# Modelling relative humidity effect on load duration of timber beams

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## Summary

Under mechanical and various climatic conditions, timber beams are subjected to deformations, damage and cracking which can drive to complete failure. A stochastic approach is used to predict (forecast) the evolutions of the air humidity which can occur in the future. The presented model firstly predicts the incubation time before cracking and secondly the time of crack propagation until the failure. Predictions of the proposed model [for stochastic variations of air humidity] are compared to delayed rupture test realized on bending of notched beams under various climatic conditions.

## 1. Introduction

Wood mechanical properties (elastic, viscous, fracture…) depend on moisture content, on temperature and, moreover, on their variations. The moisture content of wood (w) depends on the intrinsic physical characteristics of the material, the air relative humidity (RH) and the temperature of the surrounding environment. To study the lifetime of timber structures [9] in their environment, climatic variations and particularly variations of ambient humidity must be taken into account. It is known that temperature has a lower influence on the time to failure than RH and thus, it will not be discussed hereafter. Under mechanical load, a stress concentration can create cracks, which can cause failure [4, 5, 6]. The moisture content variation modifies the viscoelastic properties of wood and thus the long duration behaviour of timber elements.

Considering the climatic variations expected in the future, we have developed a stochastic model to simulate the future RH. This model is coupled with a viscoelastic crack model to predict failure under variable and extreme climates.

## 2. Stochastic modelling of relative humidity RH

The modelling of temporal variability (climate changes) or spatial variability (evolution of the spatial mechanical characteristics of wood) can be relied on the notion of stochastic processes. These processes are similar to an infinite number of random variables indexed on the coordinates of the considered deterministic parameter (time hereafter). In addition to the mean and standard deviation that may change with the indexed variables, stochastic processes are characterized by an auto-correlation function, which implies some dependence between neighbouring points of the same process. So, for instance, the relative humidity at a given time is correlated to the value measured at the previous hour but it is independent of the value RH of the previous year.

Several methods exist to generate the process: OLE, EOLE, Karhunen-Loève, polynomial chaos [7]. The Karhunen-Loève method, which is based on a spectral decomposition of the covariance function of the process, provides good results and it is less time consuming. The process describing the relative humidity RH is as follows (1):

$$RH(t) = \overline{RH}(t) + \sum_{i=1}^{\infty} \sqrt{\lambda_i} \cdot \xi_i(t) \cdot f_i(t) \qquad (1)$$

t represents time (positive real), $\overline{HR}(t)$ is the average of the process. $\xi$ (t) is a vector whose components $\xi_i$ (T) are Gaussian random centred variables. $\lambda_i$ is the eigenvalue corresponding to the eigenvector $f_i$ of the covariance function.

The relative humidity model must be sufficiently complex to describe reality as well as possible. It must also take into account several elements that are representative of the weather forecasts. The developed model generates processes for day and night according to season. It also generates seasonal processes in which dry or wet periods can be created. In order to limit the number of calculations, year is divided into two seasons: a wet season with high relative humidity level ($RH_{wet}$) and a dry season ($RH_{dry}$). Thus, the relative humidity is considered as a stochastic process whose trajectory evolves around seasonal sinusoidal average values. In the same way, day is divided into two periods: $RH_{days}$ and $RH_{night}$. For each day, the couple of average values obtained ($RH_{day}$, $RH_{night}$) generates the daily processes that are appropriate for a given season.

The relative humidity related to a moment t depends on its value at the moment t-1. These values are correlated each other by a daily process. This correlation is obtained with an average of daily RH. The length of desired correlation can be chosen in the model. The correlation between seasons has not been realised yet. Moreover, the greatest variation of RH values is obtained between seasons.

The stochastic model parameters fit to last years meteorological data of the weather in the centre of France (Clermont-Ferrand). The annual average RH is about 65%. Simulations are realised considering this average annual RH as the beginning of the calculations, with a variety of scenarios: no change in average RH in the future, a decrease or an increase of average annual RH of 10% in 50 years. The standard deviation between $RH_{wet}$ and $RH_{dry}$ is taken equal to 10% and standard deviation between $RH_{day}$ and $RH_{night}$ is equal to 20%. These values, in agreement with the meteorological observations, are supposed to be fixed in this first approach. Figure 3 presents an example of the stochastic simulation of the relative air humidity.

## 3. From the relative humidity to the moisture content of wood

The moisture content at the surface of a beam is assumed to be equal to the equilibrium moisture content ($w_{eq}$) of the material obtained from the iso-sorption curves. In this study, we use the iso-sorption curve of a reconstituted wood –spruce- (LVL, Laminated Veneer Lumber) obtained by Lasserre [8] at a temperature of 20°C (figure 1).

