Dynastic Measures of Intergenerational Mobility

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Abstract

We suggest a simple and flexible criterion to assess relative inter-generational mobility. It accommodates different types of outcomes, such as (continuous) earnings or (discrete and ordinal) education levels, and captures dynastic improvements of such outcomes at different points of the initial distribution. We provide dominance characterizations - for instance on the relative progress made by women vs. men - that are consistent with social preferences upon desirable patterns of mobility. We suggest an application on Indonesia. Using the IFLS data, we match parents observed in 1993 to their children in 2014, providing one of the rare intergenerational mobility analyses based on a long panel in the context of a developing country. Results indicate that mobility in terms of education and potential earnings was markedly at the advantage of women. The bulk of the population came out of illiteracy, possibly due to largescale education reforms, but the relative educational mobility was regressive, which considerably reduced the progressivity of mobility in terms of potential earnings.

Keywords: intergenerational mobility, education, earnings, social welfare, gender.

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Dynastic Measures of Intergenerational Mobility*

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October 30, 2022

Abstract

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1 Introduction

Inequality in a society is perhaps more bearable if it is accompanied by high economic mobility between generations. On this condition, parents can invest in their children’s education to ensure them a better future than their own. A growing academic literature aims to address this question, suggesting ways to measure and characterize patterns of intergenerational mobility yet, many studies still rely on a single-valued mobility elasticity stemming from the regression of children’s outcomes on parents’ outcomes. More general and disaggregated characterizations of intergenerational mobility are certainly needed and a few studies go in this direction by reporting more comprehensive patterns based on mobility curves. However, these measures are specifically designed according to the variable chosen to represent individual achievement or economic status, and on which mobility is calculated. So far, there is no encompassing framework for measuring intergenerational mobility throughout the initial distribution while accommodating different types of achievement measures, such as discrete versus continuous outcomes (or ordinal versus cardinal outcomes). This is an issue since comprehensive studies need to characterize the mobility patterns of different types of outcomes, for instance (continuous) earnings and (discrete) education levels, in a comparable way.

In the present paper, we address these concerns by proposing a simple criterion and a general framework to evaluate and compare intergenerational mobility processes. We also suggest an illustration for Indonesia, a country where dramatic changes in education levels must have strongly affected the earnings mobility of women and men, hence where intergenerational mobility may be interestingly assessed along both dimensions. Our framework builds upon information on mobility experienced at the level of a parents-children dynasty (defined according to the first-generation distribution) and relies on ‘dynastic curves’ to generate partial but robust rankings of mobility processes. The approach has several advantages. First, it enables us to assess if and how mobility differs at different points of the distribution. Second, it can be implemented to compare the mobility of subgroups of the population, e.g. differential mobility patterns for women and men. Third, the approach

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1 See the reviews in Van de Gaer, Schokkaert and Martinez (2001), Black and Devereux (2011), Bourguignon (2011), Brunori, Ferreira and Peragine (2013) and Jäntti and Jenkins (2015), among others. This topic also attracts a lot of attention among policy makers and international organizations (e.g. Ferreira et al., 2012, OECD, 2018) and is closely associated with (in)equality of opportunity (Ferreira and Gignoux, 2011).

is consistent with a general definition of parents’ and children’s achievement: it can be used interchangeably to evaluate mobility when the achievement variable is either cardinal or only ordinal, making it general enough to assess the mobility experienced in different domains of well-being. Our application will focus on achievements based on potential earnings and education categories in Indonesia. Fourth, we also incorporate weighting mechanisms to consider not only the position of a given dynasty in each generation but also the distance that separates it from others. Thus, the approach goes beyond re-ranking measures and additionally accounts for structural changes in the distribution. Fifth, we provide a normative justification for the implementation of our approach, closely linked to the inequality and social welfare literature. Precisely, we suggest higher-order dominance results for situations where mobility curves alone do not allow ranking mobility patterns – for instance to evaluate the progress made by women, relative to men, in our application for Indonesia.

Empirically, we also suggest one of the rare assessments of intergenerational mobility in low- and middle-income countries based on longitudinal data. For rich countries, long panel data are becoming increasingly available and provide researchers with the possibility to compare various sorts of achievements across generations. This literature often relies on long panels from administrative sources to produce increasingly refined analyses of intergenerational mobility (Black and Devereux, 2011). However, the literature remains very scant for low- and middle-income countries precisely because of the limited availability of long and robust panels. As documented hereafter, existing studies rather use retrospective information on parents’ labor market outcomes and human capital, or focus on specific households, namely those where the two generations are still cohabiting. We focus here on education and earning mobility in Indonesia primarily because it is, to our knowledge, one of the rare developing countries for which a long representative panel data exists and can be used to measure intergenerational mobility. We exploit two important features of the IFLS data, namely the long duration of this panel and its exceptionally low attrition. This enables us to match parents in 1993 with their children in 2014 in order to extract and compare outcomes of both groups in each period.

The results point to broad improvements in education levels over a generation, lifting a large part of the population out of illiteracy. These changes are very likely due to the large-scale education policies implemented in Indonesia and extensively documented in the economic literature. Several reforms have indeed provided universal primary education and expanded access to secondary education in the second generation considered in our
analysis. Mobility patterns reveal that, independently of the outcome considered, women have progressed faster than men. Yet, a striking result is that educational mobility was regressive: progress in education was more pronounced among economically advantaged dynasties. A decomposition analysis shows that these trends seriously limit the degree of progressivity observed in terms of potential earnings mobility. These results shed new light on the long-term implications of education reforms and the way they can reshape both the distribution of human capital and the distribution of potential labor market outcomes.

The structure of the paper is as follows. Section 2 suggests a brief survey of the related literatures. Section 3 outlines the approach and suggests a normative characterization of patterns of mobility. Section 4 presents the data and our empirical strategy. Section 5 discusses the results of our intergenerational mobility analysis for education and earnings. Section 6 concludes.

2 A Brief Account of the Literature

Methods to Measure Mobility. Most intergenerational mobility studies are based on a single linear parameter derived from regressions of children’s outcomes on parents’ outcomes and controls (see, among others, Corak, 2013, Neidhöfer et al., 2018, Mocetti et al., 2020, Idzalika and Lo Bue, 2020). Such intergenerational elasticity coefficients are useful single-valued summary measures of mobility. They are widely used so that estimates can be compared across studies, across countries, and over time. However, these measures also show limitations. They are not informative on whether the society has faced upward or downward mobility. Thus, comparisons between different episodes of mobility performed using these indicators need to be interpreted with caution. They are also not made to capture mobility differentials across different groups in the population or between different segments of the distribution. This can be overcome through quantile regression techniques, as used in Eide and Showalter (1999), or by testing for the existence of nonlinearities, as done in Bratsberg et al. (2007), Connolly et al. (2019), and Kurtellos et al. (2020), among others. However, these procedures, being based on parametric estimations, do not enable a sharp disaggregated evaluation as the one sought in this paper.\(^3\)

The recent literature adopts a more disaggregated approach that captures the mobility experienced by different parts of the distribution and, in doing so, provides a more detailed

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\(^3\)See also Hertz (2008) for a detailed discussion on the comparison of mobility measures across population subgroups and Chetty et al. (2014) for differential mobility rates across geographical areas.
picture about the specific features of the mobility process under analysis (see, among others, Markussen and Røed, 2019, Bratberg et al., 2017). As mentioned, however, such measures are designed according to the variable chosen to represent the achievement of an individual (and mobility thereof) while they could be more general and, for instance, distinguish between ordinal and cardinal outcomes (Klasen, 2008, Klasen and Reimers, 2017).

We suggest a tool that can be used interchangeably to evaluate the mobility of cardinal and ordinal variables while providing a normative support to our framework as well as dominance results.

**Empirical Results and Challenges.** Intergenerational mobility has been an active field of research (see the surveys by Solon, 1999, Björklund and Jantti, 2009, Black and Devereux, 2011, and Jantti and Jenkins, 2015, Deutscher and Mazumder, 2022). In particular in the case of education mobility, early studies include Bowles (1972), Blake (1985) and Spady (1967). More recently, Card et al. (2018) investigate education mobility in the US, while Hertz et al. (2008) estimate country-level mobility coefficients across 42 countries. Because of the difficulty of matching information on both parents and children, studies on low- and middle-income countries are limited. Alesina et al. (2020) use Census data on cohabiting parents and children to explore education mobility across 26 African countries. Bossuyt and Cogneau (2013) study occupational mobility in five sub-Saharan African countries. Fontep and Sen (2020) estimate intergenerational persistence of occupation and education status in Cameroon, suggesting an interesting gender comparison. Importantly, many of these studies do not rely on panel information and must find ways to retrieve information for both parents and children using specific selections (e.g. cohabiting families) or recall information.

**Normative Approaches to Assess Mobility.** Our approach aims to provide dominance results to characterize intergenerational mobility. Thus, we focus here on the

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4Bratberg et al. (2017) specifically consider changes in income rank and income share between parents and children. Markussen and Røed (2019) look at rank and class correlation between parent and children for each ventile of the parent distribution for different income and non-income variables.

5From an empirical point of view, the above contributions focus on rich countries for which exceptional data are available to characterize intergenerational mobility in a highly representative manner. Our work focuses instead on intergenerational mobility in a poor country for which only survey data can be used.

6Novel data on intergenerational mobility in 18 Latin American countries are provided by Neidhöfer et al. (2019). Several studies also focus on mobility in India (Azam and Bhatt, 2015, Emran and Shilpi 2015, Asher et al., 2018) and China (Golley and Kong, 2013, Emran and Sun, 2015, Emran et al., 2020).

7Alesina et al (2020) match individuals to their parents using data on cohabitants of different generations. Fontep and Sen (2020) retrieve parents’ information thanks to retrospective questions.

