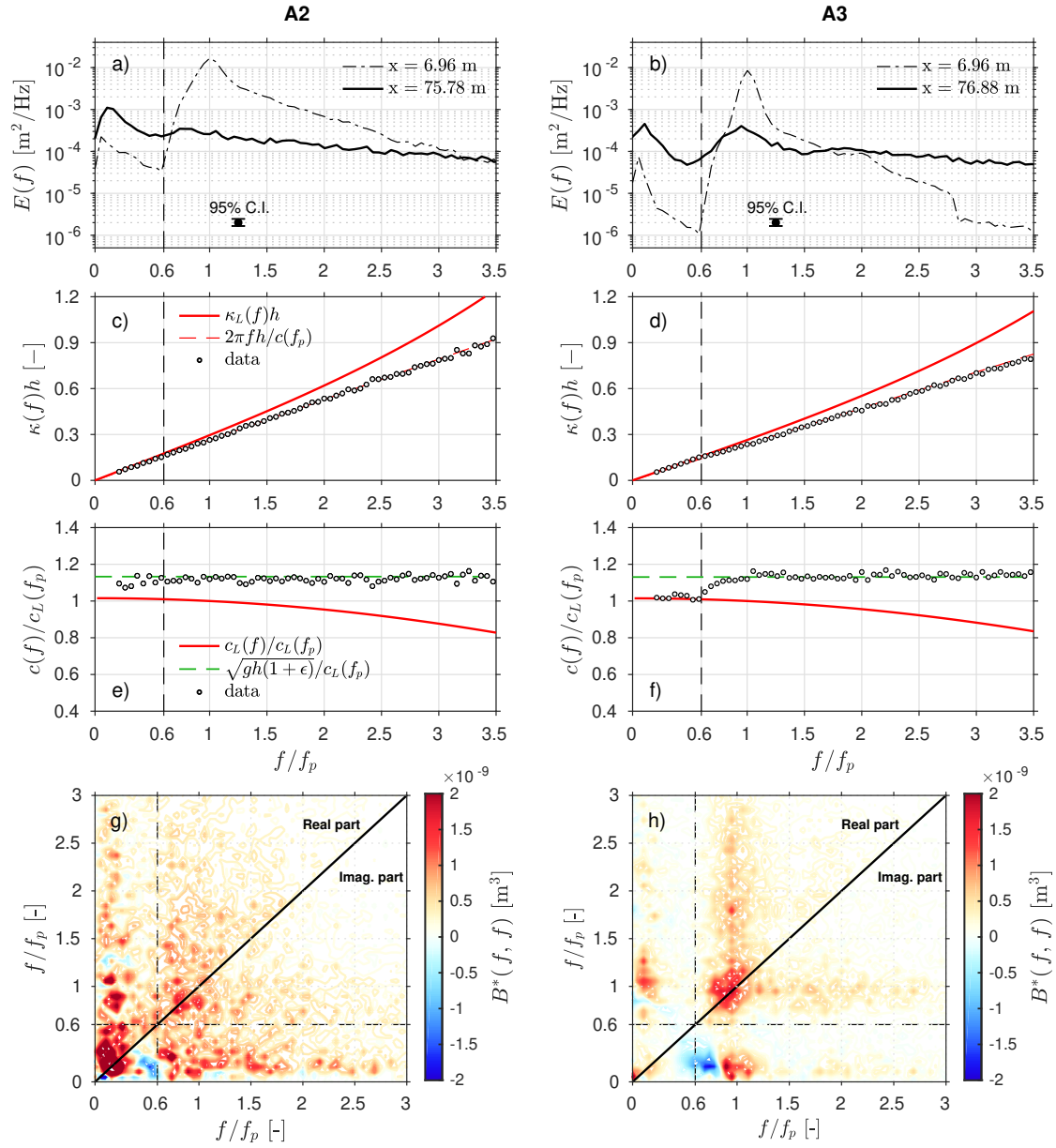


Figure 7: Similar to figure 5, but with  $U_r \sim 3.2$  (stage IV). Additionally, the green dashed lines in panels e) and f) correspond to the modified shallow water dispersion relation  $c = \sqrt{gh(1+\epsilon)}$ .

