

# Fluidization of variable short fiber/powder mixtures: hydrodynamic investigation

## CRediT authorship contribution statement

- 1<sup>st</sup> author: **Thomas DA CALVA MOUILLEVOIS**<sup>1,2</sup> (corresponding author): [dacalva@lcts.u-bordeaux.fr](mailto:dacalva@lcts.u-bordeaux.fr), Conceptualization, Methodology, Formal analysis, Investigation, Writing - Original Draft, Writing - Review & Editing, Visualization.
- 2<sup>nd</sup> author: **Matthias AUDREN-PAUL**<sup>3</sup>: [mapaul001@univ-pau.fr](mailto:mapaul001@univ-pau.fr), Investigation.
- 3<sup>rd</sup> author: **Georges CHOLLON**<sup>1</sup>: [chollon@lcts.u-bordeaux.fr](mailto:chollon@lcts.u-bordeaux.fr), Writing - Review & Editing, Supervision, Project administration.
- 4<sup>th</sup> author: **Nathalie BERTRAND**<sup>1,2</sup>: [bertrand@lcts.u-bordeaux.fr](mailto:bertrand@lcts.u-bordeaux.fr), Writing - Review & Editing, Supervision, Project administration.

<sup>1</sup> *ThermoStructural Composites Laboratory (LCTS), 3 Allée de la Boétie, 33600 Pessac, France*

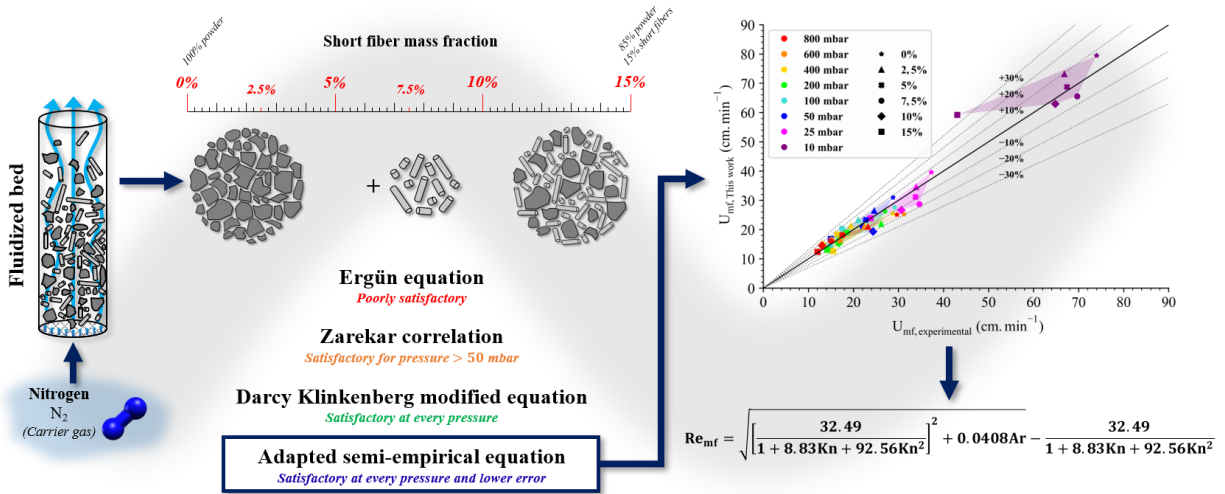
<sup>2</sup> *University of Bordeaux, 351 Cour de la Libération, 33400 Talence, France*

<sup>3</sup> *National School of Chemical and Technological Engineering, 4 Allée Emile Monso, 31030 Toulouse, France*

## Highlights

- Numerical determination of the minimum fluidization velocity as a function of the operating parameters
- Prediction of the equivalent hydraulic diameter of short fiber/powder mixtures in variable proportions
- Coefficient estimation of the Darcy-Klinkenberg hydrodynamic equation adapted to fiber/powder mixtures
- Accurate prediction ( $\pm 10\%$ ) of the minimum fluidization velocity by semi-empirical correlation with Kn
- Extension and specification of the fluidized bed expansion equation as a function of operating parameters

27 **Graphical abstract**



28  
29 **Colors for figures**

30 For a proper readability of the figures, all require color printing.

31 **Abstract**

32 The behavior of a particle mixture composed of short fibers and silicon carbide powder in variable proportions  
 33 has been experimentally investigated. The study shows that the hydrodynamic behavior of a bed initially  
 34 composed of powder is affected as soon as a few percent of short fibers are added. The minimum fluidization  
 35 velocity ( $U_{mf}$ ) is globally lowered and the intrinsic properties of the porous medium (equivalent particle  
 36 diameter, porosity, flow coefficients...) are altered. Several hydrodynamic equations describing  $U_{mf}$  as a  
 37 function of the pressure are considered and the intrinsic properties of the bed such as the hydraulic diameter or  
 38 the Darcy and Knudsen coefficients are fitted. The Darcy-Klinkenberg equation on one hand and a semi-  
 39 empirical correlation based on Ergün's equation on the other hand, are both adapted to describe the  $U_{mf}$  behavior  
 40 as a function of a limited number of initial parameters. The semi-empirical correlation leads to the best  $U_{mf}$   
 41 prediction, with an absolute error of less than 10%, using only the temperature, pressure and percentage of fibers  
 42 in the mixture as input parameters.

43 **Keywords**

44 Fluidized bed, Minimum fluidization velocity, Hydrodynamic, Particle mixture, Short fibers

45

47 **Nomenclature**

A	Semi-empirical coefficient of Re(Ar) equation	(-)
a	Coefficient for the curve fitting of $\Psi_s d_g(\%_f)$	(-)
Ar	Archimedes number	(-)
B	Semi-empirical coefficient of Re(Ar) equation	(-)
b	Coefficient for the curve fitting of $\Psi_s d_g(\%_f)$	(-)
$C_1$	Constant coefficient for $1/(\Psi_s \epsilon_{mf}^3)$	(-)
c	Gas molecular velocity	(m.s <sup>-1</sup> )
$c_0$	Adjustment factor for $K_2(Kn)$ equation	(-)
$c_1$	Adjustment factor for $K_2(Kn)$ equation	(-)
$c_2$	Adjustment factor for $K_2(Kn)$ equation	(-)
$d_g$	Average particle diameter	(m)
$d_p$	Pore diameter	(m)
$d_{sv}$	Surface volume diameter or Sauter diameter	(m)
$d_{sv}^f$	Surface volume diameter of the short fibers	(m)
$d_{sv}^p$	Surface volume diameter of the powder	(m)
$D_K$	Knudsen diffusion coefficient	(m <sup>2</sup> .s <sup>-1</sup> )
g	Gravitational force constant	(9.81 m.s <sup>-2</sup> )
H	Absolute bed height	(m)
$H^*$	Normalized bed height	(-)
$H_{mf}$	Bed height at minimum fluidization velocity	(m)
K	Permeability	(m <sup>2</sup> )
$K_1$	Semi-empirical coefficient of Ar(Re) equation	(-)
$K_2$	Semi-empirical coefficient of Ar(Re) equation	(-)
$K_B$	Boltzmann constant	(1.380649.10 <sup>-23</sup> m <sup>2</sup> .kg.s <sup>-2</sup> .K <sup>-1</sup> )
Kn	Knudsen number	(-)
$k_2$	Inertial flow coefficient	(-)
m	Bed mass	(kg)
M	Molar mass	(kg.mol <sup>-1</sup> )
n	Coefficient for the curve fitting of $\Psi_s d_g(\%_f)$	(-)
$n_1$	Bed expansion equation factor for (U-U <sub>mf</sub> )	(-)
$n_2$	Bed expansion equation factor for U <sub>mf</sub>	(-)
$n_3$	Bed expansion equation factor for $\rho_p$	(-)
$n_4$	Bed expansion equation factor for $\rho_g$	(-)
$n_5$	Bed expansion equation factor for $d_g$	(-)
P	Absolute pressure	(Pa)
$p^0$	Reference pressure	(101325 Pa)
$P_{mean}$	Average pressure in the fluidized bed	(Pa)
R	Universal constant of perfect gases	(8.314 J.K <sup>-1</sup> .mol <sup>-1</sup> )
Re	Reynolds number	(-)
$Re_{mf}$	Reynolds number at minimum fluidization velocity	(-)
$r^2$	Coefficient of determination	(-)
S	Cross section of the fluidization column	(m <sup>2</sup> )

T	Absolute temperature	(K)
$T^0$	Reference temperature	(273.15 K)
U	Surface velocity of the gas	(m.s <sup>-1</sup> )
$U_{mf}$	Minimum fluidization velocity	(m.s <sup>-1</sup> )
$\alpha$	Coefficient for the bed expansion correlation	(-)
$\beta$	Coefficient for the bed expansion correlation	(-)
$\gamma$	Coefficient for the bed expansion correlation	(-)
$\delta$	Coefficient for the bed expansion correlation	(-)
$\epsilon_{mf}$	Bed porosity at minimum fluidization velocity	(-)
$\eta_V$	Viscous tortuosity	(-)
$\eta_{V,mean}$	Mean viscous tortuosity	(-)
$\eta_K$	Knudsen tortuosity	(-)
$\eta_{K,mean}$	Mean Knudsen tortuosity	(-)
$\mu$	Viscosity	(Pa.s)
$\rho_g$	Density of the carrier gas	(kg.m <sup>-3</sup> )
$\rho_p$	Density of the particles	(kg.m <sup>-3</sup> )
$\sigma$	Molecular diameter of gas	(m)
$\Psi_s$	Particle sphericity	(-)
$\%_f$	Fiber mass fraction	(%)

48

## 49 1. Introduction

50 For more than 40 years, ceramic matrix composites (CMCs) have demonstrated their potential as  
51 thermostructural components in the hot sections of commercial and military jet engines [1–3]. Their  
52 thermomechanical properties would not be as high without an interfacial layer (or “interphase”) between the  
53 fibers and the matrix, providing the composite a pseudo-ductile behavior [4,5]. For some specific applications of  
54 CMC where traditional woven fiber reinforcements are not be applicable, short fibers could be used. However, if  
55 CMCs are to remain damage-tolerant, the deposition process of the interphase must be compatible with such a  
56 discontinuous substrate.

57 CVD is typically used to deposit the interphase because of its inherent ability to coat complex shapes and porous  
58 substrates with uniform thicknesses and controlled microstructures. The process is well established for  
59 continuous substrates, such as solid parts (for the outer protective coating) or fibrous fabrics (for the CMC  
60 matrix). However, when the substrate is discontinuous, such as a bed of particles, the CVD process must be  
61 significantly redesigned to ensure a homogeneous distribution of the gas phase around all the particles, to  
62 ultimately obtain a coating of uniform composition, structure and thickness. Conventional CVD reactors  
63 typically include a hot zone where the parts to be coated or infiltrated are carefully placed so that the gases flow  
64 and react as uniformly as possible. Such a reactor design is obviously not suitable for short fibers. Conversely, it

65 is possible to combine a traditional CVD system with a vertical fluidization system (*i.e.*, the gas precursor inlet  
66 at the bottom and the gas effluent evacuation/treatment at the top) that simultaneously acts as a hot zone, a  
67 particle/gas mixing zone, and a deposition zone.