8We keep the discussion on whether one should assess intergenerational mobility patterns – as we do for inequality, for instance – for the next section.
related normative literature, notably on the tools adopted in the context of multidimensional inequality and *intra*-generational mobility (see the very detailed exposition from Jäntti and Jenkins, 2015). Dominance checks extended to mobility patterns can find their origins in results for multivariate distributions of income (Atkinson and Bourguignon, 1982, Markandya, 1984, Gottschalk and Spolaore, 2002). The framework suggested by these authors is commonly interpreted as an aggregation of intertemporal utilities defined over two periods. All relevant mobility is captured by the changes in individuals’ ranks, making social welfare results sensitive to mobility as reversals rather than mobility as origin dependence. In other words, the utilitarian social welfare framework does not incorporate evaluations of mobility in the form of individual income growth, which is the interesting dimension for intergenerational mobility. An exception is Bourguignon (2011), who shows that the Atkinson and Bourguignon (1982) results can be applied to comparisons of alternative ‘growth processes’ when the pair of marginal distributions relating to the first period are identical (this restriction unfortunately limits the applicability of the approach).

Related to this, several contributions have defined social welfare explicitly in terms of income mobility, i.e. income changes rather than income levels. This literature, focusing again mainly on intra-generational mobility interpretations and applications, assumes that individual-level mobilities are represented by concepts of ‘distance’ between first and second-period incomes (Fields et al., 2002, van Kerm, 2009, Jenkins and van Kerm, 2016). It has been extensively used to characterize which segments of the population benefit the most from income growth using non-anonymous growth incidence curves (Grimm, 2007, Bourguignon, 2011, Van Kerm, 2009, Jenkins and Van Kerm, 2016, Palmisano, 2018, Lo Bue and Palmisano, 2020, Berman and Bourguignon, 2021). Our approach is related to this literature but adapted to the intergenerational perspective. We also go beyond the usual focus on income and suggest a flexible framework that accommodate both cardinal types of achievements (such as income) and purely ordinal ones (such as discrete education classes). In this respect, our paper contributes to the literature focused on social evaluations in a multiperiod context (see Decancq and Zoli, 2014) and gives a normative support to that part of the empirical literature that aims to evaluate multiperiod outcome distributions.

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9Aspects such as aversion to intertemporal fluctuations and to future income risk are specific to this interpretation and absent from the intergenerational perspective.

10Note that a few contributions adopt transition matrices rather than rank comparisons when individual incomes are classified into discrete classes (see in particular Dardanoni, 1993, for stochastic dominance results for rankings of mobility processes summarized by transition matrices).
3 Theoretical Framework

We first describe the methodology used to measure intergenerational mobility, then discuss welfare-related characteristics of mobility patterns.

3.1 Measuring Intergenerational Mobility

Mobility means different things to different people (Fields, 2008). We first present the definition of mobility that we adopt in this paper and propose tools to evaluate mobility in a way that is consistent with our definition. The degree of mobility is interpreted here as the intensity with which individual achievements improve or deteriorate between two generations. The term ‘individual’ means that we will be comparing the achievements of a person and his/her parents, i.e. of a specific dynasty, over the span of a generation while retaining the principle of non-anonymity. Individual ‘achievements’ will be defined in relative terms to other dynasties in each generation. We now describe in detail these different concepts and the mobility measure itself.

**Dynasties.** The first step corresponds to the definition of a rule that allows tracking individual achievements across generations, which we will refer to as a ‘dynasty’. We denote $t$ the generation of the parents and $t + 1$ that of their children. Dynasties will be defined according to an outcome $z$ used to characterize the first-generation distribution. We will use per-capita expenditure in our application, as it relates to notions of welfare (or living standards) and provides a very disaggregated picture of the population (at least compared to other variables such as income groups or education classes). Following the standard literature on mobility, a dynasty is defined as the relative position of the parents in the initial distribution of $z$. Let the $z$-distribution observed for generation $t$ be represented by its cumulative distribution function (CDF) $F_z(z_t) = P(\tilde{z}_t \in \mathbb{R}_+ : \tilde{z}_t \leq z_t)$. Each dynasty will simply be characterized by this probability, denoted $p_t \in [0, 1]$, which returns the proportion of people observed below $z_t$ in generation $t$ and can be interpreted as the social status of the parents.

**Achievements.** The second step consists in the definition of individual achievements. In a general representation, the outcome used to measure achievements may differ from $z$. For instance, we may study the mobility in terms of *educational* or *earnings* achievements at different points of the distribution of living standards $z$. Then, notice that different mobility studies adopt different definitions of achievement depending on the focus of the analysis (Fields and Ok 1999, D’Agostino and Dardanoni, 2009). Two main approaches
can be detected. The first interprets an individual’s achievement as something that is measurable in terms of level of any outcome variable. The second approach interprets an individual’s achievement as something that is only measurable in relation to the rank of an individual in the outcome distribution of interest. A measure of mobility based on an index of achievement constructed using the first approach would treat an ordinal variable (e.g., education categories) as a cardinal one. An important issue that characterizes ordinal data is that the mean is not order-preserving under scale changes (Allison and Foster, 2004), whereas distributional orderings based on comparisons of CDFs are robust to changes in the scale. This militates for the use of CDFs comparisons when dealing with ordinal data, as recently popularized by Cowell and Flachaire (2017, 2018) and Jenkins (2020, 2019). This is the line of reasoning that we adopt here and, hence, we shall follow the second approach.

We first consider a continuous outcome, denoted by $y$, which is typically the case of monetary variables such as individual earnings. We represent by $y_t(p_t) : [0, 1] \rightarrow \mathbb{R}_+$ the earnings of the parents (generation $t$) for a dynasty $p_t$. In the same way, we denote by $y_{t+1}(p_t)$ the earnings of the children (generation $t + 1$) of this dynasty $p_t$. We denote $f$ and $F$ the PDF and CDF of the outcome considered to characterize achievements.\footnote{Distinguish $F$ without subscript, i.e. the CDF of the achievement outcomes, from $F_z$, i.e. the CDF used to define dynasties.} A measure of achievement for the parents and children of any dynasty $p_t$ can respectively be written as follows:

\[
\forall p_t \in [0, 1], \quad \left\{ \begin{array}{l}
\alpha_t(p_t) = F(y_t(p_t)) \\
\alpha_{t+1}(p_t) = F(y_{t+1}(p_t)).
\end{array} \right. \tag{1}
\]

In the space of a continuous measure such as earnings, for a dynasty $p_t$, the parents’ achievement is represented by the fraction of individuals of generation $t$ with earnings equal to or below the level $y_t(p_t)$ achieved by the parents of dynasty $p_t$. Their children’s achievement is measured by the fraction of individuals of generation $t + 1$ with earnings equal to or below the level $y_{t+1}(p_t)$ achieved by these children of dynasty $p_t$.\footnote{This is a downward-looking based definition of achievement, reminiscent of the sense of pride embedded in other-regarding preference models (see Hopkins, 2008). Alternatively, one could choose the upward-looking based definition of achievement, which relates to the envy-based models of other-regarding preferences. It would be defined by the fraction of individuals with earnings equal to or above the level $y_t(p_t)$ achieved by the parents of dynasty $p_t$ in generation $t$ and, in generation $t + 1$, by the fraction of individuals with earnings equal to or above the level $y_{t+1}(p_t)$ as achieved by the children of dynasty $p_t$.}

Let us now consider a discrete outcome, which is typically the case of non-monetary vari-
ables that are ordinal but not cardinal, such as education classes, health status, occupation
types, etc. Let there be an ordered set of \( K > 2 \) classes, each class \( k \) being associated with
a latent outcome level, for instance the educational attainment. Let \( k_t(p_t) \) (resp. \( k_{t+1}(p_t) \))
be the class occupied by parents (resp. children) of dynasty \( p_t \), \( n_{k,t} \) (resp. \( n_{k,t+1} \)) the
number of individuals in this class and \( N_t \) (resp. \( N_{t+1} \)) the total number of individuals in
the population of parents (resp. children). Achievement for parents and children can be
expressed as:

\[
\forall p_t \in [0, 1], \begin{cases}
    a_t^{(2)}(p_t) &= \frac{\sum_{i=1}^{k_t(p_t)} n_{i,t}}{N_t} \\
    a_{t+1}^{(2)}(p_t) &= \frac{\sum_{i=1}^{k_{t+1}(p_t)} n_{i,t+1}}{N_{t+1}}
\end{cases}
\]  

respectively. In the space of discrete variables such as education classes, for a dynasty
\( p_t \), the parents’ achievement is represented by the fraction of individuals of generation \( t \)
who belong to the same or to a lower class than the class reached by these parents. Their
children’s achievement is measured by the fraction of individuals of generation \( t + 1 \) who
belong to the same or to a lower class than the class reached by these children of dynasty \( p_t \).
Hence, our concept of achievement is independent of scale and allows us to treat cardinal
and ordinal variables in a uniform way.

**Weighted Achievements.** We also consider complementary definitions obtained by in-
troducing weights in the achievement schemes previously defined. In the case of continuous
outcomes, these are expressed as follows:

\[
\forall p_t \in [0, 1], \begin{cases}
    a_t^{(3)}(p_t) &= \int_{y_t(p_t)}^{y_{t+1}(p_t)} f(y_t(s_i))(F(y_t(p_t)) - F(y_t(s_i)))dy(s_i) \\
    a_{t+1}^{(3)}(p_t) &= \int_{y_t(p_t)}^{y_{t+1}(p_t)} f(y_{t+1}(s_i))(F(y_{t+1}(p_t)) - F(y_{t+1}(s_i)))dy(s_i)
\end{cases}
\]  

In the case of discrete outcomes, these are written as follows:

\[
\forall p_t \in [0, 1], \begin{cases}
    a_t^{(4)}(p_t) &= \frac{\sum_{i=1}^{k_t(p_t)} n_{i,t} \sum_{s=i+1}^{k_{t+1}(p_t)} n_{s,t}}{N_t^2} \\
    a_{t+1}^{(4)}(p_t) &= \frac{\sum_{i=1}^{k_{t+1}(p_t)} n_{i,t+1} \sum_{s=i+1}^{k_{t+1}(p_t)} n_{s,t+1}}{N_{t+1}^2}
\end{cases}
\]  

For a dynasty \( p_t \), individual achievement is represented by the fraction of individuals of
generation $t$ (resp. $t+1$) attaining a level or class lower or equal to that of the parents (resp. children) of this dynasty, now weighted by the distance that separate each of these individuals from the parents (resp. children). The distance is simply the density of people between each of these individuals and the parents (resp. children) of this dynasty, measured as a difference in ranks in the continuous approach of equation \ref{eq:3} or by summing intermediary classes in the discrete approach of equation \ref{eq:4}. In our mobility measures, weighting achievements will make it possible to account for the change in the marginal distribution of outcome between the two generations. There are many possible alternative specifications for the weights but, as further discussed below, we opt for the one proposed in equations (3) and (4) because it is intuitive, consistent with the way we define mobility, and avoiding additional structure to the model.