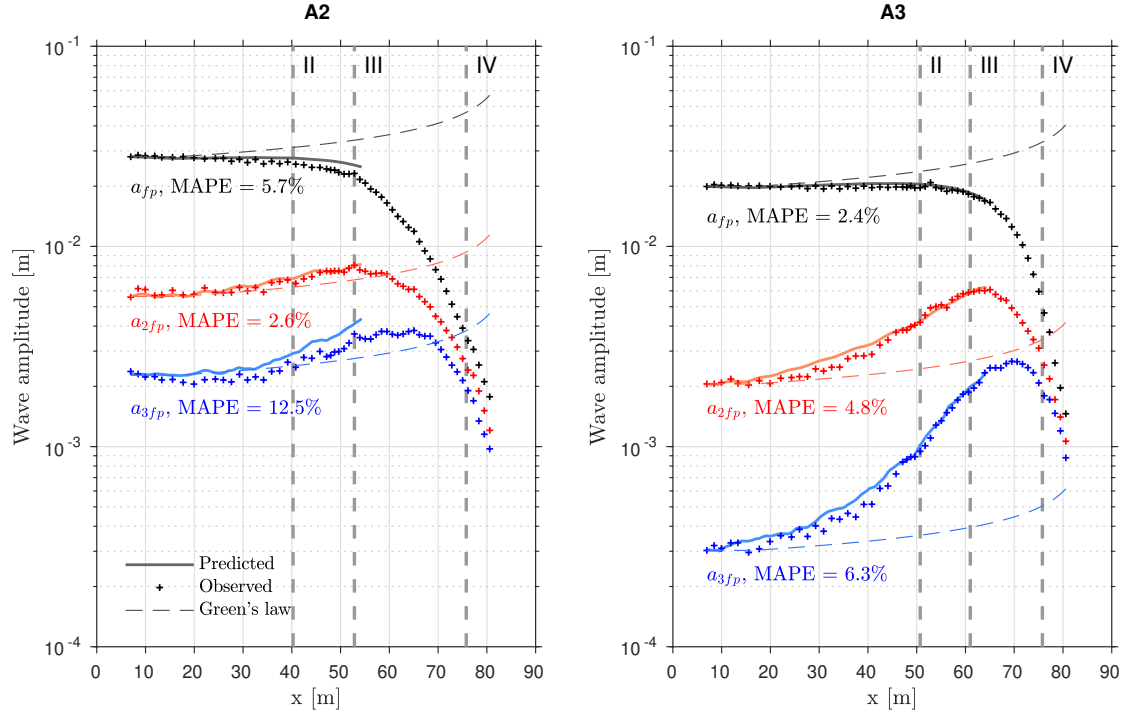


Figure 8: Comparison between observed and predicted (Eq. 13) wave amplitudes at  $f_p$ ,  $2f_p$  and  $3f_p$  ( $a_{f_p}$ ,  $a_{2f_p}$  and  $a_{3f_p}$  respectively). The cross-shore locations corresponding to the regimes of propagation discussed in section 4 are indicated by the vertical dashed lines for both wave tests. Since the present modelling approach neglects wave breaking-induced energy dissipation, modelled amplitudes are shown only until the approximate mean breaking point location (slightly after stage III). The amplitude growth predicted with Green's law is also shown.