68 The fluidization process was developed at the beginning of the last century. Initially developed for ammonia  
69 production by the Haber process, fluidization later gained visibility for coal gasification and catalytic oil  
70 cracking. Coupled with CVD techniques, the process was used in the 1960s to coat the well-known TRISO fuel  
71 particles for high-temperature gas nuclear reactors [6–8]. This process succeeded in coating particulate substrates  
72 with high performance coatings at excellent yields, taking advantage of very efficient heat and mass transfer [9].

73 This process has been naturally chosen to coat a more original substrate here: short ceramic fibers. These  
74 particles have a high shape factor and are generally not classified in the different categories of fluidizable  
75 particles. The ability of these kind of particles to be fluidized has been mainly studied through modeling [10–16]  
76 and very few studies have described the fluidization behavior of short fibers from an experimental point of view.  
77 Furthermore, no studies have addressed the fluidization of such substrates under the specific conditions  
78 investigated here for the synthesis of interphases and protective coatings. Indeed, the specific conditions of high  
79 temperature and low pressure required are not described in the literature. A hydrodynamic study would help to  
80 clarify the specificities or similarities with other studies carried out on more classical substrates.

81 Preliminary experiments have shown that the geometry and the characteristic dimensions of short fibers, as well  
82 as surface interactions, lead to difficulties in maintaining a stable fluidization of a load that would consist only of  
83 short fibers. The elongation of the particles favors collisions, while their micrometric size enhances  
84 physicochemical interactions, thus preventing a proper fluidization. Due to chemical interactions or mechanical  
85 entanglements, short fibers tend to gradually agglomerate, which may eventually lead to a fixed bed. Similar  
86 observations have been reported in the case of fine cohesive spherical particles belonging to the C category  
87 according to the Geldart's classification, *e.g.* with particle diameters below 15  $\mu\text{m}$  or with low density below  
88  $1000 \text{ kg}\cdot\text{m}^{-3}$  [17]. At these scales, the surface forces are no longer negligible, which can severely affect or even  
89 prevent fluidization. Several techniques can be used to overcome these difficulties. Among them are the  
90 mechanical stirring by rotating blades inserted within the bed [18–20], the addition of acoustic vibrations  
91 towards the bed [21–24], the mechanical vibration of the whole fluidized bed support [25–28], the application of  
92 electromagnetic pulses to magnetized particles [29,30] or the addition of easily fluidizable particles to the bed  
93 [31,32]. The last solution is obviously the easiest to adapt to our equipment. It is chosen here through the

94 addition to the short fibers of SiC powder belonging to the A/B category according to the Geldart's  
95 classification. The interaction between these particles prevents cohesion between fibers and makes the bed more  
96 fluid, acting as a "lubricant". A few numerical studies have been carried out on mixtures of high form factor  
97 particles with near-spherical powders [13,16]. However, these models do not consider ideal hydrodynamic  
98 behavior, neglect surface forces due to physicochemical interactions between particles, and even investigate  
99 millimeter rather than micrometer particles.

100 We have previously investigated the fluidization behavior of a fiber/powder mixture and compared it to that of a  
101 pure powder bed [33]. The study demonstrates the possibility of fluidizing a load with a fraction of short fibers  
102 mixed with micrometric powders. It also suggests the possibility of predicting the minimum fluidization velocity  
103 using the classical Ergün's equation [34], but also using the equation of Zarekar *et al.* [35] and a modified  
104 Darcy-Klinkenberg equation. An adjustment of the experimental data has allowed us to determine, with the help  
105 of these three equations, the intrinsic properties of the fiber/powder mixture, *i.e.*:

- 106 • The hydraulic diameter ( $\Psi_s d_g$ ), derived from the Ergün and Zarekar equations
- 107 • The Knudsen permeability (K) and diffusion coefficients ( $D_K$ ), from the modified Darcy-Klinkenberg  
108 equation.

109 In this work, the effect of the addition of short fibers to a load initially made of powder will be examined.  
110 Several mixtures are fluidized at room temperature and at pressures ranging from 10 to 800 mbar. The  
111 monitoring of the minimum fluidization velocities is confronted with different equations adapted to the  
112 conditions used and the intrinsic properties of the mixtures are determined. The study highlights the importance  
113 of using equations adapted to hydrodynamic flow conditions. Indeed, in the case of fluidization under CVD  
114 conditions (*i.e.* with the use of moderate gas velocities and low pressures) the flow tends to be rarefied rather  
115 than inertial, consequently, the vast majority of  $U_{mf}$ 's prediction equations fail in their prediction. Finally, the  
116 study will propose a prediction model of  $U_{mf}$  according to simple parameters.

## 117 **2. Materials and methods**

118 The different elements of the fluidized bed reactor and the particle characteristics of the powder and short fibers  
119 have been described in detail elsewhere [33]. The hydrodynamic study was performed on SIKA<sup>®</sup> TECH silicon  
120 carbide (SiC) powders from Fiven, and Hi-Nicalon type S SiC fibers from NGS Advanced Fibers Co., Ltd. The  
121 particle size as well as the diameter and length distributions of the SiC powder and fibers are shown in Fig. 1.

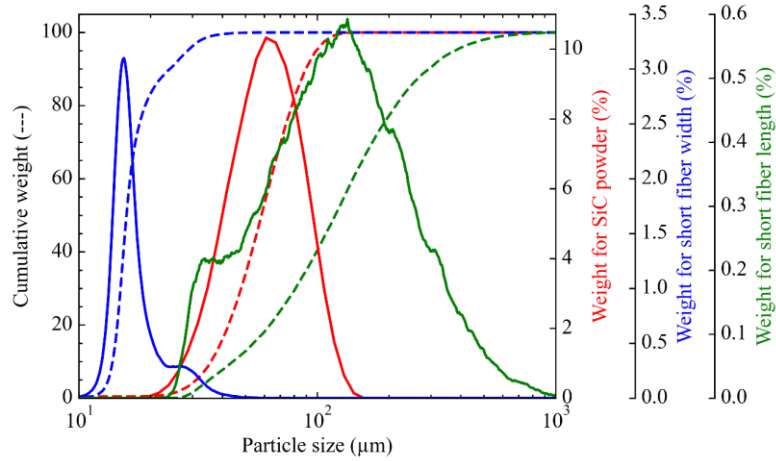


Fig. 1 : Particle sizes distributions of short fibers length, diameter and SiC powder diameter.

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124 The mixtures are homogenized in the reactor for a few minutes to ensure optimal mixing of the particles.  
 125 Fluidization tests are then performed for different short fiber/powder ratios at different pressures. For each  
 126 pressure and each fiber/powder ratio, the plot of the bed pressure drop as a function of the decreasing carrier gas  
 127 velocity (*e.g.* nitrogen) provides the minimum fluidization velocity ( $U_{mf}$ ). Furthermore, measurements of the bed  
 128 height at different velocities are used to determine the normalized expansion and the porosity of the bed at the  
 129 minimum fluidization velocity. The bed height is measured using a graduated ruler placed next to the quartz  
 130 fluidization column. The height is determined as the average of the maximum and minimum heights of the bed  
 131 along the column walls during the fluidization process.

### 132 2.1. Hydrodynamic laws

133 Several authors have been interested in predicting the minimum fluidization velocity using reference equations,  
 134 semi-empirical correlations or numerical modeling. It is difficult to compare and evaluate the relevance of all of  
 135 these studies since each author uses specific fluidization operating conditions. All correlations are specific to  
 136 particular experimental conditions, either by using different pressures, temperatures, particles with variable  
 137 physical and geometrical properties, etc... Nevertheless, a significant number of studies are based on the semi-  
 138 empirical correlation derived from the Ergün equation. This correlation proposes to relate the dimensionless  
 139 Reynolds number:  $Re_{mf} = (U_{mf}\rho_g d_g)/\mu$  and the Archimedes number:  $Ar = (g(\rho_p - \rho_g)\rho_g d_g^3)/\mu^2$  numbers by the  
 140 following Equation (1).

$$Re_{mf} = \sqrt{A^2 + BAr} - A \quad \text{Equation (1)}$$

141 Equation (1) is derived from the numerical solution of the Archimedes number equation as a function of  
 142 Reynolds number, see Equation (2).

$$Ar = K_1 Re_{mf}^2 + K_2 Re_{mf} \quad \text{Equation (2)}$$

143 Where  $K_1$  and  $K_2$  are the inertial and laminar flow coefficients, respectively, and  $A$  and  $B$  are the coefficients  
 144 resulting from the numerical solution of Equation (2), *i.e.*  $A = K_2/(2K_1)$  and  $B = 1/K_1$ . Many authors propose  
 145 values for these last coefficients, some of them are listed in Table 1.

146 **Table 1 : Values of the  $A$  and  $B$  coefficients for the calculation of the minimum fluidization.**

Authors	Parameters	
	$A$	$B$
Babu <i>et al.</i> [39]	25.25	0.0651
Bourgeois and Grenier [37]	25.46	0.0384
Chitester <i>et al.</i> [42]	28.70	0.0494
Richardson and Jerónimo [40]	25.70	0.0365
Saxena and Vogel [38]	25.28	0.0571
Tannous <i>et al.</i> [43]	25.83	0.0430
Thonglimp <i>et al.</i> [41]	31.60	0.0452
Wen and Yu [36]	33.70	0.0408

147 Under subatmospheric pressure fluidization conditions, and especially at low pressures (*e.g.*  $< 50$  mbar), other  
 148 studies [44,35,45] emphasize the relationship between the coefficient  $K_2$  in Equation (2) and the Knudsen  
 149 number:  $Kn = (K_B T)/(\sqrt{2}\pi\sigma^2 P d_p)$ . For example, Zarekar *et al.* [35] propose an inverted second-order polynomial  
 150 equation of the form  $K_2 = (7.10 \cdot 10^{-4} + 6.25 \cdot 10^{-3} Kn + 3.52 \cdot 10^{-3} Kn^2)^{-1}$ . Weesasiri *et al.* [45] propose a linear  
 151 dependence of the form:  $K_2 = 2.52 \cdot 10^3 - 7.53 \cdot 10^4 Kn$ . Finally, Llop *et al.* [44] distinguish two cases depending on  
 152 the sphericity of the particles. In the case of near-spherical particles ( $\Psi_s > 0.8$ ):  $K_2 = (6.06 \cdot 10^{-4} - 1.98 \cdot 10^{-2} Kn)^{-1}$   
 153 and in the case of non-spherical particles ( $0.4 > \Psi_s > 0.8$ ):  $K_2 = (7.41 \cdot 10^{-4} - 1.51 \cdot 10^{-2} Kn)^{-1}$ . These considerations  
 154 best approximate the peculiar behavior of the minimum fluidization velocity at low pressure, by the Knudsen  
 155 number proportional to  $P^{-1}$ .