**Mobility Measure.** The last step consists in the construction of a measure of mobility, the Dynastic Curve (DynaC hereafter):

$$d \left( a^{(s)} (p_t) \right) = a_{i+1}^{(s)} (p_t) - a_i^{(s)} (p_t), \forall p_t \in [0,1] \quad \text{and} \quad s = 1, 2, 3, 4.$$  \hspace{1cm} (5)

DynaC allows measuring relative intergenerational mobility at a disaggregated level and in a consistent way for both cardinal and ordinal outcomes. DynaC associates to every dynasty $p_t$ the difference between children’s achievement and their parents’ achievement. Since achievement is distribution-dependent, $d(a^*(p_t))$ is a relative measure of mobility, i.e. intergenerational mobility is evaluated according to how better or worse, with respect to their parents, children are positioned in the outcome distribution of their generation. This is the only criterion retained for $s = 1, 2$. Graphically, the mobility score $d(a^*(p_t))$ plotted against each $p_t$ will be zero in case of immobility. If children of a dynasty $p_t$ attain the same achievement as their parents (or progress only a little) while other dynasties progress a lot more, then children of dynasty $p_t$ dominate fewer people than their parents used to and the mobility score of dynasty $p_t$ will be negative. Thus, we shall refer to ‘negative’ (‘positive’) mobility to convey relative downward (relative upward) mobility, i.e. the fact that a dynasty attains a worse (better) rank in the second generation than in the first. Finally, if the DynaC curve is increasing (decreasing), the mobility process is deemed regressive (progressive) with respect to the variable $z$ defining quantiles $p_t$.

\footnote{Some expert readers may ask whether it is possible to relate the dynastic curve to a transition matrix. We believe it is only possible when (i) the variable used to identify dynasties is the same as the variable over which mobility is measured (i.e. $z$ y coincide) and (ii) DynaC individual measures of mobility were aggregated into a societal measure of mobility (a single numerical value).}
**Interpretation.** Assessing mobility using changes in the *relative* position of parents versus children allows focusing on comparisons that are independent from the marginal distributions. Changes in the marginal distributions can be accounted for by assessing mobility through changes in the weighted achievements, as will be explained below. Introducing some flexibility in the treatment of marginal changes may be relevant, for instance, in periods of sustained variations in inequality. Therefore, our framework provides an assessment of mobility that is generated by the real movements of individuals within the distribution and not by the structural changes of the distribution, since ranks represent a stable benchmark, in contrast to levels that may fluctuate mechanically in times of inequality variations. Besides these advantages, relative mobility is also relevant when observed from the perspective of relative concerns, positional goods and status (see Easterlin 1974), thus substantiating the importance of ranks. Higher positions within an outcome distribution can be disproportionately more important for well-being than higher absolute levels (Clark et al. 2009, Dolan and Lordan, 2020, Simandani 2018).

We also suggest an interpretation of DynaC in the weighted case, i.e. for $s = 3, 4$. Assume the child in a given dynasty obtains the same unweighted achievement in two alternative distributions, i.e. he dominates the same number of people in both distributions. His weighted achievement needs not be the same. It will be higher in the distribution where the people the child dominates are further below. This is an interesting interpretation for mobility because it means that for a positive *unweighted* DynaC, which indicates that the dynasty improves its rank between two generations, the progress made by this dynasty is all the faster as the *weighted* DynaC is large. Thus, while DynaC can be seen as a simple measure of reranking in its unweighted formulation of the continuous case, the weighted approach adds more information related to the change in the shape of the distribution.

In our example above, there is an increase in inequality below the dynasty in question. Take a somewhat opposite situation, for instance a dynasty with a zero unweighted DynaC and a negative weighted DynaC. It can be interpreted as a decline in inequality below this dynasty in the second generation, as the people dominated by this dynasty catch up and concentrate just below it. In other words, a dynasty can well preserve her rank (zero DynaC) but experience a negative weighted mobility through this reshaping of the distribution below it. As mentioned, the weights could be defined in alternative ways, for instance as a distance in *levels* of earnings or education. But our definition is consistent with the mobility definition: it captures changes in the distribution while abstracting from any form of cardinalization, using density of population between discrete groups (such as
Finally, note that by construction, our model allows for the outcome variables $y$ or $k$ used for mobility measures to differ from the variable $z$ used to identify a dynasty. This modeling choice provides a more general characterization than the setting usually adopted in the mobility literature and, thus, allows for a more flexible and versatile empirical implementation, as illustrated below. This is especially the case when the framework is applied to compare the mobility process between two subgroups of the same population – e.g., men’s and women’s education levels – since we can rank individuals on the basis of a variable that is not sub-group dependent and see how these two groups perform differently within a given dynasty.\[14\]

3.2 Normative Justification

We now provide a normative support to the use of DynaC to evaluate and compare mobility episodes. We assume that social preferences upon mobility patterns are an abstraction of some relevance, either because ‘movement’ is desirable per se (see Van de gaer et al., 2001), because it has specific properties (e.g., people tolerate long-term inequalities more when there is more mobility), or because we want to judge the relative progress of population subgroups (e.g., women vs. men). The last motivation is perhaps the one best motivated by our application hereafter.

Let $D^{(s)} = \{d(a^s(p_t)) \mid p_t \in [0,1]\}$ be the set of all possible mobility of outcome $s = 1, ..., 4$ for each dynasty $p_t \in [0,1]$ in a given population. It is interpreted as the general process of mobility between two generations in that population and we are interested in judging it from a normative perspective. We assume that a social planner is endowed with cardinal preferences over mobility processes, denoted by $W(D)$, and write $P$ the set of social preferences. A social planner with preferences $W \in P$ may assess whether

\[14\] Further work could actually explore the link with recent studies on non-anonymous growth incidence curves (NAGIC), which are changes in levels (for instance income levels) of given dynasties while DynaC is a change in ranks. Berman and Bourguignon (2021) suggest a decomposition of NAGIC into a mobility effect (reranking) and a shape effect (i.e., a change in the whole cross-sectional distribution). Our weighted DynaC adds to the rank mobility a weight based on the change in shape (but focused on the part of the distribution below each dynasty).

\[15\] It also makes more sense to assess the progressivity/regressivity of education mobility on the basis of a broad welfare measure $z$ (such as per capita consumption, in our application) than on an education-based dynasties, especially if the number of education categories is small and education is weakly correlated with $z$ (which is the case for Indonesia since a large majority of people were education-poor in the first generation, as we shall see).
the mobility of a given process, say \( D^{(s)}_\pi \), is socially superior to immobility by evaluating whether \( W(D^{(s)}_\pi) \) is larger than a benchmark process of immobility for all the dynasties. Considering two mobility processes \( D^{(s)}_\pi \) and \( D^{(s)}_\omega \), the planner may deem the first process socially preferred to the second if \( W(D^{(s)}_\pi) \geq W(D^{(s)}_\omega) \).

We reformulate the dominance in terms of observables, i.e. social preferences over mobility processes are specified as functions of the observed distributions of achievements. To represent such preferences, we adapt the rank-dependent model proposed by Yaari (1987), which offers theoretical and empirical tractability. It assumes that social welfare can be written as a weighted average of all possible realizations, where the weights are a function of the ranks of the realizations. Transposed to our mobility problem, it becomes the weighted average of mobility measures \( d(a^{(s)}(p_t)) \) over all dynasties in the population, with a weight \( w(p_t) \geq 0 \) assigned to the mobility of dynasty \( p_t \). Thus, the social evaluation of any intergenerational mobility process, indexed by \( \pi \), is written as:

\[
W(D^{(s)}_\pi) = \int_0^1 w(p_t) d(\pi(a^{(s)}(p_t))) dp_t. \quad (6)
\]

Our departing point is a class of rank-dependent social evaluation functions, which are explicitly sensitive to information about the change in the relative position of parents and children. The extent of mobility, as measured by the DynaC, is computed for one of the \( s = 1, 2, 3, 4 \) definitions of achievement introduced in the previous section. Let us rewrite \( D^{(s)}_\pi \) as \( D_\pi \) and \( d_\pi(a^{(s)}(p_t)) \) as \( d_\pi(p_t) \) hereafter to simplify notations. We will restrict to a set of social preferences

\[
P^* = \{W : w(p_t) \geq 0 \ \forall p_t \in [0, 1]\} \quad (7)
\]

such that the social marginal effect of each dynasty’s mobility is positive. We suggest four propositions, the proofs of which are reported in Appendix B.

**Proposition 1.** Given two mobility processes \( D_\pi \) and \( D_\omega \), \( D_\pi \) is socially preferred to \( D_\omega \), \( \forall W \in P^* \), if and only if

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16If the set of social preferences were known, we could directly conduct these types of assessments. We do not have this information and, hence, should in principle conduct a sensitivity analysis over a reasonable range of preferences.

17This is consistent with the growing interest in the role of individual economic positions as an indicator of social status (Hopkins and Kornienko 2009, Ray and Robson 2012) and complements the discussion of formally incorporating other-regarding preferences into a normative measurement model (Fleurbaey, 2012, Decerf and van der Linden, 2016, Treibich 2018).
\[ d_\pi(p_t) \geq d_\omega(p_t), \forall p_t \in [0, 1]. \]  

Proposition 1 characterizes the first-order dominance criterion based on DynaC. It requires checking the DynaC dominance of one mobility process over the other for every dynasty. The intuition of this proposition is simple: if there is at least one dynasty experiencing a higher mobility in process \( D_\pi \) than in process \( D_\omega \), while there is no difference for the other dynasties, then the former process is socially preferred to the latter. When assessing whether a single process yields mobility, the proposition simply becomes 

\[ d_\pi(p_t) \geq 0, \forall p_t \in [0, 1]. \]  

With our relative mobility concept, it may seem difficult to obtain dominance (between two processes or between a single process and immobility). The zero-sum nature of rank mobility implies that we can never find dominance with respect to the immobile situation when it is tested on the whole population, for any value of \( s \). However, dominance is in principle possible in all other situations. For instance, daughters may experience a higher relative mobility compared to sons so dominance may appear at least over some portion of the dynastic distribution.