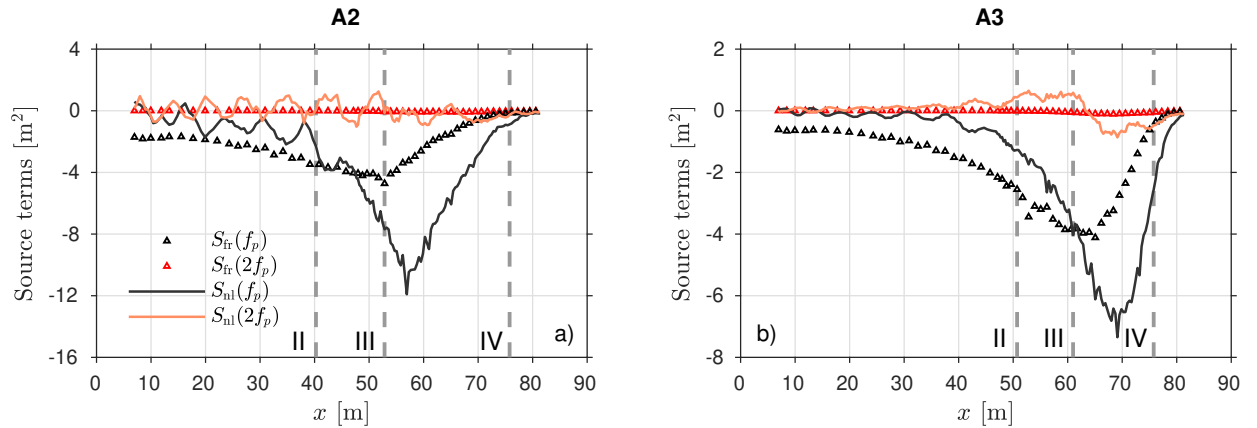


Figure 9: Cross-shore variation of the source terms for the non-linear energy exchanges between triads and for the bottom friction, used in Eq. 13. Only values at the peak frequency ( $f_p$ ) and the second harmonic ( $2f_p$ ) are shown. The cross-shore locations corresponding to the regimes of propagation discussed in section 4 are indicated by the vertical dashed lines for both wave tests.

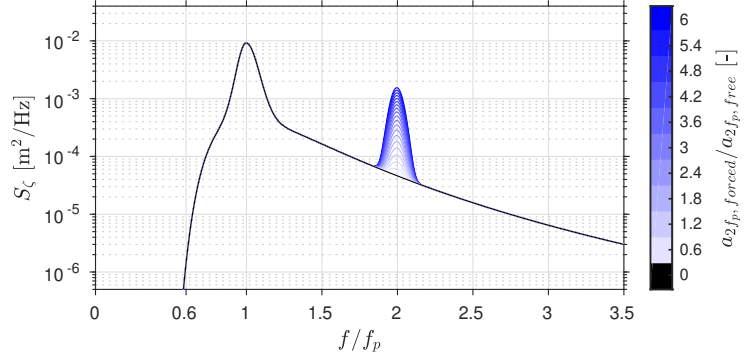


Figure A1: Energy density spectra imposed at a fictive  $x_1$  position, with different ratio of forced to free second harmonic amplitude. In the absence of forced components, the forcing corresponds to a JONSWAP spectrum with parameter similar to A3 (see Table 1).

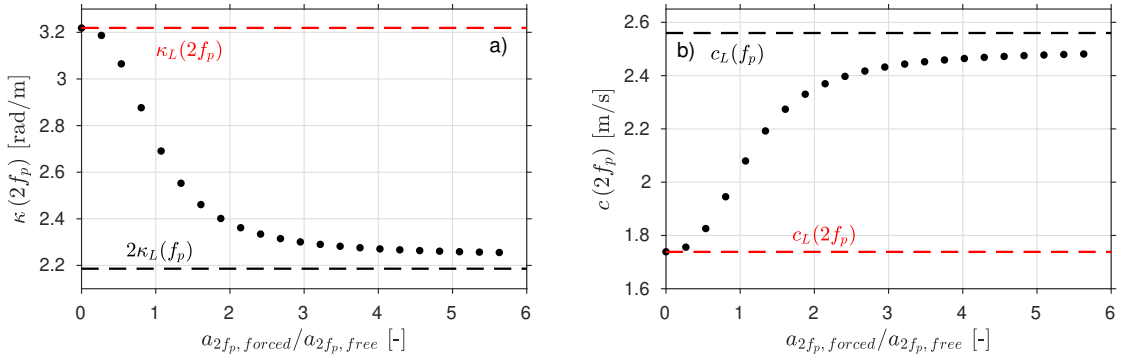


Figure A2: Wavenumber (a) and phase velocity (b) spectra computed on the synthetic surface elevation timeseries for varying ratio of forced to free second harmonic amplitudes.