156 The mathematical developments of the Ergün and modified Darcy-Klinkenberg equations have been described in  
 157 detail elsewhere [33] and the results are respectively summarized below, see Equations (3) and (5). The equation  
 158 of Zarekar *et al.* [35] has also been considered in its most complete form by introducing the squared Knudsen  
 159 term, *i.e.*  $4.96 Kn^2$ , representing the rarefied flow, which cannot be neglected at low pressure, see Equation (4).

The Ergün equation is described below:

$$U_{mf} = \sqrt{\left(\frac{150\mu(1 - \varepsilon_{mf})}{3.5\rho_g(\Psi_s d_g)}\right)^2 + \frac{\varepsilon_{mf}^3(\rho_p - \rho_g)g(\Psi_s d_g)}{1.75\rho_g}} - \frac{150\mu(1 - \varepsilon_{mf})}{3.5\rho_g(\Psi_s d_g)} \quad \text{Equation (3)}$$



As well as the Zarekar's equation:

$$U_{mf} = \sqrt{\left(\frac{25.7\mu}{\rho_g(\Psi_s d_g)(1 + 8.8Kn + 4.96Kn^2)}\right)^2 + 0.0365 \frac{(\rho_p - \rho_g)g(\Psi_s d_g)}{\rho_g}} - \frac{25.7\mu}{\rho_g(\Psi_s d_g)(1 + 8.8Kn + 4.96Kn^2)} \quad \text{Equation (4)}$$

And the Darcy-Klinkenberg equation:

$$U_{mf} = (\rho_p - \rho_g)(1 - \varepsilon_{mf})g \left[ \frac{K}{\mu} + \frac{D_K}{P_{mean}} \right] \quad \text{Equation (5)}$$

## 160 2.2. Methodology

161 Our previous study showed a limited effect of the temperature on  $U_{mf}$  compared to that of the pressure  
 162 (especially at low pressures). Therefore, in the present study, the minimum fluidization rate was evaluated only  
 163 as a function of the pressure (from 10 to 800 mbar) at room temperature. These experiments allow the direct  
 164 visualization of the fluidized bed through the quartz column. Moreover, the surface interactions between the  
 165 particles are limited at room temperature (as opposed to high temperatures where surface reactions are  
 166 activated), which allows to stabilize the fluidization behavior of the bed during the experiments. Variables such  
 167 as the density ( $\rho_g$ ) or carrier gas viscosity ( $\mu$ ) are calculated under the considered temperature and pressure  
 168 conditions. The porosity at minimum fluidization is derived from the bed expansion measurements using  
 169 Equation (6).

$$\varepsilon_{mf} = 1 - \frac{m}{\rho_p S H_{mf}} \quad \text{Equation (6)}$$

170 Where  $m$  is the mass of the fluidized bed,  $S$  is the cross-sectional area of the fluidization column,  $H_{mf}$  is the  
 171 absolute height of the fluidized bed at minimum fluidization conditions, and  $\rho_p$  is the particle density. Therefore,  
 172 for each fiber fraction in the mixture and for each pressure,  $\varepsilon_{mf}$  can be calculated. This value at minimum  
 173 fluidization velocity varies only slightly with pressure. An average value of  $\varepsilon_{mf}$  can then be deduced for each  
 174 short fiber/powder ratio, regardless of the pressure considered.

175 The Levenberg-Marquardt algorithm, developed by Levenberg [46] in 1944 and published by Marquardt [47] in  
 176 1963, provides a numerical solution to the problem of minimizing nonlinear functions dependent on one or more  
 177 variables. The algorithm, based on the Gauss-Newton methods and the gradient algorithm, minimizes the sum of  
 178 squares of the deviations between the experimental and the fitted data (least squares minimization). This method  
 179 is used here to fit the intrinsic parameters of the different particle loads, via the equations proposed above, see  
 180 Equations (3), (4) and (5). The algorithm also provides the standard deviation of each parameter.

181 The minimum fluidization velocities ( $U_{mf}$ ) are determined from the pressure drop curves at decreasing flow rates  
 182 and using an algorithm for determining the pressure drop slope at low flow rates and the plateau at the highest  
 183 flow rates. This algorithm performs two linear regressions, giving access to  $U_{mf}$  and the error associated with the  
 184 fit. From the experimental values of the minimum fluidization velocity as a function of pressure (P), a regression  
 185 of the Ergün and Zarekar equations directly yields the product  $\Psi_s d_g$ . However, it is important to note that the  
 186 Ergün equation does not take into account the experimental data at low pressure (*i.e.* where the rarefied regime is  
 187 no longer negligible). In fact, the Ergün equation considers laminar to inertial flow and therefore does not  
 188 consider the modifications of  $U_{mf}$  in the case of a flow that tends to become rarefied (which is actually the case at  
 189  $P < 100$  mbar, where the contribution of the rarefied flow is at least 20%). Conversely, the Zarekar and modified  
 190 Darcy-Klinkenberg equations integrate the rarefied flow at low pressures. Therefore, the low-pressure  
 191 experimental points should be carefully considered for variable adjustment.

192 While both Ergün's and Zarekar's equations are able to account for the evolution of the hydraulic diameter ( $\Psi_s d_g$ )  
 193 as a function of the fiber fraction in the mixtures, the Darcy-Klinkenberg equation only provides an estimation of  
 194 the permeability coefficient  $K = (\varepsilon_{mf}^3 d_g^2)/(32\eta_v(1-\varepsilon_{mf})^2)$  and the Knudsen diffusion coefficient  $D_K =$   
 195  $(\varepsilon_{mf}^2 c d_g)/(3\eta_K (1-\varepsilon_{mf}))$ , as a function of the fiber fraction. These coefficients involve two different corrective  
 196 terms, which are equivalent to the sphericity ( $\Psi_s$ ) of the particles in the Ergün and Zarekar equations: the viscous  
 197 and Knudsen tortuosities, respectively  $\eta_v$  and  $\eta_K$ . At this stage, it is difficult to introduce a sphericity of the  
 198 fibers or the mixtures, by analogy to that of the powder, in order to introduce it in the fitted hydraulic diameter  
 199 values. Sphericity is a parameter that is relevant in the case of powder alone, but the elongated and cylindrical  
 200 geometry of the short fibers may require the introduction of another more appropriate shape factor to correct the  
 201 grain diameter ( $d_g$ ).

202 Fluidized bed expansion can be followed as a function of several parameters including the gas velocity excess  
 203 ( $U-U_{mf}$ ), the minimum fluidization velocity ( $U_{mf}$ ), the particle density ( $\rho_p$ ), the gas density ( $\rho_g$ ), or even the grain  
 204 diameter ( $d_g$ ). Several authors have proposed their own empirical correlation, using some or all of the latter  
 205 parameters [42,48–50]. The general form and the coefficients proposed by some authors have been described  
 206 elsewhere [33]. The general equation can be recalled as follows in Equation (7).

$$H^* = \frac{H}{H_{mf}} = 1 + \alpha(U - U_{mf})^{n_1} U_{mf}^{n_2} \rho_p^{n_3} \rho_g^{n_4} d_g^{n_5} \quad \text{Equation (7)}$$

207 In our previous study [33], the coefficients  $n_1 = 1$  and  $n_4 = 0.2108 \pm 0.0313$  have been proposed for two different  
 208 types of load, the first one consisting only of powder and the second one of a certain fiber fraction in the mixture.  
 209 In the present case, the variation of the short fiber fraction in the mixture allows to vary the diameter of the load  
 210 while keeping the other parameters fixed, in order to deduce the exponent  $n_5$ .

### 211 3. Results

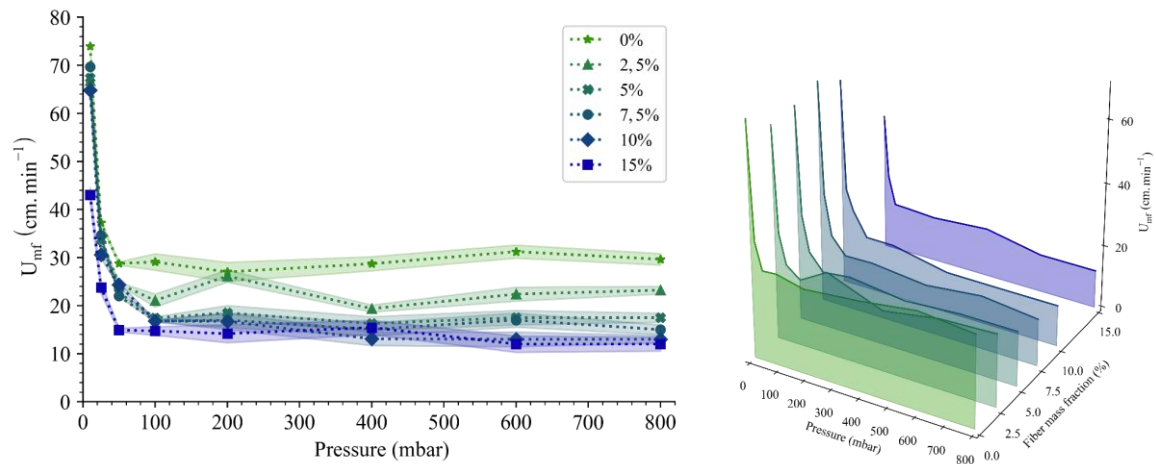
#### 212 3.1. Velocities and porosities at minimum fluidization

213 The hydrodynamic study investigates the effect of adding particles with a high shape factor (short fibers) to a  
 214 powder with a shape factor close to 1. Six different mixtures have been submitted to fluidization tests with  
 215 increasing fiber mass ratios from 0 to 15%, corresponding to bulk volume fractions of 0 to 30.1% or tapped  
 216 volume fractions of 0 to 26.1%, respectively, see Table 2. Each mixture is subjected to the gas flow at several  
 217 pressures (ranging from 10 to 800 mbar) and the pressure drop between the bottom and the top of the bed is  
 218 plotted against the decreasing gas velocity.

219 *Table 2 : Mass, bulk and tapped percentages of fibers in fiber/powder mixtures.*

Fiber mass fractions	0%	2.5%	5.0%	7.5%	10.0%	15.0%
Fiber bulk volume fractions	0%	5.9%	11.4%	16.5%	21.3%	30.1%
Fiber tapped volume fractions	0%	4.9%	9.5%	13.9%	18.2%	26.1%

220 The variation of the pressure drop with the decreasing gas velocity allows the determination of the minimum  
 221 fluidization velocity  $U_{mf}$ , for all pressures and fiber fractions considered. These minimum fluidization velocities  
 222 are plotted against pressure in Fig. 2. The contribution of the rarefied flow increases as the fluidization pressure  
 223 decreases. In fact, below 100 mbar, the minimum fluidization velocity increases rapidly (up to almost 250%).  
 224 For pressures above 100 mbar,  $U_{mf}$  remains stable. The addition of fibers to the mixture generally results in a  
 225 decrease of  $U_{mf}$ , *e.g.* in the range of 100-800 mbar:  $U_{mf} = 29 \text{ cm}\cdot\text{min}^{-1}$  without fibers, versus  $14 \text{ cm}\cdot\text{min}^{-1}$  with 15  
 226 mass % of fibers, *i.e.*, a reduction of  $U_{mf}$  of approximately 50%.