If the DynaCs of two mobility processes are crossing, it is possible to follow the social choice tradition and suggest higher-order dominance results. These are the minimal refinements on the set of admissible preferences that may lead to an unambiguous assessment of the mobility processes. We can use cumulative DynaCs to increasingly put more weight on the mobility of the poorest dynasties. We start with second-order dominance and general social weights that decline with the dynasty percentile (Proposition 2), then introduce convexity in the social weighting scheme to give an emphasis on the poorest of the poor (Proposition 3) and finally suggest a Rawlsian-type of social valuation (Proposition 4).

**Proposition 2** Given two mobility processes \( D_\pi \) and \( D_\omega \), \( D_\pi \) is socially preferred to \( D_\omega \), \( \forall W \in P^* \) for which \( \frac{\delta w(p_t)}{\delta p_t} \leq 0 \) for all \( p_t \in [0, 1] \), if and only if

\[ \int_0^{p_t} d_\pi(q_t) dq_t \geq \int_0^{p_t} d_\omega(q_t) dq_t, \forall p_t \in [0, 1]. \]  

If we allow social preferences to be more sensitive to the mobility experienced by the more disadvantaged dynasties in the initial period, a comparison between two alternative mobility processes can be carried out by testing for cumulative DynaC dominance, as suggested in this proposition. When assessing the mobility of a single process, we simply write \( \int_0^{p_t} d(q_t) dq_t \geq 0 \) (\( \int_0^{p_t} d(q_t) dq_t \leq 0 \)) for all \( p_t \) as a situation of weak relative positive
Proposition 3 Given two mobility processes $D_\pi$ and $D_\omega$, $D_\pi$ is socially preferred to $D_\omega$ for all $W \in P^*$ if and only if
\[ \int_0^{p_t} \int_0^{q_t} d_\pi(s_t) ds_t dq_t \geq \int_0^{p_t} \int_0^{q_t} d_\omega(s_t) ds_t dq_t, \forall q_t \in [0, 1] \forall p_t \in [0, 1]. \] (10)

According to this proposition, we can perform a test based on a third-order upward DynaC dominance, which finds its justification in social preferences that give more weight to the mobility experienced by the poorest of the poor. Any progressive inheritance tax (or transfer) that improves the mobility of a poor individual by taking some mobility from a middle-class individual is preferred to a transfer of mobility from a rich to a middle-class individual. We conclude with a comparison criterion of a Rawlsian flavor and that might be useful in presence of coarse data.

Proposition 4 Given two mobility processes $D_\pi$ and $D_\omega$, $D_\pi$ is socially preferred to $D_\omega$ for all $W \in P^*$ if and only if
\[ \int_0^{\bar{p}_t} \int_0^{\bar{q}_t} d_\pi(q_t) dq_t \geq \int_0^{\bar{p}_t} \int_0^{\bar{q}_t} d_\omega(q_t) dq_t. \] (11)

By selecting a threshold equal to $\bar{p}$ implies that we focus only on dynasties corresponding to households of the lower $\bar{p}$ percent of the first-generation distribution. Alternative thresholds can be selected to focus on different groups of dynasties. The test corresponding to this proposition can be interpreted as a ‘priority’ criterion, i.e. it reflects the preferences of a

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18 The second order dominance is obtained by imposing the requirement that $\frac{\delta w(p_t)}{\delta p_t} \leq 0$ for all $p_t \in [0, 1]$, which is equivalent to imposing that a progressive transfer of mobility from a more to a less advantaged dynasty will improve the social evaluation of mobility. There are both intrinsic and instrumental motivations for this requirement. First, this restriction allows accounting for the vertical equity feature of a mobility process, where the equity concern is expressed with respect to $p_t$ — the social status — of parents that may exert a strong influence on the outcome prospects of children. From a pure instrumental perspective, it may solve the problem of ambiguous ranking of social processes that usually arises when strong dominance criteria are used as in the typical case of first-order dominance.

19 Alternatively, it is possible to obtain a ranking criterion that is based on the idea of preserving the mobility of less advantaged dynasties while, at the same time, focusing on the dynamics of the richest dynasties. This alternative proposition (3’) is discussed in Appendix C.

20 If the threshold is equal to 1, the proposition reduces to the comparison of mean mobilities. When $\bar{p} = 0.5$, it is reminiscent of a recent approach introduced by Asher et al. (2018) to estimate intergenerational educational mobility with coarse data, which is typically the case of developing countries.
social planner who endorses a ‘mobility priority for the worst off’. This echoes back to the maximin criterion à la Rawls: mobility is valuable if and only if poor dynasties experience an improvement, independently of how the rest of the population performs.

4 Empirical Approach

4.1 Data

Overview. Our empirical analysis relies on the Indonesian Family Life Survey (IFLS), which is an ongoing longitudinal survey of individuals, households and communities. Based on an initial sample representing 83% of the Indonesian population, the IFLS is conducted in 13 Indonesian provinces extending across the islands of Sumatra, Java, Kalimantan, Sulawesi, Bali, and West Nusa Tenggara. It has interesting features that make it particularly suited to our analysis. First, it covers a long time period, ranging from 1993 to 2014, which is appropriate for intergenerational mobility measurement. Second, it benefits from an exceptionally low attrition rate. Indeed, extensive efforts were provided to track respondents when collecting data in each of the five waves (1993, 1997, 2000, 2007 and 2014). It has allowed reaching a recontact rate of 92% in the last wave (Strauss et al., 2016; Strauss and Witoelar, 2019), which guarantees both cross-sectional and longitudinal representativeness of our sample for intergenerational mobility calculations. Third, it includes individual and household information on a large set of socio-economic characteristics, such as education, income and consumption expenditure, in addition to standard demographic variables.

For these reasons, the IFLS has been applied to many development studies, including those focusing on intra-generational mobility (see, Grimm, 2007; Lo Bue and Palmisano, 2020). We suggest here to exploit the length of the panel for intergenerational mobility measurement. As previously discussed, the ability to link information on parents and their children when both groups are about the same age is relatively rare in the context of low- and middle-income countries. Mobility across the two generations of interest in the IFLS will necessarily reflect the impact of the aforementioned reforms on the second generation’s potential earnings but also the other changes affecting the earnings potential of men and women over time.

\[21\] It has also been used to investigate the heterogeneous effect of the INPRES school construction program on girls’ educational achievements between ethnic groups practicing bride price and the others (Ashraf et al., 2020) or the spillover effects of the program on the second generation’s human capital outcomes (Mazumder et al., 2019; Akresh et al., 2018).
Generation Matching and Selection. We use the first and last waves of the IFLS. That is, we identify the generation of fathers and mothers in 1993 using IFLS 1 (cf. Frankenberg and Karoly, 1995) and match them with their offspring observed in 2014 in IFLS 5 (cf. Strauss et al., 2016). We aim at observing both generations at similar age ranges, or at least not too radically different. It is not an issue for education outcomes since parents and children are old enough for their education levels to be fixed characteristics. Yet it would be an issue for meaningful profiles of earnings mobility, so we will also apply an age correction when predicting earnings hereafter. We select dynasties where individuals are observed in an age range of 20-40 (born 1952-1974 for parents and 1974-1994 for children), with the exception of fathers who are selected in a 20-50 age range. The asymmetry for fathers is justified as follows: males of the first generation tend to be older – and have children later – than their spouses, and restricting to 20-40 would reduce the sample size and the representativeness of the matched, dynastic sample. This leaves us with a sample of 2,164 daughters and 2,060 sons matched with 2,284 mothers and 2,284 fathers.

4.2 Achievements and DynaC Implementation

The concept \( z \) used to define dynasties is the per-capita expenditure in households observed in 1993. It is convenient to use this welfare measure as a backdrop against which we can assess individual mobility in terms of earnings or education. Our analysis focuses on two types of achievements: one based on discrete outcomes \( k \) corresponding to education classes, the other based on continuous outcomes \( y \) corresponding to potential earnings. The discrete outcome is based on the information reported in the IFLS data on the highest education level attended and on the grade completed for that level. We construct eight education classes, from ‘no education’ to ‘university degree’. These classes are used to implement baseline achievement measures of equation 2.

The continuous outcome is the potential earnings of each individual, used to implement achievements in equation 1. Potential earnings seem more relevant than actual earnings: given the number of adults without a paid job (very high for women), using actual earn-

\(^{22}\)In sensitivity checks, we apply the 20-40 age bracket to everyone or the 20-50 bracket to both mothers and fathers. We do not find radically different conclusions.

\(^{23}\)Note that with this selection, our first generation has left school after the implementation of INPRES and subsequent reforms, while the second generation is the one most affected by these large-scale policies.

\(^{24}\)We refrain from using it as an outcome. First, it has a high potential for measurement error (see Grimm, 2007). Second, for all children still living with their parents in 2014, there will be a strong persistence in this variable, which is not what we want to characterize. We rather focus on men’s and women’s individual mobility in terms of individual achievement (education, earnings potentials).

16
ings would give a very truncated picture of intergenerational mobility. Moreover, predicted earnings help to assess how education mobility translates in monetary terms. Precisely, since education is one of the key determinants of earnings equations, we can produce counterfactual earnings distribution when fixing children’s education at their parents’ level and, hence, assess, how educational mobility has contributed to earnings mobility at different points of the initial distribution.

To generate potential earnings, we simply use observed earnings and run separate earnings estimations for mothers, fathers, sons and daughters. Then, using the four sets of estimations and individual characteristics at the time of observation (except age), we predict potential earnings for all. To compare the potential earnings of sons and fathers (daughters and mothers) at about the same age, we predict 2014 earnings for sons (daughters) using the age that their father (mother) had in 1993.