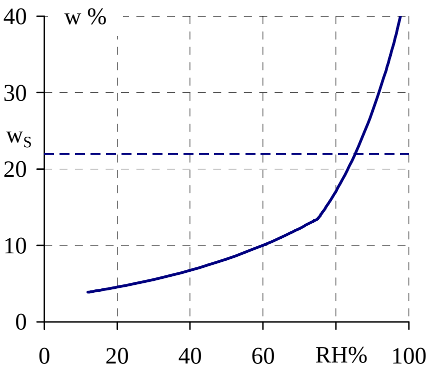

*Fig. 1 Isotherm sorption curve of LVL material at the temperature T = 20°C*

Assuming a process with only one direction of sorption (height of the beam), and assuming that the relative humidity RH is expressed by a Fourier series (periodic basis), we can approximate the value w of the moisture content of wood at a depth z as follows (2):

$$w(z) = \overline{w_{eq1}} + \sum_{i=2}^{4} \exp\left(-\left(\frac{\pi f_i}{\Delta}\right)^{1/2} z\right)\left(\overline{w_{eq\,i}} - \overline{w_{eq\,i-1}}\right) \qquad (2)$$

With $\overline{w_{eqi}}$, the equilibrium average moisture content on surface for the period rang i, $f_i$: frequency associated to the period i, $\Delta$ : sorption coefficient. Practically, the moisture content w(z) can be determined for four periods: i = 1: year; i = 2: episode 3 months; i = 3: month (30.5 days); i = 4: week (7 days).

In this study, RH and thus w are not strictly periodic functions, but they average values are periodic. As an approximation, the relationship (2) is applied to determine moisture content in a notched beam (figure 2). The absorption of LVL is mainly in its transverse direction, parallel to the layers of glue. The value of $\Delta$ is taken equal to $20.10^{-11}$ m² / s, value obtain at the US2B for spruce. The average moisture content and the moisture content at surface of the notched beam according to relative humidity is given on figure 3.

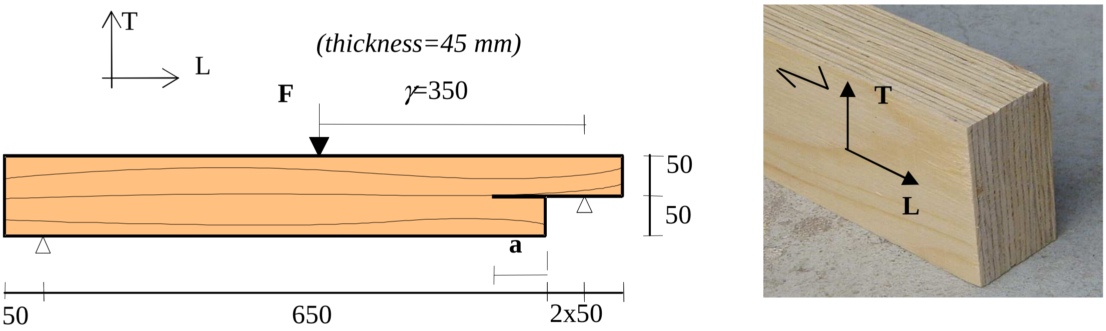

*Fig. 2 Shape of the studied Laminated Veneer Lumber notched beams (dimensions in mm)*