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**Fig. 2 : Minimum fluidization velocity ( $U_{mf}$ ) as a function of pressure for various fiber/powder mixtures (error bars appear as colored beams delimited by the low and high values for each experimental point).**

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The measurement of the normalized bed height ( $H^* = H/H_{mf}$ ) can be used to determine its porosity, especially at the minimum fluidization velocity ( $\epsilon_{mf}$ , Equation (6)). Fig. 3 shows the evolution of  $\epsilon_{mf}$  as a function of the short fiber mass fraction. The addition of fibers gradually decreases the bed porosity at the minimum fluidization velocity. The porosity  $\epsilon_{mf}$  indeed decreases from 0.633 without fibers to 0.590 with 15 mass % fibers, *i.e.* a reduction of about -7%.

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This tendency may be surprising since short fibers are more elongated than powder particles. The diameter of the fibers being lower than that of the powder particles, short fibers added at low percentages will fill the space between the particles (especially the shortest fibers between the largest particles), resulting in a decrease in porosity. On the other hand, since a pure short fiber bed has a higher porosity than a pure powder bed, it would not be surprising to find that the porosity of a high-fiber fraction mixture tends to increase with the amount of added fibers. Unfortunately, these high-fraction mixtures could not be studied because of their unsuitability for fluidization. In fact, a channeling phenomenon gradually occurred, the bed becoming static and thus the pressure drop curves could not be exploited.

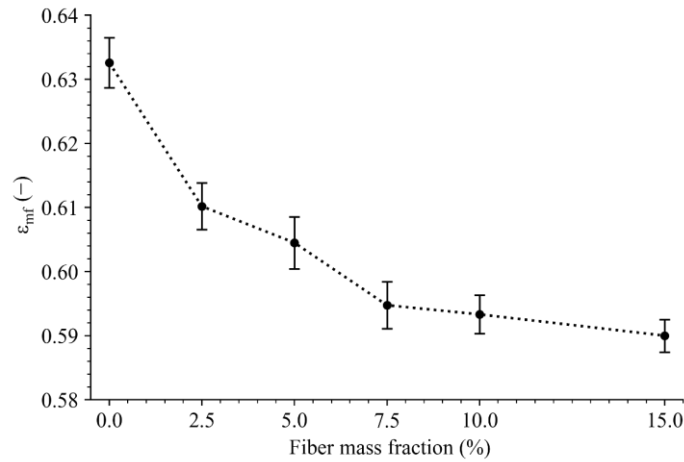


Fig. 3 : Evolution of the average porosity at minimum fluidization ( $\epsilon_{mf}$ ) as a function of the fiber fraction in the mixture.

### 3.2. Ergün and Zarekar equations

The variable  $\Psi_s d_g$  represents the hydraulic diameter and is similar to the sphere diameter of the equivalent surface-to-volume ratio:  $d_{sv}$  (or Sauter diameter).  $\Psi_s d_g$  is also the main fitting parameter for the Ergün and Zarekar hydrodynamic equations, see Equations (3) and (4). This quantity is fitted and plotted for each  $U_{mf}(P)$  curve and each fiber/powder ratio, assuming an Ergün or Zarekar type relationship, see Fig. 4. The fit, based on the Ergün equation, gives lower  $\Psi_s d_g$  values (34 to 29  $\mu\text{m}$ ) than for the Zarekar equation (61 to 43  $\mu\text{m}$ ). In both cases, the hydraulic diameter tends to decrease as the mass fraction of fibers increases. This trend indicates that in terms of fluidization, the equivalent size of the particle mixture felt by the gas stream decreases as the fiber mass fraction increases, which is consistent with the decrease of the minimum fluidization velocity observed simultaneously.

The difference between the results of the Zarekar and Ergün equations, see Fig. 4, lies in the basic considerations for applying these two equations. For instance, the Ergün's equation includes empirical corrective factors found in Equation (3), which are of the form:  $150/3.5$  for the Darcy laminar flow part and  $1/1.75$  for the Forchheimer inertial flow part, while Zarekar uses the Richardson's correlated coefficients [40] 25.7 and 0.0365, in addition to the correction for rarefied flow via the Knudsen dimensionless number, of the following expression:  $1 + 8.8\text{Kn} + 4.96\text{Kn}^2$ . These different considerations inevitably imply different results for the hydraulic diameter.

The sphericity value ( $\Psi_s$ ) acts as a corrective coefficient for the physical particle diameter ( $d_g$ ). However, it is impossible to calculate or even estimate by any means the sphericity value of a powder and short fiber mixture. Moreover, there is no evidence that sphericity as defined by Wadell [51], remains the relevant corrective factor to describe the behavior of the mixture after the addition of fibers. Assuming that sphericity is indeed the

265 corrective factor to be considered, assigning a specific value for a mixture of particles of very different  
 266 geometries (in this case a powder and fibers), simply from their respective mass fractions, may seem  
 267 inappropriate given the complexity of the problem. It was therefore decided to keep the whole  $\Psi_s d_g$  product as  
 268 the main variable of interest for the hydrodynamic calculations.

269 The hydraulic diameter can be related to the fiber mass fraction to deduce the corresponding minimum  
 270 fluidization velocity. For this purpose, it was decided to use the data derived from the fitting of the Zarekar  
 271 equation. The chosen equation is of the form:  $\Psi_s d_g = a(\%_f + b)^n$ , where a and b are positive coefficients and n a  
 272 negative coefficient with an absolute value less than 1. The hydraulic diameter  $\Psi_s d_g$  is approximated by the  
 273 Sauter diameter  $d_{sv}$ . In this case, without fibers, the hydraulic diameter is equal to the Sauter's diameter of the  
 274 powder:  $\Psi_s d_g(0\%_f) = d_{sv}^p$ , while with the fibers only it becomes:  $\Psi_s d_g(100\%_f) = d_{sv}^f$ . Under these conditions, it  
 275 comes:

$$\begin{cases} \Psi_s d_g(0\%_f) = a(\%_f + b)^n = d_{sv}^p \\ \Psi_s d_g(100\%_f) = a(\%_f + b)^n = d_{sv}^f \end{cases} \quad \text{Equation (8)}$$

276 With the respective fiber mass fractions:

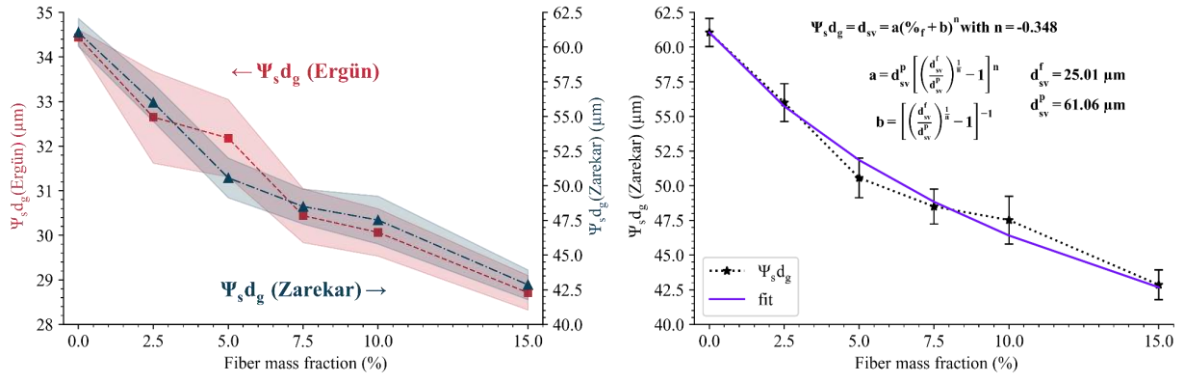
$$\begin{cases} ab^n = d_{sv}^p \\ a(1 + b)^n = d_{sv}^f \end{cases} \quad \text{Equation (9)}$$

277 That is to say:

$$\begin{cases} a = d_{sv}^p \left[ \left( \frac{d_{sv}^f}{d_{sv}^p} \right)^{n-1} - 1 \right]^{-n} \\ b = \left[ \left( \frac{d_{sv}^f}{d_{sv}^p} \right)^{n-1} - 1 \right]^{-1} \end{cases} \quad \text{Equation (10)}$$

278 The Sauter diameter of the fibers and the powder are obtained from optical microscopy images. Each image is  
 279 processed individually by a powder or fiber edge detection algorithm. The entities partially masked or cut-off by  
 280 the edges of the image frame are removed. Analysis of the closed contours provides the projected area of each  
 281 particle and allows to calculate the ratio between the measured projected area and the equivalent spherical  
 282 volume. A series of images is used to obtain an average of the measurements.

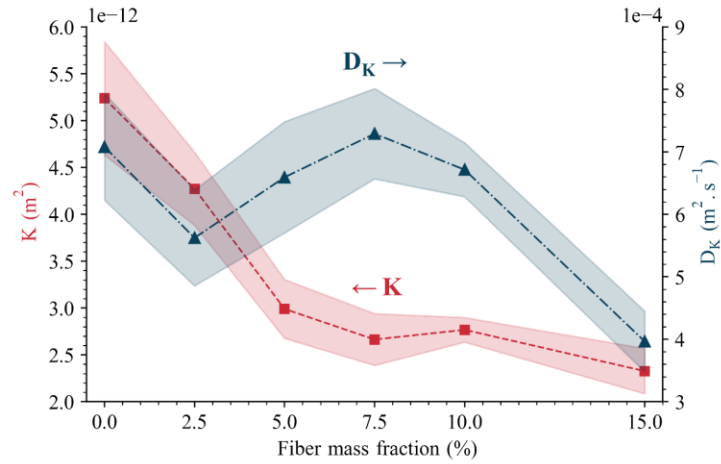
283 The fitting parameter for the variable  $n$  yields a satisfactory correlation ( $r^2 = 0.9860$ ), see Fig. 4, with  $d_{sv}^f = 25.0$   
 284  $\pm 0.8 \mu\text{m}$ ,  $d_{sv}^p = 61.1 \pm 1.0 \mu\text{m}$  and  $n = -0.35 \pm 0.01$ , which allows the prediction of the hydraulic diameter for  
 285 any ratio of fibers in the fiber/powder mixture.