Descriptive statistics are reported in Table A2. Daughters and sons are observed aged 29 on average, which is lower but close to their mother’s age (32) and father’s age (37). Figure A1 shows a broad overlap between generations. However, there are mechanical differences imposed by the matching of generations at specific points in time (1993 and 2014) and demographic particularities (e.g. the fact that men are older than their spouses). This justifies the strategy described above to neutralize the role of age when predicting earnings. Further results in Table A2 focus on achievements. It turns out that, on average, our sample of individuals in the offspring generation seems to fare relatively better than their parents. Their economic conditions have improved, as the value of real per-capita expenditure has more than doubled. Second-generation average education level is about 5.9, corresponding to 10-11 years of schooling, which is almost twice as large as their parents’ educational achievement (i.e. 3-3.6 or roughly 6 years of schooling). Their potential earnings are also much improved: in real terms, i.e. when accounting for changes in living costs over twenty years, they are 5 to 6 times as high as their parents’ potential earnings.

25These regressions account for a first-step selection equation using a relatively standard instrument, namely the total resource of other family members. Estimation results are reported in appendix Table A1: they essentially show the (monotonic) effect of education classes in the main equation (earning potentials) as well as the significant role of the instrument (and its expected negative sign) in the participation equation.

26Without this adjustment, earnings mobility would partly reflect the fact that parents are observed at older ages and hence at higher earnings levels (a differential that might vary along the cross-sectional distribution).
4.3 Education Reforms in Indonesia and Absolute Mobility

Education mobility in Indonesia is particularly interesting to examine given the large-scale education reforms implemented by this country over the past decades. Note that a broad literature analyzes the effect of some of these reforms - we highlight the fact that this is not the purpose of the present paper to attribute causality from the reforms to the mobility patterns we observe. We nonetheless provide a brief overview of these reforms, for they potentially affected the offspring in our analysis, and move to the description of educational mobility patterns.

**Education Reforms.** A series of programs have been implemented since the 1970s and have certainly contributed to boost enrollment rates and increase access to education. In 1973, the Indonesian government launched the “Sekolah Dasar INPRES” program, a large-scale school construction program, whose effectiveness on primary school enrollment has been well documented in many studies.\(^{27}\) Education enrollment expansion continued through the 1990s and the early 2000s, i.e. the years of the Asian economic ‘miracle’ marked by remarkable progress in poverty reduction and economic growth (Bolt et al., 2018, Timmer, 2018). By the early 2000s, Indonesia achieved nearly universal net primary enrollment rate while junior high school enrollment rates had reached about 60% compared to 17% in 1975 (Granado et al., 2007). In 2005, the government launched the ‘One Roof School Program’ aimed to facilitate transitions from primary to secondary school by building junior high schools on the same site as a primary schools, especially in remote areas (ILO, 2011). Efforts to increase access to education, especially among the most disadvantaged children have also been made since the late 1990s through the launch of a series of scholarship programs.\(^{28}\)

**Changes in Education Levels and Absolute Mobility.** Figure A2 shows the distribu-

\(^{27}\)See inter alia Duflo (2001), Duflo (2004), Akresh et al. (2019), Mazumder et al. (2019), Ashraf et al. (2020). Between 1973-1974 and 1978-1979, the number of primary schools in the country more than doubled, leading to a remarkable increase in enrollment rates among children aged 7 to 12, i.e. from 69% in 1973 to 83% by 1978. In our selection, the parents have been hardly affected by this reform when they were themselves children but all their children were in the right age group to fully benefit from the reforms. Indeed, those who could benefit from the early phase of INPRES schools were those aged 5 or below by 1974, which represents less than 3% (12%) of the fathers (mothers) in our sample. In contrast, all the children in our selection who were born after 1974 and hence have been at primary school age at the time of the second phase of INPRES school construction.

\(^{28}\)This includes the 1998 ‘School Grants Program’, which was effective in preventing schools drop-outs especially among primary school-aged children from poor rural families (Sparrow, 2004). Another program was implemented in 2001-2005, the ‘Compensation for Fuel Subsidy Decreased Program’, to help children from poor families through scholarships (Bantuan Khusus Murid, BKM) which covered both primary and secondary education.
tion of education levels for both generations in our selected data. The first observation is that the new generation has basically escaped illiteracy, possibly thanks to the previously discussed reforms. Around 33% of the parents in our dynastic sample had no education or incomplete primary schooling while almost none of the offsprings have no education and less than 5% have uncompleted primary schooling. The completion of junior high school has almost doubled while a majority of children now have a (senior) high school degree or go to university. We check who has benefited from these tremendous improvement in education using the matched generations. In Figure A3, we represent absolute education mobility when ranking dynasties according to parental education levels. We report the proportion of upward mobility, downward mobility or immobility (left axis). The graph essentially points to the upward mobility for all children of parents who had less than a high school degree, and a relative immobility for the top dynasties (with 80% of the university graduated parents sending their children to university as well). We also depict the average difference in education levels between the two generations (right axis). As expected, it shows larger improvement among dynasties of low-educated parents and a declining profile. A different picture emerges when we describe education mobility by percentile of per-capita consumption. In Figure A4, we first see that those with no education (or incomplete primary education) are present at all living standards (purple curve) and not just at the bottom, even if, as expected, the pattern is monotonously decreasing. We also point to a key fact for what follows: illiteracy has disappeared at all living standards levels but more so in higher percentiles (green curve).29

5 Empirical Results

We now turn to our main results based on DynaC measures. As per equation 5, the DynaC curves simply show for all dynasties the difference between children’s and parents’ relative achievements, their achievement being the proportion of people they dominate in their own distribution. We first describe DynaC and cumulated DynaC curves across dynasties to assess whether the overall mobility process over 1993-2014 has been progressive or not – both in terms of education and potential earnings – with potential implications for the role played by education reforms taking place over a generation. We also disentangle the contribution of education mobility on potential earnings mobility. Then, we present DynaC

29At the top, the 20%-30% rate of illiteracy characterizing parents completely disappears, while at the bottom, this rate was around 50%-60% and only 40%-70% of it disappears in the next generation.
dominance results when comparing the relative mobility of men and women. Our baseline relies on like-for-like comparisons (daughters to mothers and fathers to sons), but we also check the sensitivity to alternative definitions including a reference point that is common to both men and women.

5.1 Relative Mobility across Dynasties

Relative Education Mobility by Initial Education Class. We first represent education DynaCs across parents’ education classes in Figure1. The graph shows a declining pattern with ‘positive’ mobility only for dynasties of parents in the lowest education categories, i.e. classes 1 and 2 for ‘no education’ and ‘incomplete primary school’ (results are broadly similar for sons and daughters). This seems consistent with previous results indicating how education reforms have greatly reduced illiteracy in Indonesia. More than this, it tells us that dynasties where parents were at the lowest levels have progressed more than others on the education ladder. Faster education mobility among the lowest levels is expected, given the fact that the bulk of first-generations had no education or incomplete primary schooling (and, mechanically, dynasties with parents at higher level also had smaller margins of progress). While DynaC mobility by parental education class is interesting, most of the non-anonymity is lost with such a large grouping. Moreover, as seen above, uneducated parents are found at different levels of living standards. Hence, we turn to our main results, depicting DynaC curves over the range of dynasties \( p_t \) based on per-capita expenditure in first-generation households.

Relative Education Mobility by Per-Capita Consumption Percentile. For the sake of exposition, we show ventile averages of dynastic mobility measures. We want to know where relative mobility was the highest. In absolute terms, we have seen that everyone has gained. However, Figure 2(a) shows a regressive pattern of relative education mobility along the distribution of living standards, i.e. larger mobility scores among richer dynasties. This result is due to the combination of two factors. First, as discussed above, most of the first generation was uneducated: the low-educated parents were to be found in all quantiles of per-capita expenditure and not just among the consumption poor. More importantly, dynasties escaping illiteracy the most were those with a relatively higher living standard. We have documented this point before but illustrate it in Appendix Figure A5(a) using DynaC curves for dynasties of low-educated parents (classes 1 and 2) versus others.

They combine possibly very different situations across the different persons composing each ventile, in particular quite different patterns between men and women, as we shall see.
Figure 1: Relative Education Mobility (DynaC by Parent Education Levels)

Note: Educational level correspondence: 1 No Education, 2 Incomplete Primary School, 3 Completed Primary School, 4 Incomplete Junior High School, 5 Completed Junior High School, 6 Incomplete Senior High School, 7 Completed Senior High School, 8 University. Source: authors’ elaboration based on IFLS 1 and IFLS 5.

(parents with education classes 3-8). The former group shows positive mobility scores while the latter group does not progress as much, hence negative mobility scores. These results confirm that education reforms have lifted the new generation above basic education levels more rapidly in wealthier households.

Let us now consider dominance results. In the next section, we will address dominance between two groups (men and women) using propositions 1-4. At this stage, we simply examine, for the whole population, the dominance with respect to the immobility benchmark. In Figure 2(a), there is no dominance of mobility over immobility since we observe a crossing of the zero line around the third ventile as a result of the zero-sum property discussed above. Among the poorest dynasties, the mobility measure is negative, reflecting a slower pace of educational improvement compared to other dynasties. We will see that these negative values are due to men, whose relative position in terms of education tends to decline relative to that of their fathers. Moving to second-order dominance, we see in Figure 2(b) that immobility prevails over a larger segment (i.e. up to the fifth ventile) when priority is put on the poorest and further still in Figure 2(c) (up to the seventh ventile) when priority goes to the poorest of the poor.
Source: authors’ elaboration based on IFLS 1 and IFLS 5. Notes: DynaCs are represented by ventiles of per-capita expenditure. Dynasties are constructed by comparing daughters (sons) mothers (fathers).
Relative Mobility of Potential Earnings: the Role of Education. In Figure 3, we report DynaC results for the continuous outcome, i.e. potential earnings. Graph (a) depicts the DynaC curve (solid line) and a counterfactual (dashed line) obtained when setting children’s education level to their parents’ level. The main DynaC curve shows a regressive pattern (up to the 12th ventile) that turns progressive thereafter. Negative values at the top indicate that children from wealthier backgrounds progressed relatively less quickly, in terms of earnings-generating capacity, than the rest of the population. Importantly, while the overall pattern is fairly flat, the dashed curve shows that progressivity would be more pronounced if education levels had not changed between generations. In other words, the regressive nature of education reforms translates here into a reduced extent of progressivity in terms of potential earnings mobility. Given that the relative nature of our mobility measure is maybe more attractive when it comes to earnings – absolute progress in education is valuable per se, while relative mobility in earnings is the force behind a potential reduction in inequalities – this is in our view one of the most interesting empirical results of this work.