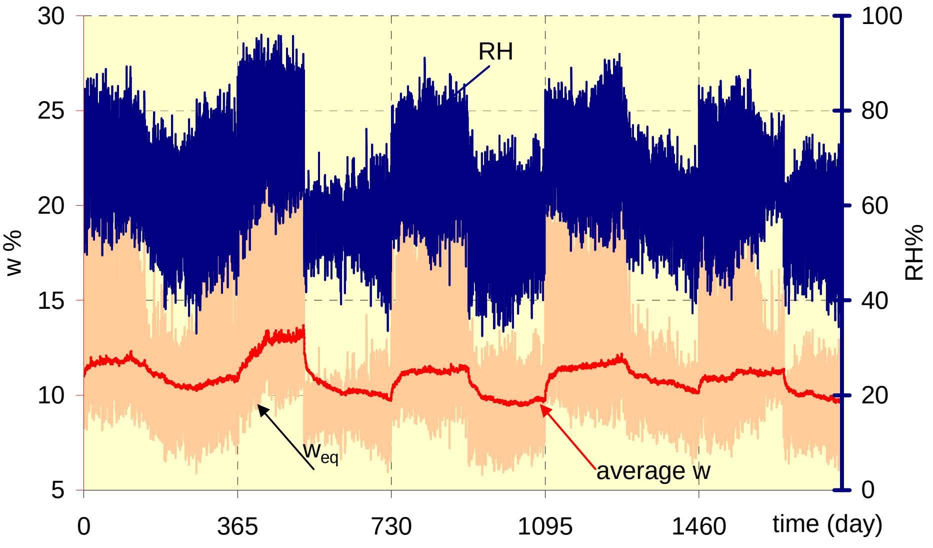

*Fig. 3 Stochastic evolutions of the relative humidity (RH), moisture content at surface $w_{eq}$ and average moisture content w in the beam. (Sorption curves for LVL at the temperature T = 20°C)*

# 4. The failure model

The performed model deals with the determination of the time to failure of timber elements whose rupture is due to the creation and the propagation of a crack (as in notched beams or in beams with a hole or in other geometric singularities). A damage model is applied to predict initiation (incubation) time: it is the time taken to create a damage area which leads to the creation of a macro-crack. The propagation of the crack is modelled by a fracture mechanics model considering that the crack grows in an orthotropic viscoelastic medium and a damage area exists at the crack tip [3].

## 4.1 Viscoelastic crack model VCM

In the incubation phase, damage is described by a characteristic parameter D ranging from 0 at the beginning of loading to $D_i$ at the crack initiation. The second model (equation 3) of Barrett and Foschi [1], with a non-linear damage evolution and a non-linear cumulative damage, has been chosen:

$$\begin{cases} \dfrac{dD}{dt} = A \cdot \left( \dfrac{F(t) - F_o}{F_s} \right)^B + C.D(t) & \text{if} \quad F(t) > F_o \\ \dfrac{dD}{dt} = 0 & \text{if} \quad F(t) < F_o \end{cases} \tag{3}$$

where□ F(□t) is the applied load at time t, Fs the static strength of the element at a reference moisture content (hereafter $w_{ref}$= 20%), $F_o$ a threshold load and A, B and C are parameters.

The crack propagation model is based on the Shapery studies [2], derived from the Barenblatt’s crack model. In the neighbourhood of the crack, the material is divided into two regions: a process zone which can be highly damaged, nonlinear and viscoelastic and a region surrounding the process zone where the material is considered as linear, viscoelastic and orthotropic. The crack length speed (da/dt) is given by the equation (4). Details on the determination of this relation are developed by Chaplain in [3].

$$\frac{da}{dt} = \frac{\pi}{2} \left[ \frac{C_2 \lambda_n}{\left( K_{Ic}^2 - K_I^2 \right)} \right]^{1/n} \frac{K_I^{2(1+1/n)}}{\left( \sigma_m I_1 \right)^2} \tag{4}$$

$K_I$ (a) is the stress intensity factor in opening mode, depending on the crack length a. $K_{Ic}$ is the critical stress intensity factor. $\sigma_m I_1$ represents the distribution of the cohesive stress at the crack tip. The parameter $\lambda_n$ is a function of the viscosity of the material. The parameter n is the power of the law used to determinate the reduced creep compliance $\kappa^v(t)$ in mode I (equation 5).

$$\kappa^v(t) = C_0 \cdot \left( 1 + C_2 t^n \right) \tag{5}$$

$C_0$ and $C_2$ are parameters calculated from viscoelastic stiffnesses of orthotropic material.

The equation (4) is integrated numerically. The time to failure is reached when $K_I = K_{Ic}$ (crack speed becomes infinite).To solve the equation (4), the relationship between the stress intensity factor $K_I$ of the structure and length “a” of the considered crack must be determined. This relation is depends on the moisture. In model, the other material parameters are also functions of the moisture content w of the element as presented in the following chapter.

## 4.2 Model parameters

In the initiation damage model, $F_o$ is taken as a function of w. The threshold stress level $SL_o=F_o/F_s$, is taken equal to 0,60 for MC=20% and $SL_o$=0,55 at w = 9% ($F_s$ is the static strength of the element at the reference moisture $w_{ref}$= 20%). These values are in accordance with the experimental. Both values are also in agreement with the short term test observations; the crack initiation time is shorter in dry beams than in wet beams [5]. We suppose that the other parameters A, B and C do not depend on w. Their values fit on the results of tests on notched beams under constant load at the reference moisture content $w_{ref}$. The deflection of notched beams is calculated by finite element method with and without a small crack (5 mm long). Considering that this displacement is proportional to the compliance of the beam, and so representative of the damage of the beam, we obtained $D_i$=0,01 for a beam with a crack length equal to 5 mm. The other parameters are fitted on the experimental results obtained on LVL specimens (Table 1).