286  
 287 **Fig. 4 : Evolution of hydraulic diameter ( $\Psi_s d_g$ ) (left) and Zarekar hydraulic diameter fit as a function of fiber mass fraction in the**  
 288 **mixture (right).**

### 289 3.3. Darcy-Klinkenberg equation

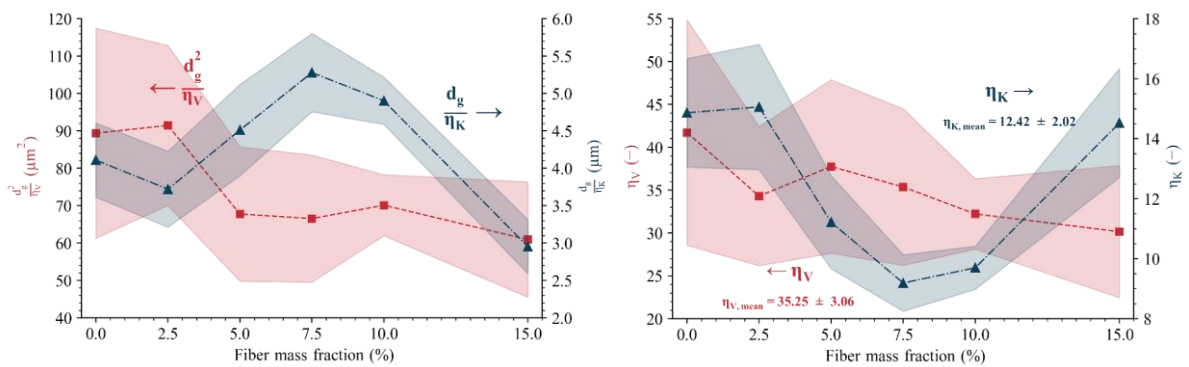
290 The variable fitting parameter of the experimental data, taking into account the Darcy-Klinkenberg equation,  
 291 allows the evaluation of the permeability coefficients  $K$  and  $D_K$  at any fiber/powder ratio, see Fig. 5. The  
 292 permeability  $K$  decreases by a factor of about 2 as the fiber mass fraction increases from 0 to 15%. In contrast,  
 293 the Knudsen coefficient  $D_K$  does not show a monotonic behavior as a function of the fiber mass fraction. This  
 294 unexpected behavior is difficult to explain, as  $D_K$  depends simultaneously on Knudsen tortuosity, molecular  
 295 velocity and pore diameter. The results also show that it might be due to experimental errors. From these  
 296 coefficients and the porosities at the fluidization minimum, it is possible to obtain the quantities:  $d_g^2/\eta_V$  with  $K$   
 297 and  $d_g/\eta_K$  with  $D_K$ , see Fig. 6. Again, these quantities do not show a clear monotonic variation with the fiber  
 298 mass fraction.



299

300 *Fig. 5 : Evolution of the permeability coefficient (K) and Knudsen coefficient (D<sub>K</sub>) as a function of the fiber mass fraction in the mixture.*

301 Like the sphericity ( $\Psi_s$ ) in the Ergün and Zarekar equations, the viscous and Knudsen tortuosities ( $\eta_V$  and  $\eta_K$ )  
 302 act as corrective factors towards the grain diameter ( $d_g$ ) in the Darcy-Klinkenberg equation. Since it is not  
 303 possible to deduce the tortuosities experimentally without a specific study of the porous medium, it was decided  
 304 to use the empirical correlation found previously for  $\Psi_s d_g$  instead of the grain diameter ( $d_g$ ) in the Darcy-  
 305 Klinkenberg equation. In this case, the tortuosities lose their corrective characters and become constant with the  
 306 variation of the fiber mass fraction. Under this condition,  $\eta_V$  and  $\eta_K$  are plotted as a function of the fiber mass  
 307 fraction, see Fig. 6. The calculated tortuosities appear relatively constant as a function of fiber mass fraction.  
 308 Therefore, the mean values  $\eta_{V,mean}$  and  $\eta_{K,mean}$  can be calculated and used in the Darcy-Klinkenberg equation,  
 309 where the grain diameter ( $d_g$ ) is replaced by the hydraulic diameter ( $\Psi_s d_g$ ) previously fitted.



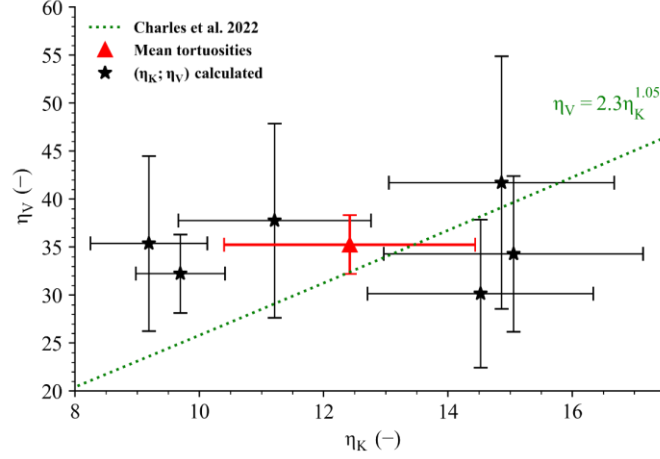
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311 *Fig. 6 : Evolution of the proportions  $d_g^2/\eta_V$  and  $d_g/\eta_K$  (left) and the tortuosities  $\eta_V$  and  $\eta_K$  (right) as a function of the mass fraction of*  
 312 *fiber in the mixture.*

313 The calculated tortuosities are also plotted against each other, see Fig. 7. Charles *et al.* [52] showed that viscous  
 314 and Knudsen tortuosities are correlated with each other in fibrous media, with the same correlation regardless of  
 315 the orientation of the fibers and the gas path. Charles *et al.* suggest a correlation between viscosity and Knudsen



316 tortuosities of the form:  $\eta_V = 2.3\eta_K^{1.05}$ . Considering the errors of  $\eta_V$  and  $\eta_K$ , the proposed correlation is in good  
 317 agreement with the experimental data. The mean tortuosities, *i.e.*  $\eta_{V,\text{mean}}$  and  $\eta_{K,\text{mean}}$ , deviate from the prediction  
 318 by only 8%.



319

320

Fig. 7 : Viscous against Knudsen tortuosities plotted with Charles et al. correlation.

321 The resulting Darcy-Klinkenberg equation can be rewritten in the following form:

$$U_{mf} = (\rho_p - \rho_g)(1 - \varepsilon_{mf})g \left[ \frac{1}{\mu} \frac{\varepsilon_{mf}^3 (\Psi_s d_g)^2}{32 \eta_{V,\text{mean}} (1 - \varepsilon_{mf})^2} + \frac{1}{P_{\text{mean}}} \frac{\varepsilon_{mf}^2 c (\Psi_s d_g)}{3 \eta_{K,\text{mean}} (1 - \varepsilon_{mf})} \right] \quad \text{Equation (11)}$$

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### 3.4. Bed expansion

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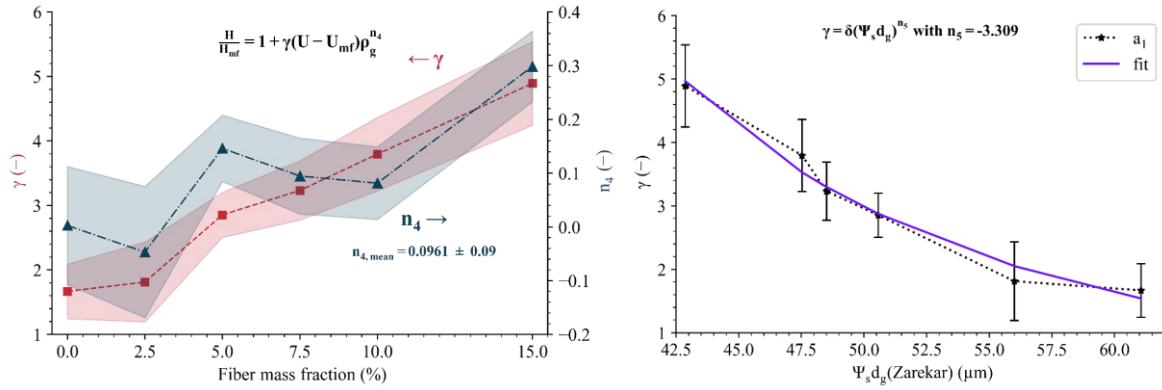
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The expansions are first fitted for each pressure and each fiber/powder ratio, with the following equation:  $1 + \beta(U-U_{mf})$ . Then, the coefficients  $\beta$  are plotted as a function of gas density for each pressure considered, for all fiber mass fractions. The curves obtained are then fitted again with a function of the type:  $\beta = \gamma \rho_g^{n_4}$ . This second fitting parameter allows to obtain the coefficients  $\gamma$  and  $n_4$  for each fiber mass fraction, see Fig. 8. There is a very minor increase of  $n_4$  as a function of the fiber mass fraction, probably indicating the effect of another parameter, not considered here. In the following, an average  $n_4$  coefficient equal to  $0.096 \pm 0.090$  will be considered. On the other hand, the coefficient  $\gamma$  contains information related to the other parameters mentioned before such as  $\rho_p$ , which remains constant here. For example, it is possible to determine the contribution of the equivalent hydraulic diameter to the bed expansion by a new variable fitting. The coefficient  $\gamma$  is thus plotted as a function of  $\Psi_s d_g$ , and a variable fitting is performed with a function of the type:  $\gamma = \delta (\Psi_s d_g)^{n_5}$ , see Fig. 8. The fitting yields results correlated with the experimental data ( $r^2 = 0.9794$ ) with an exponent  $n_5 = -3.309 \pm 0.263$  and a factor  $\delta = 1.75 \cdot 10^{-14} \pm 4.58 \cdot 10^{-14}$ .



335

336

*Fig. 8 : Evolution of  $\gamma$  and  $n_4$  coefficients as a function of fiber mass fraction in the mixture (left) and fit of  $\gamma$  as a function of  $\Psi_s d_g$  from Zarekar (right).*

337

$$H^* = \frac{H}{H_{mf}} = 1 + 1.75 \cdot 10^{-14} (U - U_{mf})^1 \rho_g^{0.096} (\Psi_s d_g)^{-3.309} \quad \text{Equation (12)}$$

338 The expansion of the bed can be summarized as follows:

339

Fig. 9 shows the fluidized bed expansions determined at different pressures and for the different fiber mass fractions considered here. The correlations proposed by Chitester *et al.* [42], Feng *et al.* [48], Lewis *et al.* [49] and Rashid *et al.* [50] are also plotted by approximating the grain diameter ( $d_g$ ) to the equivalent hydraulic diameter ( $\Psi_s d_g$ ) found via the Zarekar equation. The fluidized bed expansion determined experimentally increases with the fiber mass fraction and pressure, indicating that the coefficient  $n_4$  tends to be positive while the coefficient  $n_5$  is negative (when the fiber fraction increases, the equivalent hydraulic diameter decreases).

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Fig. 10 shows the relative errors made between the numerical calculations deduced from the different correlations and the experimentally measured fluidized bed expansions. The correlations proposed by Feng *et al.* and Rashid *et al.* deviate strongly from the measured fluidized bed expansions. For all pressures and all fiber mass fractions, the correlation of Chitester *et al.* underestimates the expansion by about 10%. The correlation of Lewis *et al.* and the one proposed here, see Equation (13), are in much better agreement with the experimental data for all pressures and all fiber mass fractions.