Relative Mobility of Potential Earnings: Dominance and Weighted DynaC. DynaCs based on continuous variables are equivalent to a change in rank between generations, as used in the recent literature focusing on re-ranking measures (cf. Section 2). Our framework goes a step further by adding dominance results and the possibility to weight achievement measures. First, we question the dominance of the mobility process over immobility. Figure 3(a) is inconclusive because the DynaC curve crosses the zero line (around the 17th ventile). Yet, if we put the emphasis on the poor, positive mobility scores prevail all along the distribution. This second-order dominance over immobility is shown in Figure 3(b). There is a fortiori dominance of the third order in Figure 3(c).

Then, weighted DynaC curves based on the achievement formulas of equation 3 allow us to account for structural changes in the distribution between parents and their children. Dynastic mobility is all the larger as the rank improvement across generation is larger. Weighted and unweighted DynaCs are presented together in appendix Figure A6, but only to inspect patterns (mobility levels are not comparable). Adding weights leads in this case to flatter trends, with in particular a lower degree of regressivity in the first half of the living standard distribution. As discussed in the theory section, if a dynasty is characterized by a zero weighted DynaC but a negative unweighted Dynac, there is a reduction in inequality below this dynasty, i.e. the distance between this dynasty and its followers shrinks as the latter are catching up. This is the situation we observe for instance at the 17th ventile, i.e.
Figure 3: Relative Mobility of Potential Earnings and the Role of Education (DynaCs)

Source: authors’ elaboration based on IFLS 1 and IFLS 5. Notes: DynaCs are represented by ventiles of per-capita expenditure. Dynasties are constructed by comparing daughters (sons) mothers (fathers).
5.2 Gender Heterogeneity and Dominance in Relative Mobility

Relative Education Mobility. We now move to the decomposition of the DynaC results by gender. Results are reported in Figure 4. In graph (a), DynaC mobility is only slightly progressive for men up to the 12th ventile and steadily regressive for women. These joint trends are consistent with the overall regressivity in Figure 2. As before, we can also inspect results by parental education level: Appendix Figures A5(b) and (c) confirm that for both men and women, those from richer families progress faster in the bulk of low-educated families.

Figure A7 in the Appendix shows that overall, boys and girls now attain very similar education levels while mothers were less educated than fathers. Hence, when daughters are compared to mothers and sons to fathers in our baseline (like-for-like matching), it is expected that the relative position of women increases more than that of men. This is indeed what we observe in Figure 4(a). From Proposition 1, we obtain a clear dominance of women’s mobility over men’s. Daughters have systematically improved their relative position compared to their mothers while only the sons of the top five ventiles are better ranked than their fathers. Because proposition 2 (resp. 3) is based on a sub-set of social preferences compared to proposition 1 (resp. 2), dominance of the first order implies dominance of second and third orders, as can be checked in Figure 4(b) and (c).

Relative Education Mobility with a Common Reference. A normative dimension – not discussed in the theory section because it is very much related to the empirical implementation – pertains to whom sons and daughters are compared to. With the like-for-like matching, we exacerbate gender differences in mobility. As discussed, girls’ ranks are compared to those of their mothers, who had the lowest education achievement in the previous generation. Thereby, daughters are deemed very mobile in this configuration. Alternatively, we can use a first-generation achievement that is common to both daughters and sons, for instance the mean education of both parents. We expect that girls’ mobility scores mechanically decrease and those of boys increase compared to the baseline. This is indeed the case in Figure 5(a). Importantly, what changes is the level, while DynaC patterns are similar to what we saw in the baseline, confirming the overall regressivity of educational mobility. DynaCs of men and women are crossing so there is no longer an unambiguous ranking of their mobility processes. With men now achieving more mobility,
than women in the lowest ventiles, dominance of male mobility goes further in graph (b) and even more in graph (c) so that there is almost full dominance of the third order.

Figure 4: Relative Education Mobility (DynaCs)

Source: authors’ elaboration based on IFLS 1 and IFLS 5. DynaCs are represented by ventiles of per-capita expenditure (PCE). In this baseline, daughters (sons) are compared to mothers (fathers).

This reversal in our conclusions about mobility dominance between men and women is not a concern: as indicated, it is a normative characterization that is not absolute but rather dependent to a large extent on the first-generation achievement of reference that is used.
Figure 5: Relative Education Mobility (DynaC by dynasty of per-capita expenditure, same reference point for sons and daughters)

Note: Parental achievement refers here to the rank of the best education level between the father and the mother. Source: authors’ elaboration based on IFLS 1 and IFLS 5.
Figure 6: Relative Mobility of Potential Earnings and the Role of Education (DynaC by dynasty of per-capita expenditure)

Source: authors’ elaboration based on IFLS 1 and IFLS 5.
in the analysis. Other results are stable and in particular the conclusion about the overall regressivity of educational mobility. It holds when using many alternative reference points, such as the father’s achievement or the highest achievement among the parents.\footnote{The educational advantages provided by parents are often defined by the highest level of human capital within the family (Erikson 1984). It is also possible to argue that the parent with less education is more relevant for children’s attainment, under the assumption that family dynamics adjust to the lowest common denominator in terms of schooling resources. Robustness checks using these different variants are available from the authors.}

We end our analysis of educational mobility with the dominance conditions defined in Proposition 4, for which results are reported in Table A3 in the appendix. This proposition reflects strong preferences for mobility among the ‘worst-off’ in an absolute sense. We suggest four different thresholds corresponding to ventiles 1, 3, 5 and 10 of the per-capita expenditure dynasties. In line with graphical results, we observe that the mobility profiles of daughters are significantly better than those of sons at all these cutoffs, and especially at the higher ones, in the like-for-like matching (panel A). Using the mean education achievement of the parents as a common reference point, we observe the same reversal as on the graphs when all the weight is put on lower segments of the dynastic distribution. In the presence of social preferences over mobility profiles à la Rawls, one can judge the mobility of men to be superior to that of women (Panel B) in this case.\footnote{In the presence of utilitarian social preferences, one would probably judge the mobility of women as superior to that of men.}

**Relative Mobility of Potential Earnings.** We report DynaC curves for potential earnings in Figure 6(a). Patterns are broadly regressive for women (except at the top) and mildly progressive for men. We disentangle again the role of education by comparing the DynaC curves to the counterfactuals where children’s education is set to their parents’ level. We obtain the same result as in Figure 3 for both men and women: regressivity in educational mobility tends to limit the extent of progressivity in potential earnings mobility. For women, mobility would be relatively neutral (and still progressive at the top) if girls had the same education levels as their mothers. For men, progressivity would be accentuated if they had the education levels of their fathers. These results translate in monetary terms our findings regarding educational mobility (and possibly the underlying role of education reforms). Figure 6 also points to an unequivocal dominance of women’s mobility patterns over men’s. As for education, this conclusion depends on the reference point used in the first generation.\footnote{Additional (unreported) results show a similar reversal when the father’s or the highest earnings achievement is used as a common reference for both men and women. This reversal is even more radical}
6 Conclusion

We provide a new tool for the comparison and evaluation of intergenerational mobility processes. Our theoretical framework is specifically tailored to provide a general characterization of intergenerational mobility. It can be implemented to generate partial but robust rankings of mobility processes for different subgroups of the population (e.g. gender, ethnic group, etc.) and using all types of individual achievements (independently of the cardinal or ordinal scale of those outcomes). Our approach can incorporate a weighting scheme which – differently from classic measures of rank changes – can account for structural changes in the parents’ and offsprings’ distributions.

We apply our theoretical framework to the IFLS data to empirically analyze the intergenerational patterns of mobility in Indonesia, both in terms of educational attainment and potential earnings, between 1993 and 2014. For both outcomes, relative mobility patterns were markedly to the advantage of women. A large part of the population was lifted out of illiteracy, possibly due to the large-scale education reforms implemented in Indonesia. However, our DynaC approach also shows that educational mobility was regressive: dynasties that have progressed the most were those of the low-educated parents at the top of the living standard distribution. These patterns also tend to seriously limit the degree of progressivity of the mobility in terms of potential earnings.

The theoretical and empirical results proposed in this paper are encouraging and open new avenues of research. From a theoretical perspective, three extensions seem particularly promising. One concerns the assessment of intergenerational mobility across more than two generations. The other extension would consist in adapting the model to allow for a multidimensional evaluation of mobility. The last extension is more challenging and would consist in providing a full characterization of a mobility measure based on weighted achievements. From an empirical perspective, the mobility scheme proposed in this paper could be applied to other countries for which long panels exist (mainly rich countries). Cross-country comparisons would be of particular interest, for instance to characterize how the relative mobility of poor dynasties (in terms of education, earnings or health) varies across countries and to identify the policies that may have contributed to strong relative mobility in some countries.

than in the case of education due to the fact that distributions of potential earnings are even more contrasted across groups than those of education. As can be seen in Figure A8, potential earnings of fathers are markedly larger than those of mothers while those of sons and daughters broadly overlap.
References


A Appendix A: Additional results
Figure A1: Age Distribution by Cohort and Gender

Source: Authors’ elaboration based on IFLS 1 and IFLS 5 data. Sample size: 4,955 for offsprings, 2,558 for fathers and 2,492 for mothers.
Figure A2: Distribution of Educational Levels

Source: authors’ elaboration based on IFLS 1 and IFLS 5. Note: Educational level correspondence: 1 No Education, 2 Incomplete Primary School, 3 Completed Primary School, 4 Incomplete Junior High School, 5 Completed Junior High School, 6 Incomplete Senior High School, 7 Completed Senior High School, 8 University. Source: authors’ elaboration based on IFLS 1 and IFLS 5.
Figure A3: Absolute Educational Mobility by Education Class

Source: authors’ elaboration based on IFLS 1 and IFLS 5. Note: Educational level correspondence: 1 No Education, 2 Incomplete Primary School, 3 Completed Primary School, 4 Incomplete Junior High School, 5 Completed Junior High School, 6 Incomplete Senior High School, 7 Completed Senior High School, 8 University. Source: authors’ elaboration based on IFLS 1 and IFLS 5.
Figure A4: Absolute Educational Mobility by Per-Capita Consumption Percentile
Figure A5: Relative Education Mobility, DynaCs by First-Generation Education Level

Source: authors’ elaboration based on IFLS 1 and IFLS 5.
Figure A6: Relative Potential Earnings Mobility, **Weighted** DynaC

Source: authors’ elaboration based on IFLS 1 and IFLS 5.
Figure A7: Education Distribution by Cohort and Gender

Note: Educational level correspondence: 1 No Education, 2 Uncompleted Primary School, 3 Completed Primary School, 4 Uncompleted Junior High School, 5 Completed Junior High School, 6 Uncompleted Senior High School, 7 Completed Senior High School, 8 University. Source: Authors’ elaboration based on IFLS 1 and IFLS 5 data. Sample size: 4,955 for offsprings, 2,558 for fathers and 2,492 for mothers.