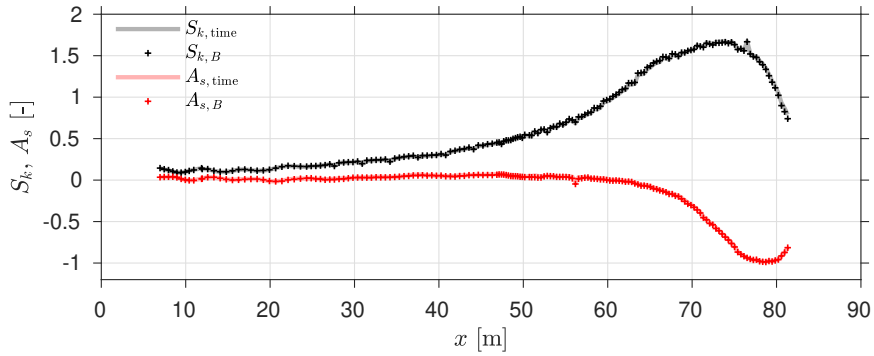


Figure B1: Cross-comparison of third-order wave parameters during A3 computed using the statistical (Eq. 1-2, 'time' subscript) and the bispectrum (Eq. B.1-B.2, 'B' subscript) definitions. No separation between infragravity and short wave frequency bands or between incoming and outgoing wave fields was performed for this comparison.

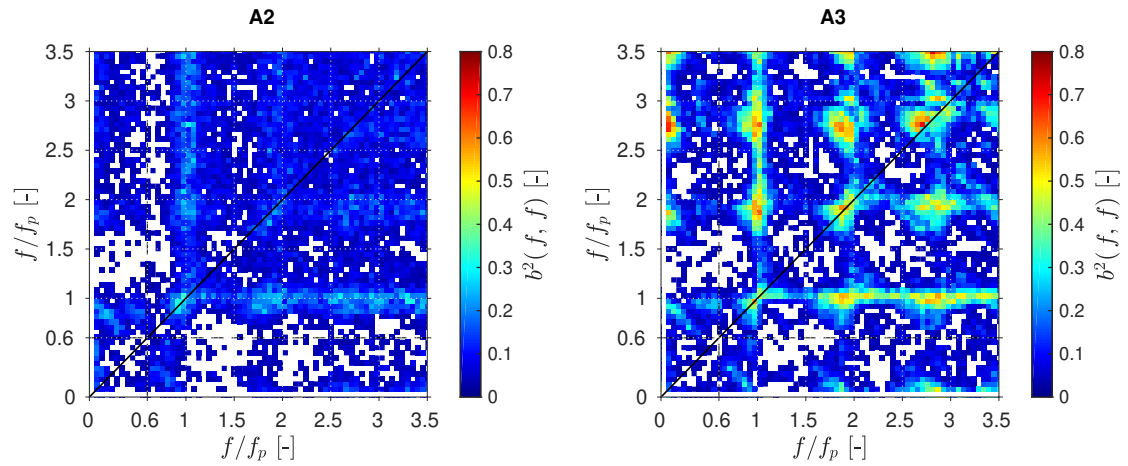


Figure C1: Squared bicoherence  $b^2$  computed using the definition of Hagihira et al. (2001) for wave test A2 (left) and A3 (right). The octant above the 1:1 diagonal corresponds to the bicoherence for stage II while stage III is that under the diagonal. Due to the symmetric properties of the bispectrum, only one octant is shown for each stage. The separation between infragravity and short-wave frequencies ( $0.6 f_p$ ) is shown as the dashed black line. Only bicoherence values greater than the 95% significance level as defined by Haubrich (1965) ( $b_{95\%}^2 \geq 6/\text{d.o.f.}$ , with d.o.f. the equivalent number of degrees of freedom) are shown.