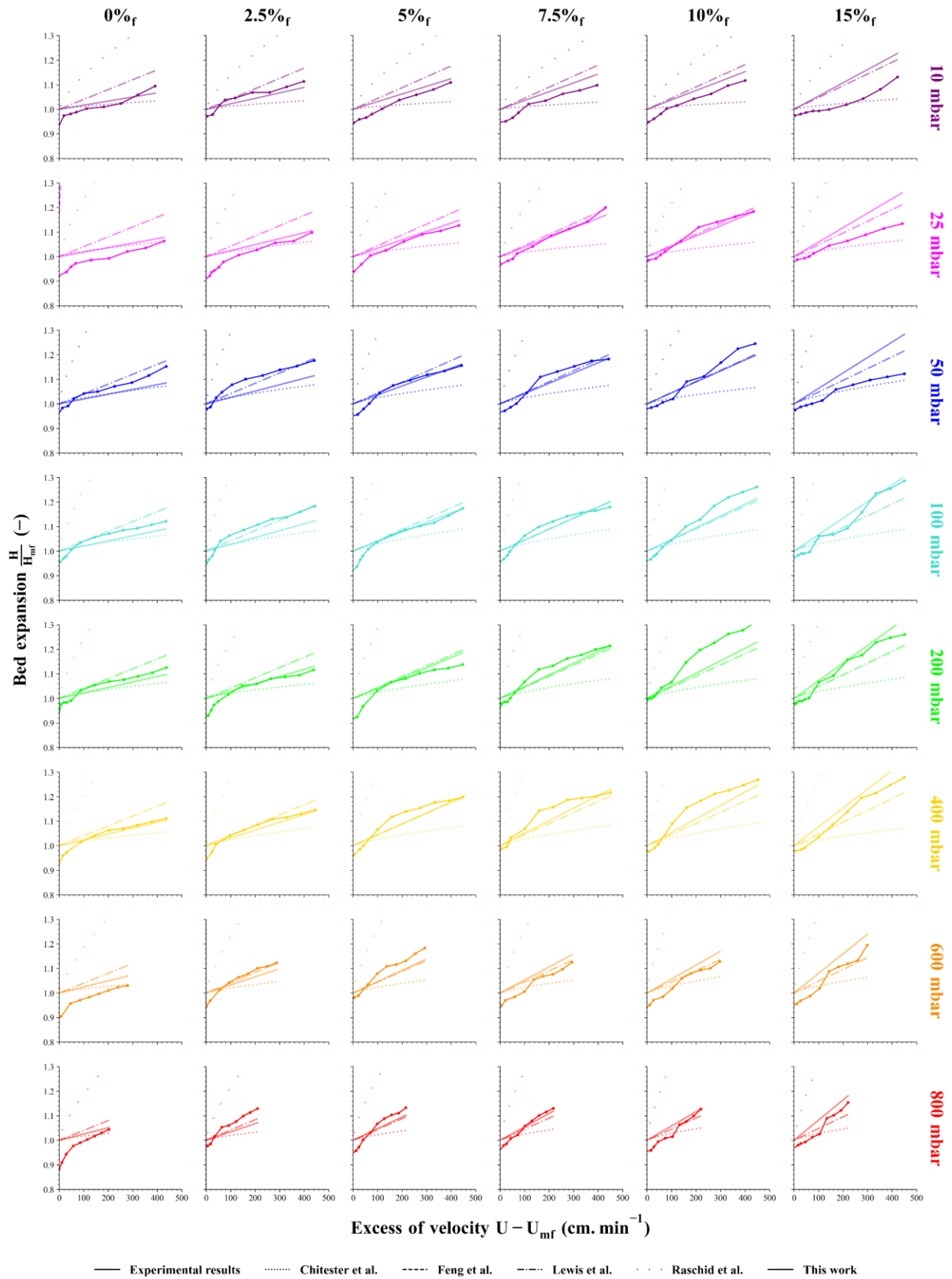
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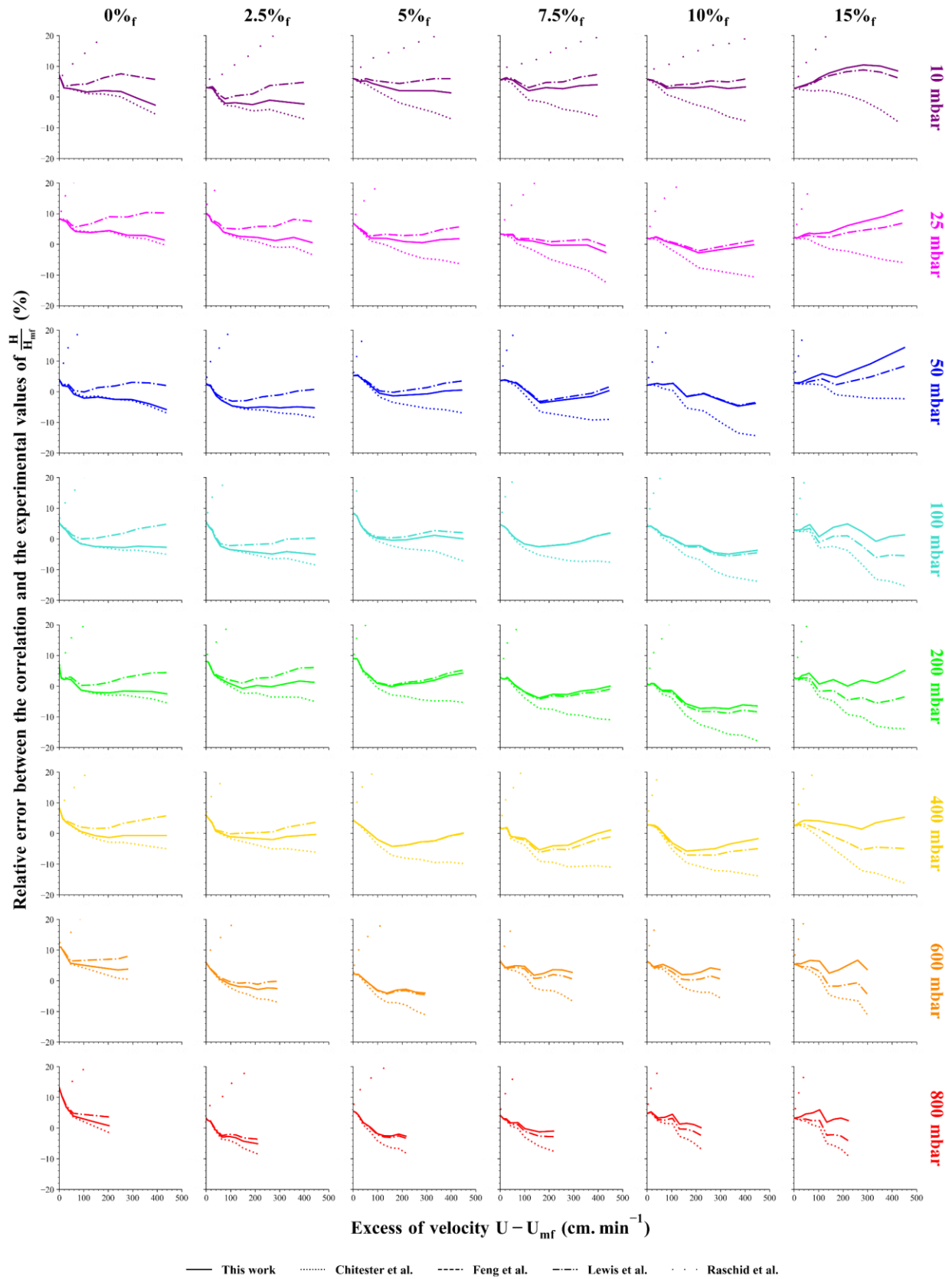
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Fig. 9 : Expansions  $H/H_{mf}$  as a function of gas velocity.



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*Fig. 10 : Relative error of the expansions  $H/H_{mf}$  as a function of gas velocity for Chitester, Feng, Lewis, Raschid and this work.*

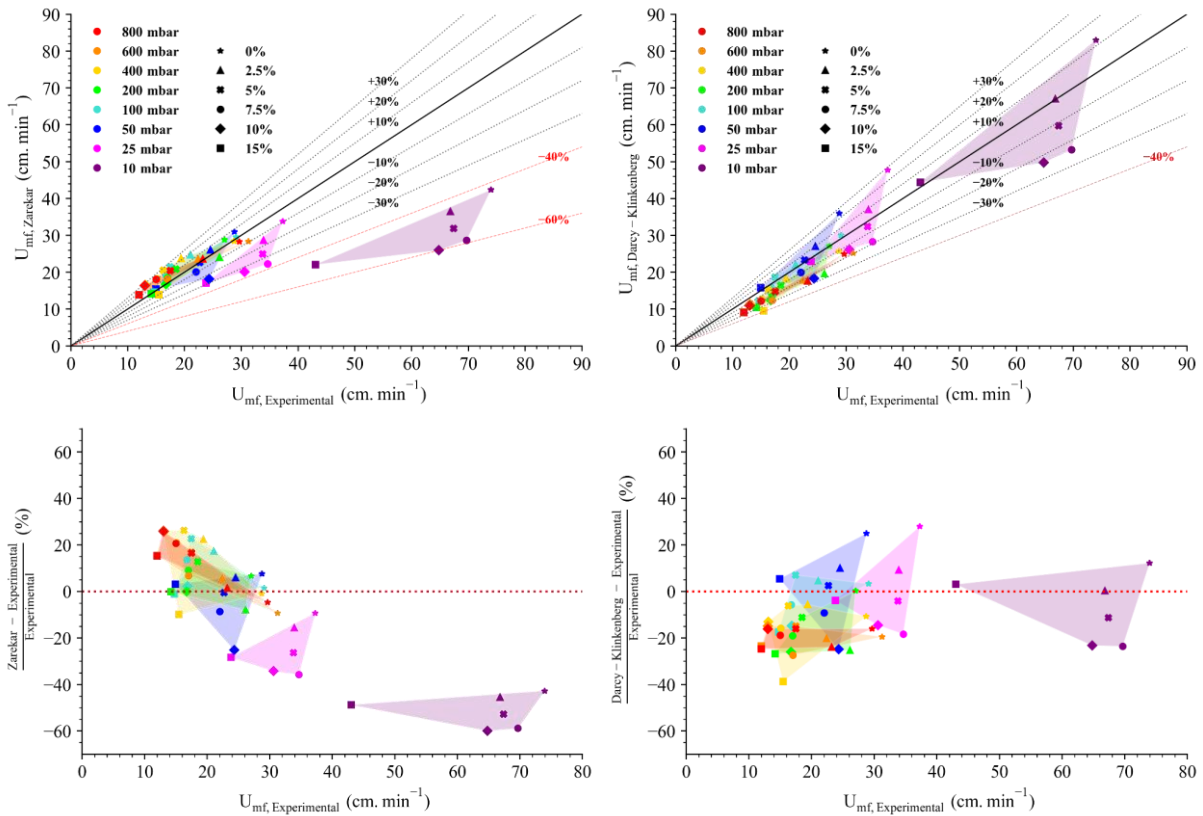
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## 356 4. Discussion

### 357 4.1. Prediction of the minimum fluidization velocity with Zarekar and Darcy-Klinkenberg

358 From the previously found fitting parameter of  $\Psi_s d_g$  as a function of the fiber mass fraction and the experimental  
359 determination of the porosity at the minimum fluidization ( $\epsilon_{mf}$ ), it is possible to determine the minimum  
360 fluidization velocity for each mixture considered using the Ergün and Zarekar equations. Nevertheless, the  
361 Ergün's equation will not be considered here because of the assumptions about the flow that it requires. Indeed,  
362 Ergün bases his semi-empirical equation on Darcy and Forchheimer flows, *i.e.* laminar to inertial flows. In  
363 reality, under the usual conditions of the fluidized bed and the types of load experienced in this study, the flows  
364 involved are rather laminar-rarefied than laminar-inertial.