The fracture mechanics model parameters were determined experimentally for two moistures from characterization tests on LVL (creep and cracking). Their values are given in Table 1.

| w (%) | A [$h^{-1}$] | B | C [$h^{-1}$] | $SL_o$ | $C_2$ [$s^{-n}$] | n | $\lambda_n$ | $\sigma_m I_1$ [MPa] | $K_{Ic}$ [MPa√mm] |
|---|---|---|---|---|---|---|---|---|---|
| 9 | $2{,}11.10^{14}$ | 30 | 0,075 | 0,55 | $9{,}51\ 10^{-3}$ | 0,41 | 0,63 | 67 | 23 |

*Table 1 Parameters used in the VCM model*

For the presented crack models, it is necessary to know the relationship between the stress intensity factor $K_I$ and the crack length (given by a calibration function g). This function generally depends on the mode of failure, on the elastic properties of a given orthotropic material and on the crack length "a". It also depends on the specimen geometry, and on the type of load (tension, bending). $K_I$ is given by equation (6) :

$$K_I = \frac{F}{b\sqrt{\gamma}} g\left(\frac{a}{\gamma}\right) \tag{6}$$

$\gamma$ is a characteristic length of the specimen (Fig. 2); g is an adimensional calibration function.

A finite element calculation resulted in expressions of g for two moistures w (w = 9% and 20%) [5]. The elastic behaviour of LVL used to obtain the calibration function by finite elements calculation has been obtained from tensile and bending tests. Figure 4 presents the evolution of the calibration function for these LVL notched beams for both moistures w.

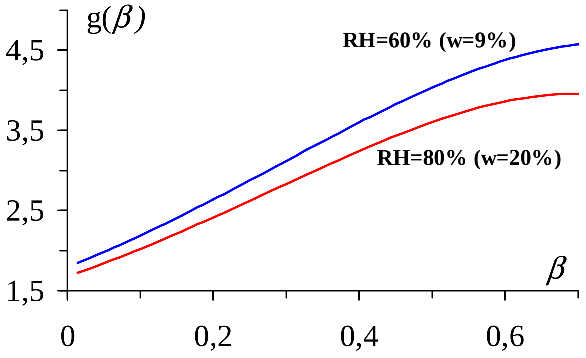

*Fig. 4 Calibration function g versus the reduced crack length $\beta=a/\gamma$ ($\gamma$=350 mm)*

When RH varies, all parameters are supposed to vary linearly with the average moisture content (variation deduced from the values of table 1), excepted for $\sigma_m$ and $I_1$. As the crack tip is supposed to be at the equilibrium moisture $w_{eq.}$, it was considered that $\sigma_m$. $I_1$ varies linearly with $w_{eq.}$. When the moisture content is greater than saturation moisture (here fixed at $w_s$ = 22%), the parameters take the values corresponding to the moisture at saturation $w_s$.

## 5. Results and Discussion

Firstly, the predictions of the model VCM are compared to the experimental results for given constant moistures. Knowing the uncertainty regarding to the determination of SL, the model gives quite good results for both w = 9% and w = 20%, as can be seen on figure 5.

Secondly, thousand relative humidity stochastic scenarios are computed and integrated into the model VCM. The average relative humidity is fixed to 65% in simulations and the average moisture is close to 11%. So, extreme and average values of time to failure of notched beams - discrepancy band - are predicted and given on figure 5, too. The time to failure predictions present important dispersion. For instance, at SL = 0.9, failure can occur between 6 and 36 days.

Experimental time to failure obtained during two perturbed seasons (Spring and Autumn) are compared to the model predictions. The experimental values are outside the predicted discrepancy band. Particularly, for smaller stress level, VCM gives longer time to failure compared to Autumn observations. For the moment, we have not realise enough simulations with various stochastic climates (because calculating time is long), which can explain theses optimistic predictions.