Figure A8: Potential Earnings Distribution by Cohort and Gender

Source: Authors’ elaboration based on IFLS 1 and IFLS 5 data. Sample size: 4,955 for offsprings, 2,558 for fathers and 2,492 for mothers.
Table A1: Heckman Regressions

<table>
<thead>
<tr>
<th></th>
<th>Daughter</th>
<th>Son</th>
<th>Mother</th>
<th>Father</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earnings</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>0.00864</td>
<td>0.0146**</td>
<td>0.00362</td>
<td>0.00553</td>
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<tr>
<td></td>
<td>(0.00686)</td>
<td>(0.00625)</td>
<td>(0.00593)</td>
<td>(0.0106)</td>
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<tr>
<td>Education: Primary School</td>
<td>0.153</td>
<td>0.0221</td>
<td>0.0410</td>
<td>0.0980</td>
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<td></td>
<td>(0.141)</td>
<td>(0.110)</td>
<td>(0.0735)</td>
<td>(0.191)</td>
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<tr>
<td>Education: Junior High School</td>
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<td>0.325***</td>
<td>0.334***</td>
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<td></td>
<td>(0.135)</td>
<td>(0.102)</td>
<td>(0.116)</td>
<td>(0.270)</td>
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<td>Education: Senior High School</td>
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<td>0.642***</td>
<td>0.504***</td>
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<td></td>
<td>(0.123)</td>
<td>(0.0870)</td>
<td>(0.105)</td>
<td>(0.229)</td>
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<td>Education: University</td>
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<td>(0.123)</td>
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<td><strong>Selection</strong></td>
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<tr>
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Note: Standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01
### Table A2: Summary statistics

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<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
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<td>Daughter’s age</td>
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<td>Log annual PCE</td>
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<tr>
<td><strong>Parents</strong></td>
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<td>Father’s level of education</td>
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<tr>
<td>Mother’s log predicted annual earnings</td>
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<td>12.1</td>
<td>14.7</td>
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<td>Father’s log predicted annual earnings</td>
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<td>Log annual PCE</td>
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<td>0.6</td>
<td>12.1</td>
<td>18.3</td>
<td>4224</td>
</tr>
</tbody>
</table>

Source: Authors’ elaboration based on IFLS 1 and IFLS 5 data.

PCE: per capita expenditure (IDR, real, base: 2002).

### Table A3: Education Mobility: Dominance Results of Proposition 4

<table>
<thead>
<tr>
<th>Panel A: Baseline (like-for-like)</th>
<th>Sons</th>
<th>Daughters</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold: $\bar{p} = 1$</td>
<td>-0.009</td>
<td>-0.001</td>
<td>-28.315</td>
</tr>
<tr>
<td>Threshold: $\bar{p} = 3$</td>
<td>-0.010</td>
<td>0.009</td>
<td>-33.950</td>
</tr>
<tr>
<td>Threshold: $\bar{p} = 5$</td>
<td>-0.015</td>
<td>0.009</td>
<td>-37.968</td>
</tr>
<tr>
<td>Threshold: $\bar{p} = 10$</td>
<td>-0.034</td>
<td>0.047</td>
<td>-59.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Same Reference Point (mean educ.)</th>
<th>Sons</th>
<th>Daughters</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold: $\bar{p} = 1$</td>
<td>-0.001</td>
<td>-0.006</td>
<td>28.742</td>
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<tr>
<td>Threshold: $\bar{p} = 3$</td>
<td>0.005</td>
<td>-0.004</td>
<td>27.653</td>
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<td>Threshold: $\bar{p} = 5$</td>
<td>0.005</td>
<td>-0.012</td>
<td>36.820</td>
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<tr>
<td>Threshold: $\bar{p} = 10$</td>
<td>0.013</td>
<td>0.006</td>
<td>11.462</td>
</tr>
</tbody>
</table>

Note: T-test of statistical significance obtained through 300 bootstrap replications.

Same Reference Point refers to father’s and mother’s average education level.

Source: Authors’ elaboration based on IFLS 1 and IFLS 5 data.
Proof of Proposition 1
We seek sufficient and necessary conditions such that:

\[ \Delta W = \int_0^1 w(p_t)(d_\omega(p_t)dp_t - d_\pi(p_t)dp_t) \geq 0, \text{ for all } W \in P^* \] (12)

Let \( \delta(p_t) = d_\omega(p_t) - d_\pi(p_t) \) so equation (12) is rewritten as:

\[ \Delta W = \int_0^1 w(p_t)\delta(p_t)dp_t \geq 0 \] (13)

For the sufficiency condition, note that \( w(p_t) \geq 0 \) for all \( p_t \in [0,1] \), so that \( \delta(p_t) \geq 0 \) for all \( p_t \in [0,1] \) implies that \( \int_0^1 w(p_t)\delta(p_t)dp_t \geq 0 \). For the necessary condition, let \( \Delta W \geq 0 \), but assume that \( \delta(p_t) < 0 \) for some \( p_t \in [0,1] \). Following Lemma 1 in Chambaz and Maurin (1998), there exists a set of values \( z(p) \in V^+ \) and \( \rho(p) \in V^+ \) such that \( \int_0^1 z(p)\delta(p_t)dp_t \leq 0 \). Define \( z(p_t) = w(p_t) \), since \( z(p_t) \in V^+ \) (hence \( z(p_t) > 0 \) for all \( p_t \)), substituting in equation (13) gives \( \Delta W \leq 0 \), which is a contradiction.

Proof of Proposition 2
We look for sufficient and necessary conditions such that:

\[ \Delta W = \int_0^1 w(p_t)dp_t \geq 0, \text{ for all } W \in P^* \] (14)

for which \( \frac{\delta w(p_t)}{dp_t} \leq 0 \) for all \( p_t \in [0,1] \). For the sufficiency part, we integrate equation (14) by parts:

\[ w(p_t = 1) \int_0^1 \delta(p_t)dp_t - \int_0^1 w'(p_t) \int_0^{p_t} \delta(q_t)dq_t dp_t \] (15)

Since \( w(p_t = 1) \geq 0 \) for all \( p_t \in [0,1] \), \( \int_0^{p_t} \delta(q_t)dq_t \geq 0 \) for all \( p_t \in [0,1] \) implies \( w(p_t = 1) \int_0^1 \delta(p_t)dp_t \geq 0 \). Furthermore, since \( w'(p_t) \leq 0 \) for all \( p_t \in [0,1] \), we have \( \int_0^1 w'(p_t) \int_0^{p_t} \delta(q_t)dq_t dp_t \leq 0 \). Thus, \( \Delta W \geq 0 \). For the necessity part let \( \Delta W \geq 0 \), but assume that \( \int_0^{p_t} \delta(q_t)dq_t < 0 \) for some \( p_t \in [a,b] \subset [0,1] \). Rewrite equation (15) as follows:

\[ w(p_t = 1) \int_0^1 \delta(p_t)dp_t + \int_0^1 -w'(p_t) \int_0^{p_t} \delta(q_t)dq_t dp_t \] (16)
Denote $-w'(p_t) = \alpha(p)$. By Lemma 2 in Chambaz and Maurin (1998), \( \int_0^1 \alpha(p) \int_0^{p_t} \delta(q_t) dq_t \leq 0 \) for all \( \alpha(p) \in V^+ \) and \( p_t \in [a, b] \subset [0, 1] \). Suppose \( \int_0^{p_t} \delta(q_t) dq_t \geq 0 \) for all \( p_t \in [0, 1] \setminus [a, b] \), the second term of equation 15 becomes negative. Then it is always possible to find combinations of \( w(p_t) \) and \( \delta(p_t) \) such that:

\[
\left| w(p_t = 1) \int_0^1 \delta(p_t) dp_t \right| < \left| \int_0^1 -w'(p_t) \int_0^{p_t} \delta(q_t) dq_t dp_t \right| \tag{17}
\]

which results in \( \Delta W < 0 \), a contradiction.