365 The experimental data are confronted with the data calculated by the Zarekar equation, using the experimentally  
366 determined porosities at the fluidization minimum and the hydraulic diameter obtained previously by fitting, see  
367 Fig. 11. Similarly, with the Darcy-Klinkenberg equation and using  $\epsilon_{mf}$ ,  $\Psi_s d_g$ , and the mean tortuosities  $\eta_{v,mean}$   
368 and  $\eta_{k,mean}$ , it is possible to calculate the minimum fluidization velocities as a function of the pressure and the  
369 fiber mass fraction, see Fig. 11. In the case of the Zarekar equation, the calculated minimum fluidization  
370 velocities agree with the experimental data, except for the lowest pressures (10 and 25 mbar). In the latter case,  
371 the calculated minimum fluidization velocities underestimate the experimental velocities by about 40%, with a  
372 maximum error of about 60% at 10 mbar, for a fiber mass fraction of 10%. For the Darcy-Klinkenberg equation,  
373 the calculated and experimental data match for the entire pressure range and for all fiber fractions. Below 50  
374 mbar, the Darcy-Klinkenberg equation clearly gives a better prediction than the Zarekar equation.

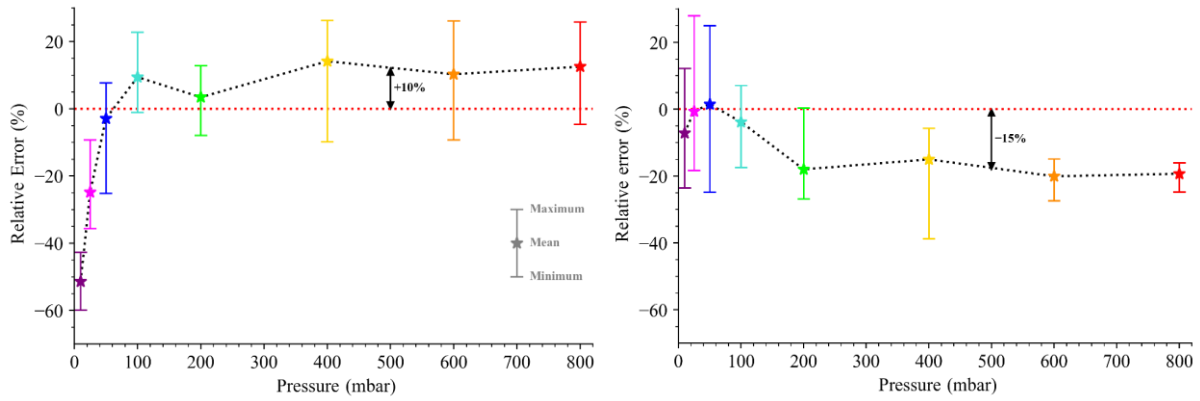


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376

377 **Fig. 11 : Comparisons of experimental and calculated minimum fluidization velocities (top) with Zarekar (left) and Darcy-Klinkenberg**  
 378 **(right). Relative errors made by the calculation compared to the experimental values (bottom) with Zarekar (left) and Darcy-Klinkenberg**  
 379 **(right).**

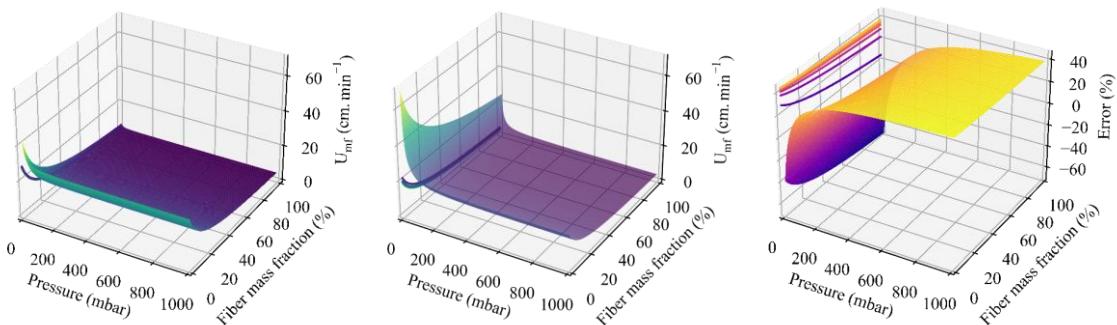
380 The average errors as well as the extreme errors (minimum and maximum) made between calculated and  
 381 experimental values are plotted as a function of the working pressure for both Zarekar and Darcy-Klinkenberg  
 382 equations, see Fig. 12. Again, the Darcy-Klinkenberg equation is more efficient in describing  $U_{mf}$  at low  
 383 pressures ( $< 100$  mbar), although the two equations are very similar in terms of the relative error at pressures  
 384 greater or equal to 100 mbar. The only notable difference is the overestimation or underestimation of the value of  
 385 the minimum fluidization velocity. In fact, while the Zarekar equation seems to slightly overestimate  $U_{mf}$  by  
 386 10%, the Darcy-Klinkenberg equation underestimates  $U_{mf}$  by about 15%. The Zarekar's equation is then quite  
 387 satisfactory for pressures greater than or equal to 100 mbar and has the advantage of not requiring any  
 388 assumption about the tortuosity values of the porous medium.



389

390 **Fig. 12 : Average errors with minimum and maximum errors of the calculations with Zarekar (left) and Darcy-Klinkenberg (right) for**  
 391 **each pressure.**

392 While the Ergün equation is inadequate for predicting the minimum fluidization velocity at reduced pressures  
 393 and rarefied flows, the Zarekar and Darcy-Klinkenberg equations better describe the variations of  $U_{mf}$ , see Fig.  
 394 13. At pressures above 100 mbar, using the Zarekar equation, see Equation (4), instead of the modified Darcy-  
 395 Klinkenberg equation, see Equation (11), results in an average error of 33%. At 10 mbar, for any fiber fraction,  
 396 the error increases and reaches an average of -61%, *i.e.* a severe underestimation of the minimum fluidization  
 397 velocity.

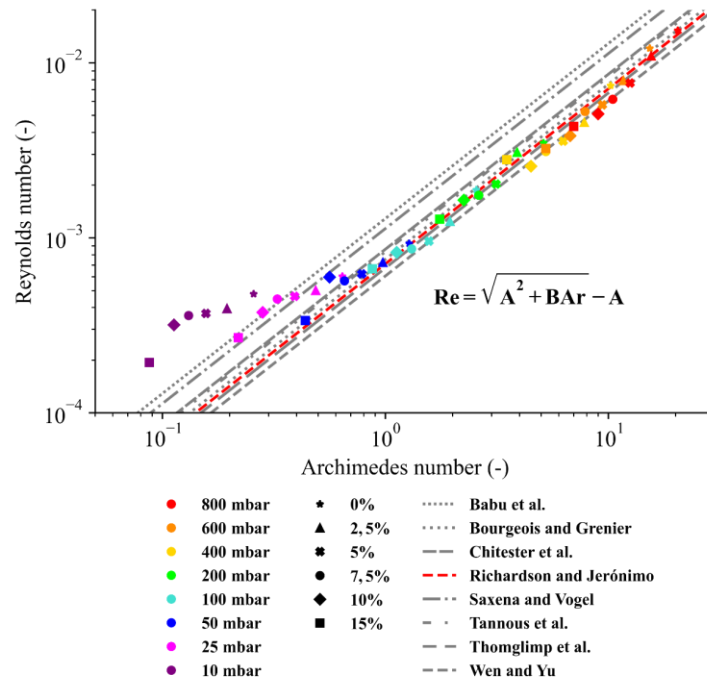


398

399 **Fig. 13 : Predictions of  $U_{mf}$  as a function of pressure and fiber mass fraction with Zarekar (left); Darcy-Klinkenberg (middle) and the**  
 400 **error made by considering the Zarekar equation rather than the Darcy-Klinkenberg one with  $\epsilon_{mf} = 0.6$  (right).**

401 From the values of  $U_{mf}$ , the temperature/pressure conditions, and the  $\Psi_s d_g$  values, it is possible to calculate the  
 402 Reynolds (Re) and Archimedes (Ar) numbers. These two dimensionless numbers are plotted against each other  
 403 in Fig. 14. In addition to these data, the semi-empirical correlations listed in Table 1 are also plotted. The  
 404 hydrodynamic considerations of the semi-empirical equations are obviously not valid below 50 mbar. The points  
 405 derived from the minimum fluidization values measured at low pressures ( $\leq 50$  mbar) deviate from the behavior

406 expected by the semi-empirical correlations proposed by the different authors. Beyond this deviation, it is the  
 407 form of Equation (1) itself that no longer allows to describe the particular behavior observed at low pressure.



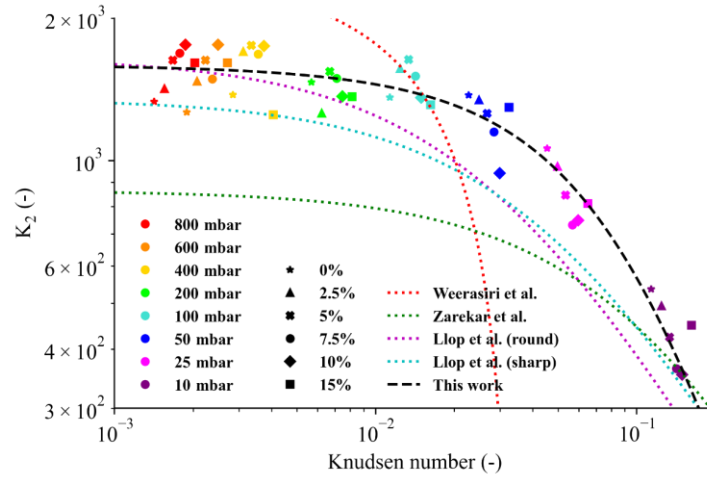
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409 *Fig. 14 : Reynolds number:  $Re_{mf} = (U_{mf}\rho_g \Psi_s d_g)/\mu$  as a function of the Archimedeian number:  $Ar = (g(\rho_p - \rho_g)\rho_g(\Psi_s d_g)^3)/\mu^2$ .*

410 **4.2. Prediction of the minimum fluidization velocity with semi-empirical correlation**

411 Previous studies [35,44,45] have highlighted the dependence of the  $K_2$  coefficient on the Knudsen number. Here  
 412 again, it is possible to observe a dependence and establish a correlation between these two values, see Fig. 15. A  
 413 nonlinear regression of an inverted second-degree polynomial was then proposed, in the form:  $K_2 = (c_2Kn^2 +$   
 414  $c_1Kn + c_0)^{-1}$ . Other forms of fitting parameters have been tested, but they were either insufficient to describe the  
 415 low-pressure points, or too complex (overfitting) and therefore physically inconsistent (*e.g.* non-monotonic). The  
 416 results of this regression are plotted in Fig. 15 as a black dashed line, along with the correlations proposed by  
 417 Weesasiri, Zarekar and Llop.