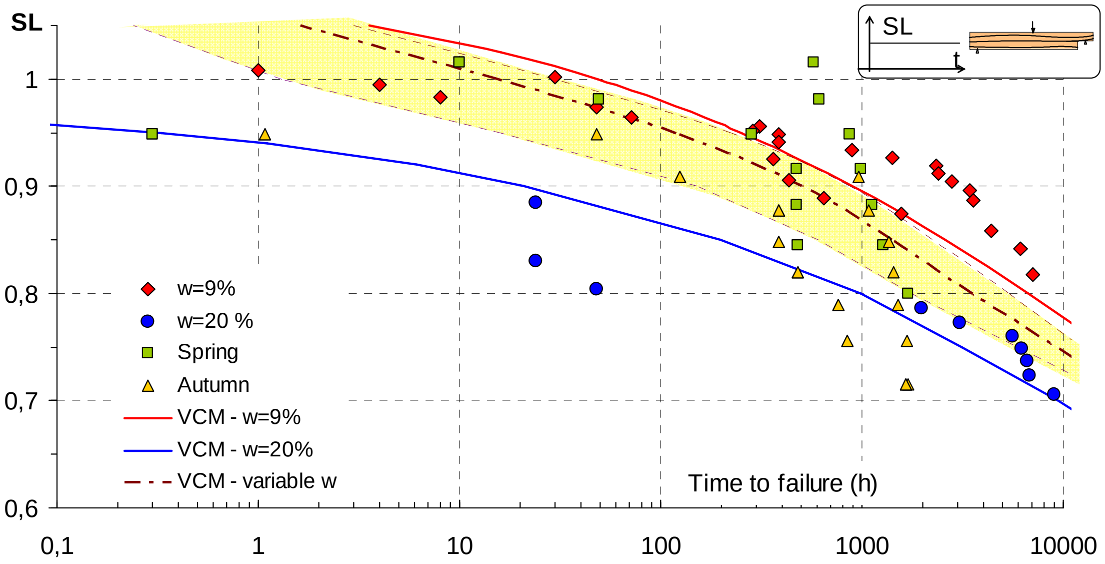

*Fig. 5 Duration of load curve for constant and for variable relative humidity. Experimental results and theoretical simulations – Discrepancy band of predicted lifetime according to climatic scenarios*

The failure probability is calculated and presented on figure 6 for a standard stress level of 55%, value generally accepted for design codes (EC5). Under this stress level, if the average annual humidity is equal to 65%, the failure probability will be 5.6% at 50 years and 36% at 60 years. If the annual average moisture varied from 65% to 55% in 50 years, the probability of failure would be 6.8% at 50 years and 47% at 60 years. This drastic increase of the failure probability between the two simulated climates is due to the incubation time: in variable climate, the average w is occasionally less than 9%. It must be recalled that, in the model, when w remains above 9%, there is no damage because the threshold stress level is not reached, so no crack can be created and time to failure is theoretically infinite. This would be the case if the relative humidity increased in the future. In the opposite, a dry period will facilitate the establishment of a macro crack and a following wet period will encourage the propagation of the crack. These types of scenarios can occur in the future - drought followed by rainy seasons - and they will be studied.

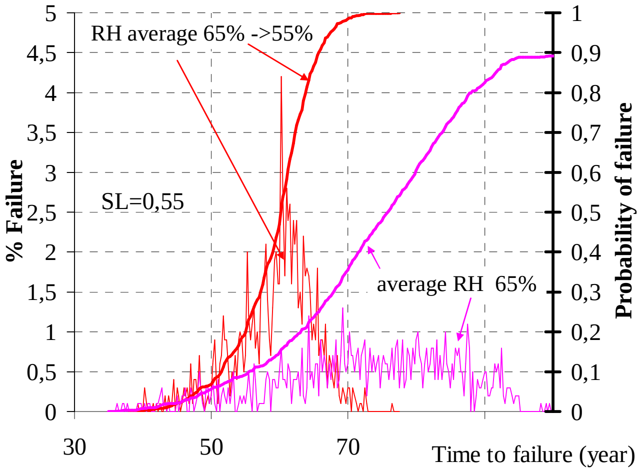

*Fig. 6 Failure and failure probability curves for two climates: annual RH is stable (65%) - annual RH decreases from 65% to 55% in 50 years*

## 6. Conclusions

The moisture content influences the time to failure of timber structures. In the case of a cracking failure of a notched beam, initiation and propagation time must be distinguished. It is found experimentally that the lower the moisture content is, the smaller the initiation time and the longer the propagation time are. For a given stress level, we observe a time to failure slightly higher under smaller moistures. The proposed model provides good forecasts in the light of our experimental results.

Monte Carlo simulations coupled with the method of Karhunen-Loève were able to generate a large number of RH scenarios in order to forecast future climate. For each simulation set, lifetime is estimated by the model VCM for different stress levels providing the probability distribution of the failure time of notched beams. Based on these simulations, a relative humidity decrease will lead to a longer lifetime, with a higher risk of damage (emergence of micro cracks), but with a slower crack propagation. However, these conclusions must be tempered as temperature was not included in this study. Although RH is the most influent parameter on the moisture, the air temperature modifies temperature of the beam and so its moisture content and therefore its mechanical properties.

## 7. References

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