**Proof of Proposition 3**

We seek sufficient and necessary conditions for:

\[
\Delta W = \int_0^1 w(p_t)\delta(p_t) dp_t \geq 0 \tag{18}
\]

\( \forall W \in P^* \) for which \( \frac{\delta w(p_t)}{\delta p_t} \leq 0 \) and \( \frac{\delta^2 w(p_t)}{\delta p_t^2} \geq 0 \) for all \( p_t \in [0, 1] \). For sufficiency, consider equation 15 and use the following notation \( \Psi(p_t) = \int_0^{p_t} \delta(q_t) dq_t \). Integrating by parts the second component:

\[
w(1)\Psi(1) - w'(1) \int_0^1 \Psi(p_t) dp_t + \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \tag{19}
\]

Since \( w''(p_t) \geq 0 \forall p_t \in [0, 1] \), \( \int_0^{p_t} \Psi(q_t) dq_t \geq 0 \) for all \( p_t \in [0, 1] \) implies \( \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \geq 0 \); since \( w(1)' \leq 0 \), it also implies that \( -w(1)' \int_0^1 \Psi(p_t) dp_t \geq 0 \); last, given that \( w(1) \geq 0, w(1)\Psi(1) \geq 0 \). Thus, \( \int_0^{p_t} \Psi(q_t) dq_t \geq 0 \) for all \( p_t \in [0, 1] \) is sufficient for \( \Delta W \geq 0 \). For the necessity part, let \( \Delta W \geq 0 \), but assume that \( \int_0^{p_t} \Psi(q_t) dq_t < 0 \) for some \( p_t \in [\alpha, \beta] \subset [0, 1] \). \( \int_0^1 w(p_t)'' \int_0^1 \Psi(q_t) dq_t dp_t \leq 0 \) for all \( w(p_t)'' \in V^+ \) and \( p_t \in [\alpha, \beta] \). Assuming that \( \int_0^{p_t} \Psi(p_t) dq_t \geq 0 \) for all \( p_t \in [0, 1] \setminus [a, b] \), then \( -w'(1)\Psi(1) + \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \leq 0 \). Now, it is always possible to find a combination of \( w(1) \) and \( \Psi(1) \) such that \( |w(1)\Psi(1)| < \left| -w'(1) \int_0^1 \Psi(p_t) dp_t + \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \right| \), which would result in \( \Delta W < 0 \), a contradiction.

**Proof of Proposition 4**

We want to find sufficient and necessary conditions for

\[
\Delta W = \int_0^1 w(p_t)\delta(p_t) dp_t \geq 0 \tag{20}
\]

\( \forall W \in P^* \) for which \( w(p_t) = w(q_t) > 0 \ \forall p_t, q_t \in [0, \bar{p}] \) and \( w(p_t) = 0 \ \forall p_t \in [\bar{p}, 1] \).
For a given $\bar{p}_t$, since $w(p_t) = w(q_t) > 0$ for all $p_t, q_t \in [0, \bar{p}]$ and $w(p_t) = 0$ for all $p_t \in [\bar{p}_t, 1]$, rewrite Equation 20 as follows:

$$\Delta W = w \int_0^{\bar{p}_t} \delta(p_t) dp_t \geq 0$$

(21)

Given $w > 0$, $\int_0^{\bar{p}_t} \delta(p_t) dp_t \geq 0$ is necessary and sufficient for $\Delta W \geq 0$.

C Appendix C: An additional proposition

**Proposition 3’**: Given two mobility processes $D^{(t+1)}_\pi$ and $D^{(t+1)}_\omega$, $D^{(t+1)}_\pi$ is preferred to $D^{(t+1)}_\omega \ \forall W \in P^*$ for which $\frac{\delta w(p_t)}{\delta p_t} \leq 0$ for all $p_t \in [0, 1]$, $\frac{\delta^2 w(p_t)}{\delta p_t^2} \geq 0$ for all $p_t \in [0, \bar{p}]$, $\frac{\delta^2 w(p_t)}{\delta p_t^2} \leq 0$ for all $p_t \in [\bar{p}, 1]$ if and only if

$$(i) \int_0^{p_t} \int_0^{q_t} \delta_w(s_t) ds_t dq_t \geq \int_0^{p_t} \int_0^{q_t} \delta_w(s_t) ds_t dq_t, \forall q_t \in [0, 1] \forall p_t \in [0, \bar{p}]$$

(22)

$$(ii) \int_{\bar{p}_t}^{1} \int_0^{q_t} \delta_w(s_t) ds_t dq_t \geq \int_{\bar{p}_t}^{1} \int_0^{q_t} \delta_w(s_t) ds_t dq_t, \forall q_t \in [0, 1] \forall p_t \in [\bar{p}, 1].$$

(23)

This is an alternative to Proposition 3. It suggests a test based on third-order upward (downward) DynaC dominance for all dynasties ranked lower (higher) or equal to $\bar{p}$. It finds its justification in the presence of a social planner that wants to preserve the mobility of the poorest among the poor while, at the same time, avoiding that the distances among the richest dynasties growth further apart (see Aaberge (2009) for a discussion on the application of this principle in standard inequality measurement).

**Proof of Proposition 3’**

We seek sufficient and necessary conditions for:

$$\Delta W = \int_0^{1} w(p_t)\delta(p_t) dp_t \geq 0$$

(24)

$\forall W \in P^*$ for which $\frac{\delta w(p_t)}{\delta p_t} \leq 0$ for all $p_t \in [0, 1]$, $\frac{\delta^2 w(p_t)}{\delta p_t^2} \geq 0$ for all $p_t \in [0, \bar{p}]$, $\frac{\delta^2 w(p_t)}{\delta p_t^2} \leq 0$ for all $p_t \in [\bar{p}, 1]$.

For a given $\bar{p}_t$, rewrite equation 24 as follows:

$$\Delta W = \int_0^{\bar{p}_t} w(p_t)\delta(p_t) dp_t + \int_{\bar{p}_t}^{1} w(s_t)\delta(s_t) ds_t \geq 0$$

(25)
For sufficiency, use the notations \( \Psi(p_t) = \int_0^{p_t} \delta(q_t) \, dq_t \), \( \Psi(\vec{p}_t) = \int_0^{\vec{p}_t} \delta(p_t) \, dp_t \) and \( \Theta(s) = \int_{s_1}^{s} \delta(r_t) \, dr_t \) and \( \Theta(1) = \int_{\vec{p}_t}^{1} \delta(r_t) \, dr_t \). We integrate equation (25) by parts twice to have:

\[
w(\vec{p}_t)\Psi(\vec{p}_t) - w'(\vec{p}_t) \int_0^{\vec{p}_t} \Psi(p_t) \, dp_t + \int_0^{\vec{p}_t} w''(p_t) \int_0^{p_t} \Psi(q_t) \, dq_t \, dp_t = w(1)\Theta(1) - w'(1) \int_{\vec{p}_t}^{1} \Theta(s_t) \, ds_t + \int_{\vec{p}_t}^{1} w''(s_t) \int_{s_t}^{r_t} \Theta(r_t) \, dr_t \, ds_t.
\]

The last component of the above equation can be rewritten as follows:

\[
\int_{\vec{p}_t}^{1} w''(s_t) \left[ \int_{s_t}^{r_t} \Theta(r_t) - \int_{s_t}^{1} \Theta(r_t) \right] \, dr_t \, ds_t = \int_{\vec{p}_t}^{1} w''(s_t) \int_{s_t}^{1} \Theta(r_t) \, dr_t \, ds_t - \int_{\vec{p}_t}^{1} w''(s_t) \int_{s_t}^{r_t} \Theta(r_t) \, dr_t \, ds_t.
\]

Noting that \( \int_{\vec{p}_t}^{1} w''(p_t) \, dp_t = w'(1) - w'(\vec{p}_t) \), for \( w'(1) = 0 \) we have:

\[-w'(\vec{p}_t) \int_{\vec{p}_t}^{1} \Theta(r_t) \, dr_t - \int_{\vec{p}_t}^{1} w''(s_t) \int_{s_t}^{1} \Theta(r_t) \, dr_t \, ds_t.
\]

\( \Delta W \) can now be rewritten as follows:

\[
w(\vec{p}_t)\Psi(\vec{p}_t) - w'(\vec{p}_t) \int_0^{\vec{p}_t} \Psi(p_t) \, dp_t + \int_0^{\vec{p}_t} w''(p_t) \int_0^{p_t} \Psi(q_t) \, dq_t \, dp_t = w(1)\Theta(1) - \int_{\vec{p}_t}^{1} w''(s_t) \int_{s_t}^{1} \Theta(r_t) \, dr_t \, ds_t
\]

Since \( w''(s_t) \leq 0 \) for all \( s_t \in [\vec{p}_t, 1] \), \( \int_{s_t}^{1} \Theta(r_t) \, dr_t \geq 0 \) for all \( s_t \in [\vec{p}_t, 1] \) implies \(-\int_{\vec{p}_t}^{1} w''(s_t) \int_{s_t}^{1} \Theta(r_t) \, dr_t \, ds_t \geq 0 \). Hence, \( \int_{s_t}^{1} \int_{\vec{p}_t}^{s} \delta(r_t) \, dr_t \, ds_t \geq 0 \) for all \( s \in [\vec{p}_t, 1] \) is sufficient for the sum of the last two component of equation (26) to be positive. The sufficiency for the positivity of the first three terms has been proved in Proposition 3, for this proposition just assume that \( p_t \in [0, \vec{p}_t] \). Putting together the arguments: \( \int_0^{p_t} \int_0^{\alpha} \delta(x_t) \, dx_t \, dq_t \geq 0 \) for all \( p_t \in [0, \vec{p}_t] \) and \( \int_{s_t}^{1} \int_{\vec{p}_t}^{s} \delta(r_t) \, dr_t \, ds_t \geq 0 \) for all \( s \in [\vec{p}_t, 1] \) imply \( \Delta W \geq 0 \). For the necessity part, let \( \Delta W \geq 0 \), but assume that \( \int_{s_t}^{1} \Theta(r_t) \, dr_t < 0 \) for some \( s_t \in [\alpha, \beta] \subset [\vec{p}_t, 1] \). \( \int_{\vec{p}_t}^{1} w(p_t)'' \int_{s_t}^{1} \Theta(r_t) \, dr_t \, ds_t \leq 0 \) for all \( w(p_t)'' \in V^+ \) and \( p_t \in [\alpha, \beta] \). Assuming that \( \int_{s_t}^{1} \Theta(r_t) \, dr_t \geq 0 \) for all \( s_t \in [\vec{p}_t, 1] \setminus [a, b] \), then \( \int_{\vec{p}_t}^{1} w''(s_t) \int_{s_t}^{r_t} \Theta(r_t) \, dr_t \, ds_t \leq 0 \). Now, it is always possible to find a combination of \( w(1) \) and \( \Theta(1) \) such that \( |w(1)\Theta(1)| < \left| \int_{\vec{p}_t}^{1} w''(p_t) \int_0^{p_t} \Psi(q_t) \, dq_t \, dp_t \right| \). Putting together these results with those obtained for proposition 3 (letting them holding for all \( p \in [0, \vec{p}] \)) would result into \( \Delta W < 0 \), a contradiction.
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