418

419

*Fig. 15 : Variable  $K_2$  as a function of the Knudsen number:  $Kn = (K_B T) / (\sqrt{2} \pi \sigma^2 P d_p)$ .*

420 The graph clearly shows that the correlations proposed by other authors are not adequate to describe the

421 experimental data found in this study. Conversely, the fitting procedure proposed here is much more accurate.

422 The values of the coefficients are given in Table 3.

423

*Table 3 : Values and errors of the fitting parameters of the  $K_2$  equations as a function of  $Kn$ .*

$K_2 = (c_2 Kn^2 + c_1 Kn + c_0)^{-1}$	$c_2$	$c_1$	$c_0$	$r^2$
Value	$5.81 \cdot 10^{-2}$	$5.55 \cdot 10^{-3}$	$6.28 \cdot 10^{-4}$	0.88432
Absolute standard error	$2.64 \cdot 10^{-2}$	$1.80 \cdot 10^{-3}$	$1.49 \cdot 10^{-5}$	

424 For further calculations, considering  $K_1 = C_1 k_2$ , where  $C_1 = 1 / (\Psi_s \epsilon_{mf}^3) = 14$ , according to Wen and Yu [36] and

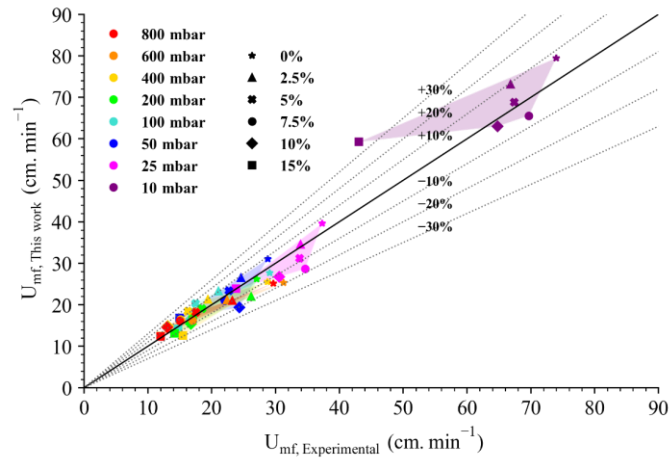
425  $k_2 = 1.75$ , according to Ergün [34] (the coefficient for the inertial regime), we get:  $B = 1/K_1 = 0.0408$ . For the

426 coefficient A, the equation of  $Re_{mf}$  as a function of  $Ar$  depends on the  $K_2(Kn)$  parameter used for fitting. If we

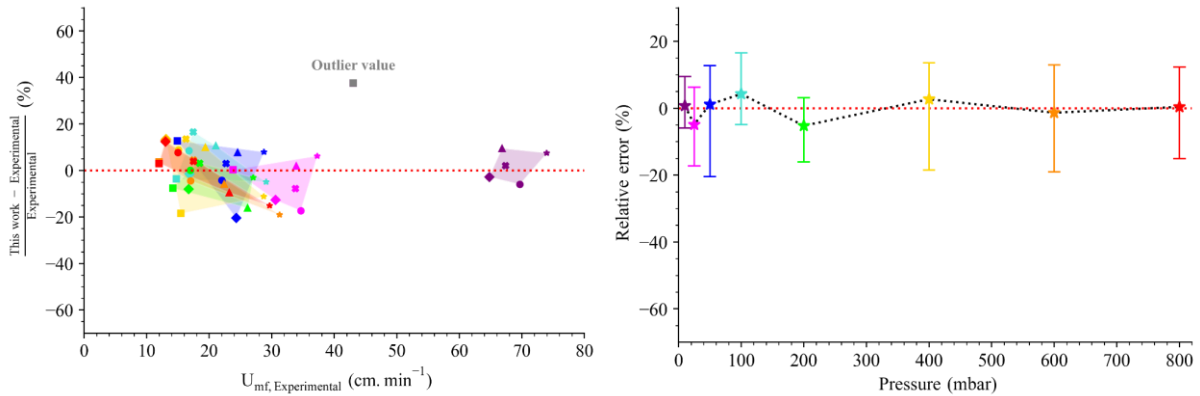
427 consider  $K_2 = (c_2 Kn^2 + c_1 Kn + c_0)^{-1}$ , then we get:

$$Re_{mf} = \sqrt{\left[ \frac{32.49}{1 + 8.83Kn + 92.56Kn^2} \right]^2 + 0.0408Ar} - \frac{32.49}{1 + 8.83Kn + 92.56Kn^2} \quad \text{Equation (14)}$$

428 *i.e.* a coefficient  $A = 32.49 / (1 + 8.83Kn + 92.56Kn^2)$ , with an asymptotic value of 32.49 when  $Kn$  tends to 0.



429



430

431 **Fig. 16** : Comparison of minimum fluidization velocities from experiments and calculations with Equation (14) (top). Relative errors  
 432 made by the calculation compared to the experimental values with Equation (14) (bottom left) and average errors with the minimum and  
 433 maximum errors of the calculations with Equation (14) (bottom right) for each pressure.

434 Fig. 16 displays the comparison between the experimental data and the  $U_{mf}$  values calculated with Equation (14).  
 435 The results are very satisfactory since the numerical prediction agrees with the experimental data, with an  
 436 average absolute error of 8.3%. The point at 15% fiber mass fraction and 10 mbar (purple square point), which  
 437 deviates from the prediction, seems to be an outlier. For all the other cases, the semi-empirical correlation with  
 438 the inverse polynomial offers the best prediction for all fiber mass fractions.

## 439 5. Conclusion

440 Previous modeling studies have investigated the effect of adding short fibers to a powder load. This study  
 441 proposes a description of the behavior of a bed composed of a mixture of short fibers and powder at variable  
 442 ratios, and two semi-empirical correlations to describe the minimum fluidization velocity. It is obvious that short  
 443 fibers, even when added in very small proportions, alter the hydrodynamic behavior of the mixture. With an  
 444 elongated shape and a form factor greater than 10, these entities modify both the fluidization capacity of the

445 mixture and its intrinsic properties such as the porosity at minimum fluidization or the minimum fluidization  
446 velocity.

447 The present study investigates the effect of adding short fibers of micrometric dimensions to powders of the  
448 same chemical nature (SiC), also micrometric. Beyond a fiber mass fraction of 15%, fluidization becomes  
449 problematic at room temperature and reduced pressure. The charge tends to be cohesive and channeling occurs  
450 through a bed that becomes static. If an effective stirring is maintained for several tens of minutes it may stop for  
451 longer periods of time, the particles gradually agglomerating to form a static bed crossed by channels of  
452 preferential gas flow.

453 The minimum fluidization velocity was described by the Zarekar and Darcy-Klinkenberg equations. After a  
454 variable fitting parameter, it is possible to propose tortuosity constants and introduce an equivalent hydraulic  
455 diameter, which is itself fitted as a function of the proportion of short fibers. The modified Darcy-Klinkenberg  
456 equation correlates with the experimental data and provides an estimate of the minimum fluidization velocity  
457 with a mean absolute error of 14.9% and a standard deviation of 8.8%, compared to a mean absolute error of  
458 18.2% and a standard deviation of 15.8% for the Zarekar equation. The discrepancy is even more pronounced at  
459 low working pressures, with a mean absolute error of 51.4% for the Zarekar equation and 12.3% for the Darcy-  
460 Klinkenberg equation.

461 The use of semi-empirical correlation has proven to be much more effective in describing  $U_{mf}$  as a function of  
462 the operating conditions. The values of the coefficients can be used and provide a prediction with an absolute  
463 error of less than 10% for all short fiber fractions in the mixtures. The numerical determination of this minimum  
464 fluidization velocity remains very important for the choice of fluidization operating conditions in the case of  
465 CVD coatings prepared at low pressures. This approach cannot be extended to other particles with other  
466 dimensions or aspect ratios, but it allows to propose a new methodology for determining  $U_{mf}$  with simple  
467 mathematical tools and little preliminary experimental work. It should be useful, for example, to select the flow  
468 rate of the gases injected into the reactor, to maximize the gas-solid exchange, a key parameter for the  
469 homogeneity of the coatings and the overall yield of the fluidized bed CVD process. These equations describing  
470 the fluidization behavior can also be integrated into a numerical simulation approach to model the fluidization of  
471 such unusual loads.

472

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477 **Declaration of interests**

478 The authors declare that they have no known competing financial interests or personal relationships that could  
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- 604



Thomas Da Calva Mouillevois is a Ph.D. student in the development of ceramic coatings on short fibers by Fluidized Bed CVD process in the ThermoStructural Composites Laboratory since 2020. Graduated from the National Engineering School of Tarbes, France in the speciality of Structural Materials and Processes Engineering, Thomas Da Calva also holds a double degree in Materials Processing Characterization and Surface Treatments in partnership with the National School of Chemical and Technological Arts Engineers and the Paul Sabatier University of Toulouse.



Matthias Audren-Paul is an engineer in materials science graduated from ENSIACET (INP-Toulouse). He worked as an intern for 6 months on pyrolytic carbon and boron nitride coatings on short fiber and powder mixtures by FB-CVD in the ThermoStructural Composites Laboratory. He is also graduated from the University of Sherbrooke in nanomaterials and materials characterization.



Georges Chollon is research scientist at the French National Center for Scientific Research (CNRS) since 1999. He graduated from the National School of Physics and Chemistry of Bordeaux in 1990 and obtained a Ph.D. in Materials Science from the University of Bordeaux in 1995. He completed two post-doctoral fellowships, at EMPA-Dübendorf, Switzerland, in 1995-1996 and NIMCR-Tsukuba, Japan, in 1996-1998. At LCTS, he is investigating the relationships between synthesis (mainly by CVD), composition, structure and properties of carbon and non-oxide ceramics.



Nathalie Bertrand is an Assistant Professor in the Chemistry Department of the University of Bordeaux since 2003. She completed a Ph.D. in Chemical engineering in 2001 at Toulouse INP. Her main topics of interest concern Gaseous Fluidization, Transport phenomena and Chemical Vapor Deposition. Her current research activities focus on the use of Fluidized Bed-CVD process on short fibers.